Localization of underwater tone noise sources using instantaneous frequency rate estimate

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***Abstract*—Doppler effect can be used to estimate the movement parameter of an acoustic source. As one of these parameters, the beam moment is the time at closet point of approach, and it corresponds to the position of acoustic source on the moving object. We introduced a new time-frequency rate distribution referred to as L-phase distribution (LPD), which can accurately estimate the instantaneous frequency rate (IFR) of high-order polynomial phase signal (PPS). The iteration procedure is given to realize the discrete-time LPD. After applying the discrete-time LPD to estimate the IFR of Doppler signal caused by relative motion between the moving acoustic source and the measurement hydrophone, the abeam time is obtained according to the minimum of the estimated IFR, and then the positions of the tone noise sources on underwater moving object can be localized. Localization errors were analyzed using simulations, and the sea experiment results showed the validity of the proposed method.**

***Index Terms*—Doppler effect, underwater moving object, localization of tone noise sources, instantaneous frequency rate, L-phase distribution**

I. INTRODUCTION

Due to relative motion between the moving acoustic source and receiver, the received signal component is not a pure tone which is emitted by the tone source. This is the well-known Doppler effect, and it is often used to estimate movement parameters of a passive target, which include the resting frequency, the relative speed, the closet distance between the source and receiver, and the abeam moment at closet approach. As the abeam moments of each source are different, the positions of the radiated-noise sources on the moving object can be obtained with a known speed. Thus, the localization of noise sources on the moving object turns to the estimate of the abeam moment.

Ferguson et al. [1-3] use the Doppler shift frequency to analyze the speed and closet distance of a turbo-prop aircraft, and the abeam moment is also obtained. However, these methods in general require initial estimations of several movement parameters. The abeam moment could be also estimated by using the information of time-varying amplitude [4] or instantaneous Doppler phase [5], but the estimation accuracy is low. As the abeam moment is just equal to the instant of the highest rate of change of the Doppler frequency, it can be obtained by using the Doppler frequency rate [6]. The precondition of this method is to estimate the IFR of the Doppler signal.

Since Doppler signal is a kind of non-stationary signal, it can be expressed as polynomial phase signal (PPS) according to the Stone-Weierstrass theory. As an important content of PPS, the estimation of instantaneous frequency rate (IFR) has attracted the attentions of many researchers. O'Shea [7,8] is the first one who defined the IFR for PPS and also proposed a bilinear match transform called CPF to estimate the IFR. CPF has the best concentrate in time-frequency rate plain for cubic phase signal, and the computation is also less due to only one-dimensional search. However, there would be inner artifacts when CPF is applied to a high-order PPS, such as the Doppler signal. This is similar to the situation when WVD is used to estimate instantaneous frequency of high-order PPS. High-order statistics is a main approach for reducing or eliminating the inner artifacts of WVD, such as PWVD [9] and LWD [10]. HPF [11] is a method that CPF takes advantage of PWVD. It could estimate the IFR of PPS with arbitrary order, and only a one-dimension search is needed. However, the threshold of signal-to-noise ratio (SNR) increases with the number of product terms of autocorrelation, which is in proportion to the order of PPS. Moreover, the coefficients in signal kernel of HPF are calculated according to the phase order, and the higher the order, the more complex the coefficients are. So, HPF is indeed applied to the PPS whose order is not more than 5.

Based on the formulation of LWD, a new time-frequency rate distribution referred to as L-phase distribution (LPD) is defined. The discrete-time LPD is realized by an iterative procedure referred to S-Method [12], and it can effectively estimate the IFR of high-order PPS in low SNR condition. Then, the discrete-time LPD is applied to estimate the IFR of the received Doppler signal, and the abeam moment can be obtained according to the minimum of the estimated IFR. Combining the speed and reference source, the positions of tone noise sources on the moving object can be localized.

