DOA Estimation Using Second Order Differential of Invariant Noise MUSIC (SODIN-MUSIC)

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***Abstract*- Among the most well-known methods employed for the direction of arrival (DOA) estimation is the multiple signal classification (MUSIC). MUSIC can resolve closely spaced sources in good propagation environments; however, it fails to estimate the DOAs in severe environments like low signal-to-noise ratio (SNR) and small number of snapshots. In this paper, we propose a new approach for DOA estimation with higher resolution capabilities than MUSIC. The proposed approach is based on computing the second order differential of invariant noise-MUSIC (SODIN-MUSIC) spectrum. SODIN-MUSIC benefits from extracting useful information provided by the second order differential of MUSIC spatial spectrum and also utilizing the invariance property of noise subspace with respect to changing the power of the emitted signals. Our method is developed to estimate both (1-D) and (2-D) DOA. Throughout computer simulations, we compared SODIN-MUSIC with both MUSIC and other newly developed MUSIC – based methods. The results show that SODIN-MUSIC was the only method able to resolve closely spaced sources in relatively poor environments.**

I. INTRODUCTION

Direction of arrival (DOA) estimation is the problem of determining the wavenumber or the angle of arrival of a wave emitted by a radiating source [1]. In the last three decades, DOA has attracted significant attention due to its widespread applications such as radar, sonar, seismic systems, electronic surveillance, medical diagnostics and treatment, and radio astronomy [1-2]. Historically, the first method used for DOA estimation is the delay and sum (DAS) method usually known as conventional beamforming (CBF). DAS is based on finding the DOA by measuring the beamformer power at each possible angle of arrival and choosing the angles corresponding to the maximum power [3]. This method is known to be of poor resolution method because DAS can't determine the DOA of signals sent out by closely spaced sources [4]. Consequently, several high resolution techniques [5-8] were developed to improve the spatial resolution. The majority of these techniques are called the subspace based methods [9] and they utilize the properties of the eigenstructure of the spatial covariance matrix.

Among the subspace based methods, the most popular are multiple signal classification (MUSIC) [10], Root-MUSIC [11] and estimation of signal parameters via rotational invariant techniques (ESPRIT) [12]. ROOT-MUSIC and ESPRIT reduce the computations and storage costs required by MUSIC [9, 13]. However, they cannot be directly applied to non-uniform arrays [14], thus we have focused on MUSIC and its recently modified versions because they do not impose specific constraints on the form of the array manifold.

MUSIC algorithm is widely used due to its ability to resolve multiple sources [8]. The main principle of MUSIC is based on decomposing the sample space into two complementary and mutually orthogonal signal subspace and noise subspace. Thus, the searching procedure for the signal direction can be restricted in the signal subspace rather than searching in the whole sample space. The signal subspace usually has a much lower dimension than the sample space, thereby significantly reducing the effect of noise and improving the resolution capability [15, 16]. In fact, MUSIC affords super- resolution and high accuracy in estimating the DOAs at relatively large signal-to-noise ratio (SNR) and data samples [8, 13]. However, its performance degrades in case of low SNR and small number of samples where it totally fails to resolve the closely spaced sources. Recently, various methods were introduced to mitigate the limitations of the conventional MUSIC.

One of these methods is developed in [17, 18]. This method exploits that the conventional MUSIC spectrum has convex downward shape around the DOAs. Consequently, the second differentiation for the conventional MUSIC spectrum provides sharper peaks than that of the conventional MUSIC spectrum. This method is quoted here as second order differential MUSIC or abbreviated to (SOD-MUSIC). Another method, proposed in [19], is based on that the noise subspace is invariant of the powers of the sources. In other words, the covariance matrix formed due to changing the sources' powers has the same noise eigenvalues and its corresponding eigenvectors of the original covariance matrix. This method is referred here as invariant noise MUSIC or abbreviated to (IN-MUSIC). Both SOD-MUSIC and IN-MUSIC provide higher resolution than the conventional MUSIC.

In this paper, we introduce a method combines the benefits of both SOD-MUSIC and IN-MUSIC and is called second order differential of invariant noise MUSIC spectrum or (SODIN-MUSIC). We developed it to estimate one and two dimensional DOAs. Moreover, it can work with any arbitrary geometry array.