The rest of this paper is organized as follows. We first show the measurement model and describe the localization approach in Section II. The process of estimation of IFR based on LPD is given in Section III. Section IV presents the

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simulation analysis. Finally, the experiment results are given to demonstrate the validity of the proposed localization method.

II. Model And Method

Fig. 1 shows the model for measuring radiated noise of an underwater moving object, where there are several tone noise sources with different resting frequencies. The measured object is assumed to move along a straight trajectory *MN* with a constant speed v. The closest distance between the sources and the measurement hydrophone located at S is R, and the abeam moment which is the time at closest point of approach O is t_0. Due to relative motion between the sources and receiver, the amplitude and frequency of the received signal will vary, and the varying frequency is called Doppler shift.

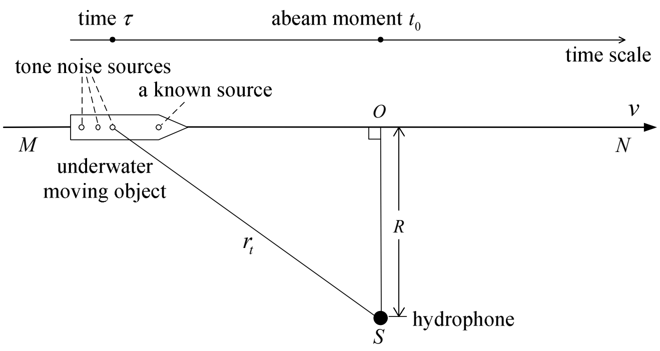


Fig. 1. Model for measuring radiated noise of underwater moving object.

As a pure tone with frequency f_0 is radiated by one source on the moving object at time τ, and the tone arrives the measurement hydrophone at time t, the received signal is expressed as

$$s(t) = A(t)\exp\left[\mathrm{j}(2\pi f_0 \tau + \phi_0)\right] \tag{1}$$

where $A(t)$ is the amplitude of the received signal at time t, ϕ_0 is the initial phase of the radiated tone noise. The relation between the emitted time τ and received time t is [13]

$$t = \tau + r_t/c \tag{2}$$

where r_t/c is the time delay for the sound to propagate at a constant sound velocity c between the tone noise source and the measurement hydrophone that are separated by a distance r_t at time t, and this slant range is given by

$$r_t = \sqrt{R^2 + v^2(t_0 - \tau)^2} \tag{3}$$

Substituting (3) into (2), then solving for τ, it is shown as

$$\tau = \frac{c^2 t - v^2 t_0 - \sqrt{R^2(c^2 - v^2) + v^2 c^2 (t - t_0)^2}}{c^2 - v^2} \tag{4}$$

The instantaneous frequency of the Doppler signal received at time t is given by

$$f_r(t) = f_0 \frac{d\tau}{dt} = \frac{f_0 c^2}{c^2 - v^2}\left[1 - \frac{v^2(t - t_0)}{\sqrt{R^2(c^2 - v^2) + c^2 v^2 (t - t_0)^2}}\right] \tag{5}$$

Then the IFR of the received signal is obtained as

$$\mathrm{IFR}(t) = \frac{\partial f_r(t)}{\partial t} = -\frac{f_0 c^2 v^2 R^2}{\left[R^2(c^2 - v^2) + c^2 v^2 (t - t_0)^2\right]^{3/2}} \tag{6}$$

It is easily known that the IFR of Doppler signal has a minimum at the time t_0, i.e., when $t = t_0$, $\mathrm{IFR}(t_0) = \mathrm{IFR}_{\min}(t)$, so we can get the estimated value of t_0 after searching the minimum of the estimated IFR.

Fixing a known reference source at the position L_r on the underwater moving object, the abeam moment of the reference source t_r and the relative speed of the moving object v could be known. Based on the IFR of the Doppler signal in (6), the abeam moment of each tone noise sources t_0 can be estimated. Then, the positions of tone noise sources on the object L_n are obtained as follow

$$L_n = v(t_0 - t_r) + L_r \tag{7}$$

III. Proposed IFR Estimator

A. The definition of LPD

The Doppler signal $s(t)$ can be expressed in form of PPS:

$$s(t) = A\exp\left[\mathrm{j}\phi(t)\right] = A\exp\left[\mathrm{j}\sum_{k=0}^{K}\left(a_k t^k\right)\right], |t| \le \frac{T}{2} \tag{8}$$

where T is the duration of signal, A is the amplitude, $\phi(t)$ is the instantaneous phase (IP), K is the order of phase and $\left\{a_k |_{k=0}^{K}\right\}$ are the polynomial phase coefficients. For this PPS, the IFR is defined as the second-order derivative of the IP

$$\mathrm{IFR}(t) = \frac{\mathrm{d}^2\phi(t)}{\mathrm{d}t^2} = \sum_{k=2}^{K} b_k t^{k-2} \tag{9}$$

where $b_k = k(k-1)a_k$. For a PPS with order $K = 3$, i.e., a quadratic FM signal, using cubic phase function (CPF), we can get

$$\begin{aligned}\mathrm{CPF}(t,\Omega) &= \int_0^{+\infty} s(t+\tau)s(t-\tau)\exp\left(-\mathrm{j}\Omega\tau^2\right)\mathrm{d}\tau \\ &= A^2\exp\left[\mathrm{j}2\left(a_0+a_1t+a_2t^2+a_3t^3\right)\right]\int_0^{+\infty} e^{\mathrm{j}\left[2(a_2+3a_3t)-\Omega\right]\tau^2}\mathrm{d}\tau\end{aligned} \tag{10}$$

where τ is the time delay. It is not hard to see that $\mathrm{CPF}(t,\Omega)$ peaks along the curve $\Omega = 2(a_2+3a_3t)$, which is the IFR of signal. However, the energy of CPF is not completely concentrated along the IFR of a PPS with order $K \geq 4$. To solve this problem, a new time-frequency rate distribution is defined, and it has good time-frequency rate aggregation for high-order PPS, such as the Doppler signal.

The L-phase distribution (LPD) of the signal $s(t)$ is defined as

$$\begin{aligned}\mathrm{LPD_s}(t,\Omega) &= \int_{-\infty}^{+\infty} R_s^{L^2}(t,\tau)\exp\left(-\mathrm{j}\Omega\tau^2\right)d\tau \\ &= \int_{-\infty}^{+\infty} s^{L^2}\left(t+\frac{\tau}{L}\right)s^{L^2}\left(t-\frac{\tau}{L}\right)\exp\left(-\mathrm{j}\Omega\tau^2\right)d\tau\end{aligned} \tag{11}$$

where $R_s^{L^2}(t,\tau)$ is the high-order instantaneous autocorrelation function. It is shown below that LPD linearizes the phase of the signal by a factor which is a power of *L*. As the value of *L* in LPD increases, it suppresses the non-linear terms. Consider the high-order instantaneous autocorrelation function

$$R_s^{L^2}(t,\tau) = A^{2L^2}\exp\left\{\mathrm{j}L^2\left[\phi\left(t+\frac{\tau}{L}\right)-\phi\left(t-\frac{\tau}{L}\right)\right]\right\} \tag{12}$$

Expanding $\phi\left(t\pm\frac{\tau}{L}\right)$ into a Taylor series around time *t* [14]

$$\begin{aligned}\phi\left(t\pm\frac{\tau}{L}\right) &= \phi(t)\pm\frac{\phi'(t)\tau^1}{L^1 1!}+\frac{\phi^{(2)}(t)\tau^2}{L^2 2!} \\ &\quad \pm\frac{\phi^{(3)}(t)\tau^3}{L^3 3!}+\frac{\phi^{(4)}(t)\tau^4}{L^4 4!}\pm\cdots\end{aligned} \tag{13}$$