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This research proves the perfection of the proposed method in different scenarios through simulation work.

II. Signal Model

Suppose Q narrowband signals emitted from Q far-field sources of directions Θ, $\Theta = [\theta_1, \theta_2 \dots, \theta_Q]$ impinging on M isotropic sensors such that $M - 1 \geq Q$ and θ_q represents the azimuth angle for the q^{th} source as shown in Figure 1. In case of (2-D) DOA estimation, the elevation angles $\phi_q, q = 1,2, \dots Q$ are also considered. Therefore the directions of the sources can be expressed as $\Theta = [(\theta_1, \phi_1), (\theta_2, \phi_2) \dots \quad, (\theta_Q, \phi_Q)]$ and is illustrated in Figure 2. The standard model applied in narrowband array processing is given as follows:

$$\bar{x}(t) = A(\Theta)\, \bar{s}(t) + \bar{w}(t), \qquad \text{t=1, 2,..., N} \tag{1}$$

Where N denotes the number of the data snapshots, $\bar{x}(t) = [x_1(t), x_2(t) \dots, x_M(t)]^T$ is the observed data vector by M sensors at time t, $(.)^T$ denotes the vector transpose , $x_m(t)$ is the observed data by the m^{th} sensor at time t, $\bar{s}(t) = [s_1(t), s_2(t) \dots, s_Q(t)]^T$ is the emitted signals vector by Q sources at time t, $s_q(t)$ is narrowband signal emitted from the q^{th} source at time t, $\bar{w}(t) = [w_1(t), w_2(t) \dots, w_M(t)]^T$ is noise vector added to the signals received by each array sensor, $w_m(t)$ is the additive noise at the m^{th} sensor at time t, $A(\Theta) = [\bar{a}(\theta_1), \bar{a}(\theta_2), \dots \dots, \bar{a}(\theta_Q)]^T$ is the $M \times Q$ array steering (response) matrix and $\bar{a}(\theta_q)$ is the array steering vector corresponding to the q^{th} source for the (1-D) DOA estimation . It should also be noticed that the steering vector is expressed as $\bar{a}(\theta_q, \phi_q)$ in case of (2-D) DOA estimation. It is assumed that the additive noise is zero mean spatially white noise with variance σ^2 at each sensor, therefore

$$E\{\bar{w}(t)\bar{w}(t)^H\} = \sigma^2 I, \tag{2}$$

Where I is an $M \times M$ identity matrix, $(.)^H$ denotes the vector hermitian transpose and $E\{.\}$ denotes expected value. Moreover, both the signal and the noise are considered to be uncorrelated. Consequently;

$$E\{\bar{s}(t)\bar{w}(t)^H\} = E\{\bar{w}(t)\bar{s}(t)^H\} = 0 \tag{3}$$

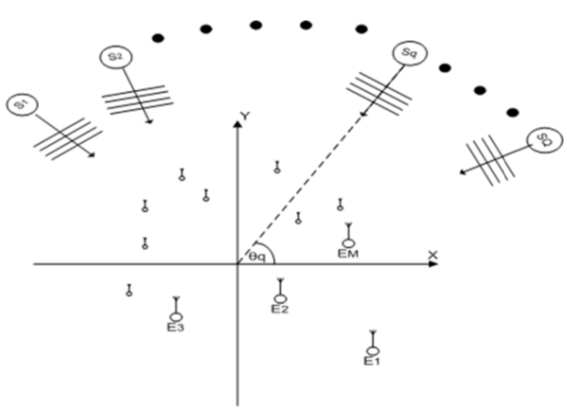


Figure 1. (1-D) DOA estimation using M elements planar array. θ_q is the azimuth angle of q[th] source.