Substituting (13) into (12) yields

$$\begin{aligned}R_s^{L^2}(t,\tau) = A^{2L^2}\exp\left\{\mathrm{j}L^2\left[2\phi(t)+\frac{2\phi^{(2)}(t)\tau^2}{L^2 2!}\right.\right. \\ \left.\left.+\frac{2\phi^{(4)}(t)\tau^4}{L^4 4!}+\cdots\right]\right\}\end{aligned} \tag{14}$$

So the LPD of signal $s(t)$ is

$$\begin{aligned}\mathrm{LPD}_s(t,\Omega) &= \int_{-\infty}^{+\infty} A^{2L^2}\exp\left\{\mathrm{j}L^2\left[2\phi(t)+\frac{2\phi^{(2)}(t)\tau^2}{L^2 2!}\right.\right. \\ &\quad \left.\left.+\frac{2\phi^{(4)}(t)\tau^4}{L^4 4!}+\cdots\right]\right\}\mathrm{e}^{-\mathrm{j}\Omega\tau^2}\mathrm{d}\tau \\ &= A^{2L^2}\exp\left[\mathrm{j}2L^2\phi(t)\right]\int_{-\infty}^{+\infty}\exp\left[\mathrm{j}\phi^{(2)}(t)\tau^2\right]\cdot \\ &\quad \exp\left[\mathrm{j}\sum_{l=4,6,\cdots}^{+\infty}\phi^{(l)}(t)\frac{2\tau^l}{L^{l-2}l!}\right]\mathrm{e}^{-\mathrm{j}\Omega\tau^2}\mathrm{d}\tau \\ &= A^{2L^2}\exp\left[\mathrm{j}2L^2\phi(t)\right]\int_{-\infty}^{+\infty}\exp\left[\mathrm{j}\phi^{(2)}(t)\tau^2\right]\cdot \\ &\quad \mathrm{e}^{-\mathrm{j}\Omega\tau^2}\mathrm{d}\tau\otimes_\Omega G(\Omega)\end{aligned} \tag{15}$$

where $\otimes_\Omega$ denotes the convolution operator with respect to Ω, and $G(\Omega)=\int_{-\infty}^{+\infty}\exp\left[\mathrm{j}\sum_{l=4,6,\cdots}^{+\infty}\phi^{(l)}(t)\frac{\tau^l}{L^{l-2}l!}\right]\mathrm{e}^{-\mathrm{j}\Omega\tau^2}\mathrm{d}\tau$. For the fourth derivative of phase, i.e., *L*=4, $\phi^{(4)}(t)$ is attenuated by more than 24dB and the other high-order derivative terms would be attenuated even more. Therefore, for sufficiently large values of *L*, the LPD of a general polynomial phase signal $s(t)$ can be expressed as

$$\mathrm{LPD}_s(t,\Omega) \simeq A^{2L^2}\exp\left[\mathrm{j}2L^2\phi(t)\right]\int_{-\infty}^{+\infty}\exp\left[\mathrm{j}\phi^{(2)}(t)\tau^2\right]\mathrm{e}^{-\mathrm{j}\Omega\tau^2}\mathrm{d}\tau \tag{16}$$

Then, it is not hard to obtain the IFR estimation of Doppler signal $s(t)$ by one-dimensional peak search

$$\hat{\mathrm{IFR}}(t) = \arg\max_\Omega\left\{\mathrm{LPD}_s(t,\Omega)\right\} \tag{17}$$

B. The realization of discrete-time LPD

From the definition of LPD in Eq. (11), it is well known that the realization of discrete-time LPD requires over-sampling or interpolation of $s(t)$ to avoid aliasing errors. But over-sampling will increase computation load, and interpolation method will decrease estimation accuracy. However, the realization of discrete-time LPD may be effectively done using recursive formula similar to S-Method. Using variable substitution, the form of LPD can be written as

$$\begin{aligned}\mathrm{LPD_s^{L^2}}(t,\Omega) &= \int_{-\infty}^{+\infty} s^{L^2}\left(t+\frac{\tau}{L}\right)s^{L^2}\left(t-\frac{\tau}{L}\right)\exp\left(-\mathrm{j}\Omega\tau^2\right)d\tau \\ &\overset{\tau=L\tau'}{=} L\int_{-\infty}^{+\infty} s^{L^2}(t+\tau')s^{L^2}(t-\tau')\exp\left(-\mathrm{j}\Omega L^2\tau'^2\right)d\tau'\end{aligned} \tag{18}$$

It is different from the conventional Fourier transform (FT), yet it is a specific form of local polynomial Fourier transform (LPFT) [15]. Using Eq. A4 in appendix, LPD can be expressed as

$$\begin{aligned}\mathrm{LPD}_{\mathrm{s}}^{L^2}(t,\Omega)&=\frac{1}{\pi L}\mathrm{LPD}_{\mathrm{s}}^{L^2/2}(t,2\Omega)\otimes_{\Omega}\mathrm{LPD}_{\mathrm{s}}^{L^2/2}(t,2\Omega)\\&=\frac{1}{\pi L}\int_{-\infty}^{+\infty}\mathrm{LPD}_{\mathrm{s}}^{L^2/2}(t,\Omega+\theta)\left(\mathrm{LPD}_{\mathrm{s}}^{L^2/2}\right)^{*}(t,\Omega-\theta)d\theta\end{aligned}\tag{19}$$

where "$*$" denotes the conjugate operator. Applying a finite rectangle window $P(\theta)=\mathrm{rect}(\theta),-W\le\theta\le W$ to Eq. (19), we can get

$$\begin{aligned}\mathrm{LPD}_{\mathrm{s}}^{L^2}(t,\Omega)=\frac{1}{\pi L}\int_{-W}^{+W}P(\theta)\mathrm{LPD}_{\mathrm{s}}^{L^2/2}(t,\Omega+\theta)\cdot\\\left(\mathrm{LPD}_{\mathrm{s}}^{L^2/2}\right)^{*}(t,\Omega-\theta)d\theta\end{aligned}\tag{20}$$

The first iteration of LPD is computed with L=1:

$$\mathrm{LPD}_{\mathrm{s}}^{1}(t,\Omega)=\int_{-\infty}^{+\infty}s(t+\tau)s(t-\tau)\exp\left(-\mathrm{j}\Omega\tau^2\right)\mathrm{d}\tau\tag{21}$$

In the discrete-time domain, LPD can be iteratively computed using the formulas below

$$\begin{aligned}\mathrm{LPD}_{\mathrm{s}}^{L^2}(n,\Omega)=\frac{1}{\pi L}\sum_{\theta=-W}^{W}\mathrm{LPD}_{\mathrm{s}}^{L^2/2}(n,\Omega+\theta)\cdot\\\left(\mathrm{LPD}_{\mathrm{s}}^{L^2/2}\right)^{*}(n,\Omega-\theta)\end{aligned}\tag{22}$$

$$\mathrm{LPD}_{\mathrm{s}}^{1}(n,\Omega)=\sum_{m=-(N-1)/2}^{(N-1)/2}s(n+m)s(n-m)\exp\left[-\mathrm{j}\Omega m^2\right]\tag{23}$$

where N is the number of samples and it is an odd integer. As Eq. (22) and (23) are computed without FT, there is no need to satisfy the Nyquist sampling theorem.