From the signal model in (1) and by applying the assumptions formulated in (2) and (3), the covariance matrix $M \times M$ of the observation can be given as:

$$R_{\bar{x}\bar{x}} = E\{\bar{x}(t)\bar{x}(t)^H\} = A(\Theta) R_{\bar{s}\bar{s}} A^H(\Theta) + \sigma^2 I \tag{4}$$

Where $R_{\bar{s}\bar{s}} = E\{\bar{s}(t)\bar{s}(t)^H\}$. Assuming that the number of the sources P is known, the eigen-decomposition for the covariance matrix represented by (4) and can be rewritten as:

$$R_{\bar{x}\bar{x}} = \sum_{m=1}^{M} \lambda_m \bar{e}_m \bar{e}_m^H \tag{5}$$

Where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_Q \geq \lambda_{Q+1} = \cdots = \lambda_M = \sigma^2$ are the eigenvalues of $R_{\bar{x}\bar{x}}$ and $\bar{e}_m$ is the orthonormal eigenvector associated with λ_m for *m=1,2,..,M*. The noise subspace $E_N = [\bar{e}_{Q+1} \dots \bar{e}_M]$ are orthogonal to the signal subspace $E_S = [\bar{e}_1 \dots \bar{e}_Q]$.

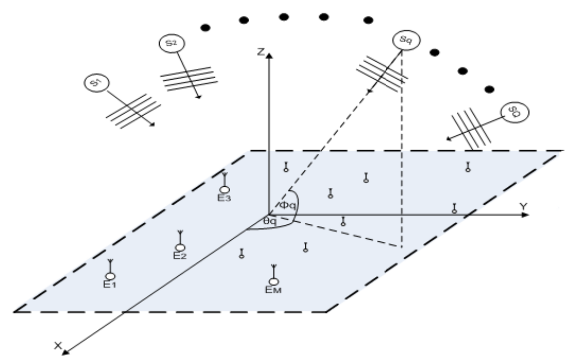


Figure 2. (2-D) DOA estimation using M elements planar array. θ_q, ϕ_q are the azimuth and the elevation angles of q[th] source.

III. SODIN-MUSIC Algorithm

SODIN-MUSIC algorithm can be considered as a combination of both SOD-MUSIC and IN-MUSIC. Our approach is stimulated from the fact that IN-MUSIC spectrum looks like the conventional MUSIC spectrum (i.e. convex downward around the DOAs); Thus the second order differentiation utilized by SOD-MUSIC can be applied for the IN-MUSIC spectrum and hence it can improve the overall performance. This way we can augment the strength of both the IN-MUSIC and SOD-MUSIC. First, we introduce the proposed algorithm for the (1-D) DOA estimation and then indicate the differences in case of (2-D) DOA estimation.

A. One Dimensional SODIN-MUSIC

Given the observed data vector $\bar{x}(t)$, t=1,2,….,N; the estimated covariance matrix $\hat{R}_{\bar{x}\bar{x}}$ is obtained as:

$$\hat{R}_{\bar{x}\bar{x}} = \frac{1}{N} \sum_{t=1}^{N} \bar{x}(t)\bar{x}(t)^H \tag{6}$$

The eigenvalues of the estimated covariance matrix $\hat{R}_{\bar{x}\bar{x}}$; $\hat{\lambda}_m, m = 1,2, \dots \dots M$ are evaluated in descending order where $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \cdots \geq \hat{\lambda}_M$. Subsequently, a new matrix $\hat{F}^{\theta}$ is computed at each possible direction θ according to the following equation [19]:

$$\hat{F}^{\theta} = \hat{R}_{\bar{x}\bar{x}} + \frac{tr\left(\hat{R}_{\bar{x}\bar{x}}\right)}{M}\bar{a}(\theta)\bar{a}^{H}(\theta) \quad (7)$$

Where $tr(.)$ denotes trace of a matrix. $\hat{\mu}_m^{\theta}$, m=1,...,M of the new matrix $\hat{F}^{\theta}$. Where $\hat{\mu}_1^{\theta} \geq \hat{\mu}_2^{\theta} \geq \cdots \geq \hat{\mu}_M^{\theta}$ .The spatial spectrum of IN-MUSIC is then established and it is given as follows:

$$P_{IN-MUSIC}(\theta) = \frac{1}{\sum_{m=Q+1}^{M}(\hat{\mu}_m^{\theta} - \hat{\lambda}_m)} \quad (8)$$

The second order difference of IN-MUSIC or the so called SODIN-MUSIC spectrum is then calculated using the following expression [17]:

$$P_{SODIN-MUSIC}(\theta) = \frac{-P_{IN-MUSIC}(\theta - \Delta\theta) + 2P_{IN-MUSIC}(\theta) - P_{IN-MUSIC}(\theta + \Delta\theta)}{(\Delta\theta)^2} \quad (9)$$

Where $\Delta\theta$ is the interval of discrete azimuth angles. By searching for the largest Q positive peaks in the $P_{SODIN-MUSIC}(\theta)$ spectrum, the DOAs can be estimated.