C. Optimization of the estimated IFR curve

After the peak search of $\mathrm{LPD}_{s}^{L^2}(n,\Omega)$ in Eq. (22), the curve of estimated IFR can be obtained. However, due to the noise in the Doppler signal, the curve of estimated IFR will fluctuate. So, it is necessary to fit the curve of estimated IFR. The discrete-time form of Eq. (17) can be written as

$$\hat{\mathrm{IFR}}(n)=\arg\max_{\Omega}\left\{\mathrm{LPD}_{s}^{L^2}(n,\Omega)\right\}\tag{24}$$

The vector $\hat{\boldsymbol{\Psi}}=\begin{bmatrix}\hat{\Omega}_1 & \hat{\Omega}_2 & \cdots & \hat{\Omega}_N\end{bmatrix}^T$ is used to represent the estimated IFR at each time t in the above equation, here N is the sampling number. From Eq. (9), we know that the estimated vector for the polynomial phase coefficients $\hat{\mathbf{b}}=[\hat{b}_2\quad\hat{b}_3\quad\cdots\quad\hat{b}_K]^T$ satisfy the following matrix equation:

$$\mathbf{X}*\hat{\mathbf{b}}=\hat{\boldsymbol{\Psi}}\tag{25}$$

where

$$\mathbf{X}=\begin{bmatrix}1\times2 & 2\times3\times t_1 & \cdots & K\times(K-1)\times t_1^{K-2}\\1\times2 & 2\times3\times t_2 & \cdots & K\times(K-1)\times t_2^{K-2}\\\vdots & \vdots & \cdots & \vdots\\1\times2 & 2\times3\times t_N & \cdots & K\times(K-1)\times t_N^{K-2}\end{bmatrix}\tag{26}$$

It is an $N\times(K-1)$ matrix, where K is the order of phase. When $N=K-1$, the vector can be solved as $\hat{\mathbf{b}}=\mathbf{X}^{-1}\hat{\boldsymbol{\Psi}}$. As the common situation $N>K-1$, Eq. (25) is a overdetermined equation, the vector of phase coefficient can be estimated by Least Square method:

$$\hat{\mathbf{b}}=\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\hat{\boldsymbol{\Psi}}\tag{27}$$

where "T" denotes the transposition operator. Then the fitting curve of estimated IFR can be obtained by using Eq. (9). Here, it is necessary to know the order K in the fitting process. The order K could be determined by the fitting error Δ_{IFR} defined as follow [16]

$$\Delta_{\mathrm{IFR}}=\frac{1}{N}\sum_{n=1}^{N}\left|\mathrm{IFR}_{PPS}(n)-\mathrm{IFR}_{real}(n)\right|\le\eta\tag{28}$$

where IFR_{real} is the real value of IFR, IFR_{PPS} is the equivalent IFR in Eq. (9), and η is the threshold. The value of K is the minimum integer satisfy the above inequality.

IV. PERFORMANCE ANALYSIS WITH SIMULATION

In the numerical simulations, the movement parameters are considered as $v=4.12\,\mathrm{m/s}$, $c=1500\,\mathrm{m/s}$, $R=40\mathrm{m}$, $f_0=200\mathrm{Hz}$, and $t_0=2.5\mathrm{s}$. The sampling rate $f_s=20\mathrm{Hz}$ and the duration of Doppler signal is 30s (from -15s to 15s).

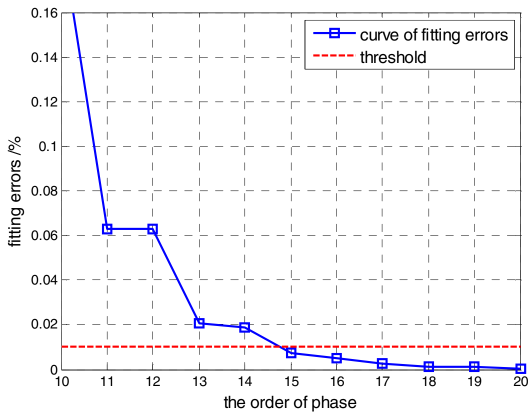


Fig. 2. Fitting errors versus the order of phase.