B. *Two Dimensional SODIN-MUSIC*

On the other hand, The (2-D) SODIN-MUSIC algorithm is similar to the one used for the (1-D) provided that we recall the elevation angle ϕ. Therefore the new matrix $\hat{F}^{\theta,\phi}$ is evaluated as follows [19]:

$$\hat{F}^{\theta,\phi} = \hat{R}_{\bar{x}\bar{x}} + \frac{tr\left(\hat{R}_{\bar{x}\bar{x}}\right)}{M}\bar{a}(\theta,\phi)\bar{a}^{H}(\theta,\phi) \quad (10)$$

The eigenvalues $\hat{\mu}_m^{\theta,\phi}$, m=1,...,M of the new matrix $\hat{F}^{\theta,\phi}$is then evaluated Where $\hat{\mu}_1^{\theta,\phi} \geq \hat{\mu}_2^{\theta,\phi} \geq \cdots \geq \hat{\mu}_M^{\theta,\phi}$. Consequently the IN-MUSIC spectrum will be computed as follows [19]:

$$P_{IN-MUSIC}(\theta,\phi) = \frac{1}{\sum_{m=Q+1}^{M}(\hat{\mu}_m^{\theta,\phi} - \hat{\lambda}_m)} \quad (11)$$

Finally the SODIN-MUSIC spectrum in (1-D) DOA estimation is obtained by [18]:

$$P_{SODIN-MUSIC}(\theta,\phi) = \frac{-P_{IN-MUSIC}(\theta,\phi - \Delta\phi) + 2P_{IN-MUSIC}(\theta,\phi) - P_{IN-MUSIC}(\theta,\phi + \Delta\phi)}{(\Delta\phi)^2} \quad (12)$$

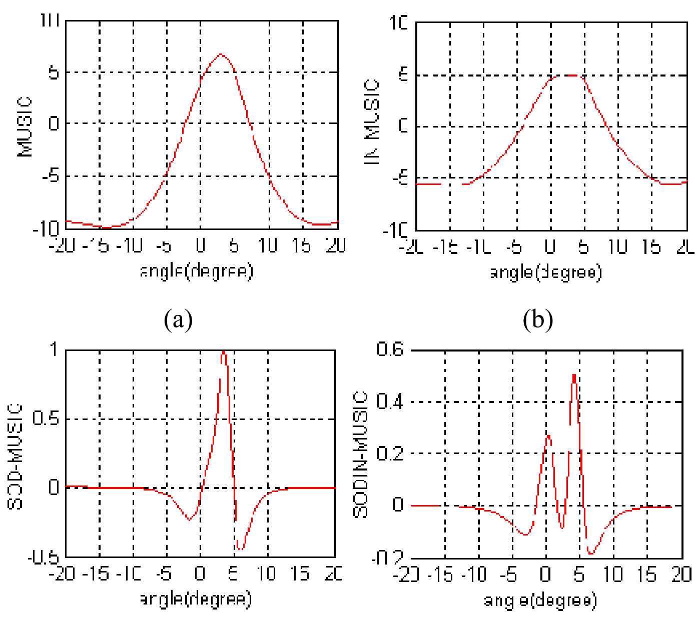
The partial differentiation in equation (12) is with respect to elevation angles because in case of horizontally positioned planar array, the IN-MUSIC spectrum as well as conventional MUSIC has poor resolution in the direction of elevation angle rather than the azimuth angle [18,20].

IV. SIMULATION WORKS

In this section we compare SODIN-MUSIC with the conventional MUSIC, IN-MUSIC and SOD-MUSIC algorithms in both (1-D) and (2-D) DOA estimation.

A. *One Dimensional DOA Estimation*

In this simulation, two uncorrelated sources impinging from $\theta_1 = 0^o$and $\theta_2 = 5^o$ to 10 elements uniform linear array (ULA) with half wavelength radius. The number of snapshots was selected 100 samples and the SNR was assumed -5dB. Figure 3 compares the spatial spectrum of the four methods and it shows the usefulness of SODIN-MUSIC. It can be depicted in Figures 3(a),(b) and (c) that the spectrums of the conventional MUSIC, IN-MUSIC and SOD-MUSIC have only one peak and therefore these methods fail to resolve the two sources. On the contrary, Figure 3(d) shows that the spectrum of SODIN-MUSIC demonstrating the capability of the new method with a spectrum depicting the two peaks at the sources' locations.



(a) (b) (c) (d)

Figure 3. an example of (1-D) DOA estimation using (a) conventional MUSIC (b) IN-MUSIC (c) SOD-MUSIC (d) SODIN-MUSIC

B. *Two Dimensional DOA Estimation*

For the case of (2-D), We assumed two incident signals coming from two sources located at $(\theta_1, \phi_1) = (15^o, 15^o)$, and $(\theta_2, \phi_2) = (25^o, 25^o)$ to 7- elements uniform circular array (UCA) with half wavelength radius. The number of snapshots was selected 100 samples and the SNR was assumed 3dB.

Similar to (1-D) case, (2-D) SODIN-MUSIC is able to detect and accurately estimate the bearings of the two sources that manifest themselves with the two peaks shown in Figure 4(d). The other methods could not separate the two sources and only one peak was detected as indicated in Figures 4(a), (b) and (c). It is obvious how the second order derivative imposed to both the conventional MUSIC and IN-MUSIC spectrum improves the performance (i.e. enhances the spatial resolution) presented by SOD-MUSIC and SODIN-MUSIC, respectively.

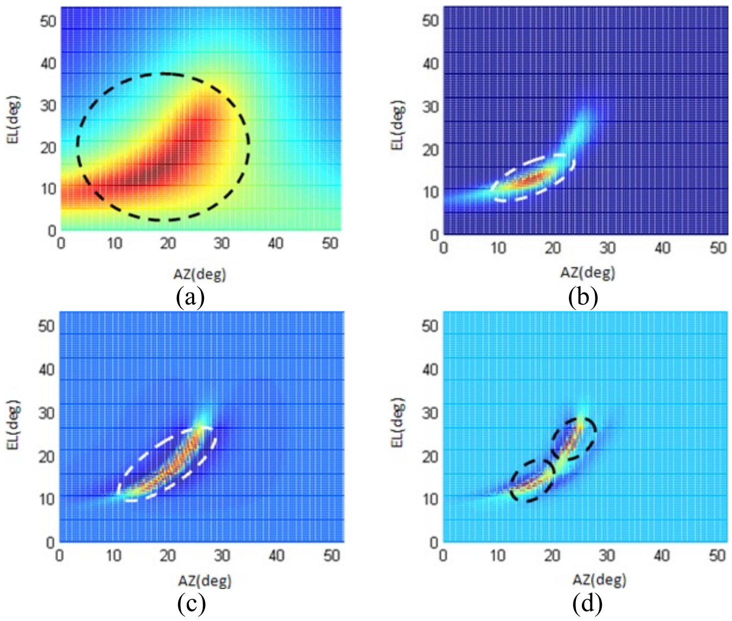


Figure 4. an example of (2-D) DOA estimation using (a) conventional MUSIC (b) IN-MUSIC (c) SOD-MUSIC (d) SODIN-MUSIC

V. Conclusion and Future Work

This paper suggested a new approach named SODIN-MUSIC to estimate the DOA with super- resolution capabilities. This method works with all array configurations moreover it is developed to estimate (1-D) (i.e. azimuth only) and (2-D) (i.e. azimuth and elevation) DOAs. It was shown via simulation scenarios how SODIN-MUSIC beats the conventional MUSIC and its recent versions like IN-MUSIC and SOD-MUSIC in the challenging environments including relatively low SNR and small numbers of snapshots. The price of this advantage is that it has more computational burden compared to conventional MUSIC and SOD-MUSIC. However, SODIN-MUSIC has almost the same computational load of IN-MUSIC because the computation of second order differentiation is negligible with respect to the computation of the IN-MUSIC spectrum. In the future work, we will perform a comprehensive study on SODIN-MUSIC to assess its probability of resolution and root mean square error versus all the environment parameters such as SNR, number of snapshots, angular separation and correlation between the sources.

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