Fig. 2 depicts the relation between the fitting errors Δ_{IFR} and the order K of the equivalent PPS. It can be obviously noticed that Δ_{IFR} is little when K is higher than 15. So K is set to be 15 and its corresponding threshold η is equal to 0.01%. As a general rule, this value of K can satisfy most of Doppler signals in our application.

The iterations of discrete-time LPD are chosen to be 4, i.e., $L^2 = 2^4$, and the search scope of Ω is between -5Hz/s to 2Hz/s with 0.005Hz/s increment interval. The length of convolution window W in Eq. (20) of discrete-time LPD is discussed firstly. Fig. 3 shows the estimated IFRs of pure Doppler signal with varying W which is respectively 10, 20, 30 and 40. One can observe that the estimated IFR is more close to the real IFR with the increasing W, and it indicates that the discrete-time LPD is a biased estimate for the IFR of Doppler signal. This is because that Eq. (20) is an approximate expression for Eq. (19), and the degree of approximation is in proportion to W. It can be also seen from Fig. 3 that the difference between the estimated and real IFR is less when W is above 30. This value can be accepted in simulation analysis, and it is also adopted for the data analysis of sea experiment.

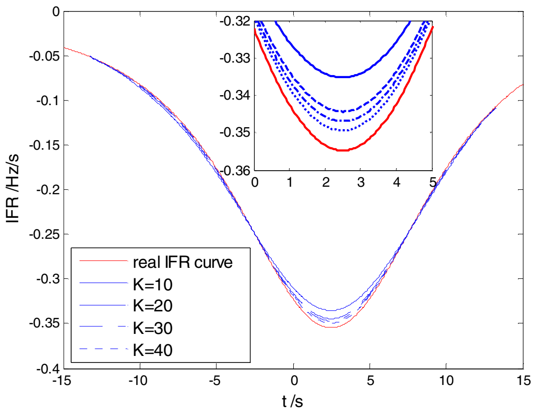
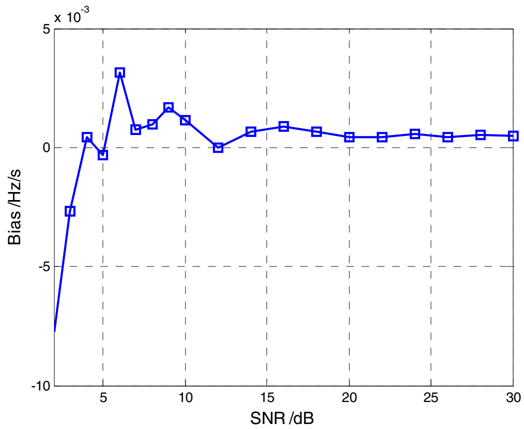
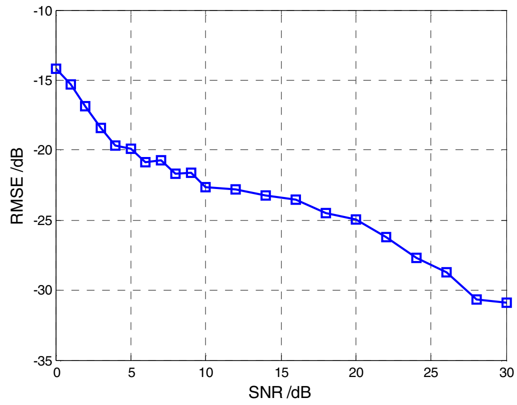


Fig. 3. Estimated IFR curves using varying length window.

To analyze the statistical characters of the estimated IFR and the abeam moment $\hat{t}_0$ obtained by using the discrete-time LPD, 100 runs of Monte-Carlo simulations are performed with different SNR, which is varied from 0 to 10dB in 1dB and 12 to 30dB in 2dB increment intervals. SNR is defined as the average power ratio of the Doppler signal to the power of white Gaussian noise.



(a) Biases of estimated IFR versus SNR

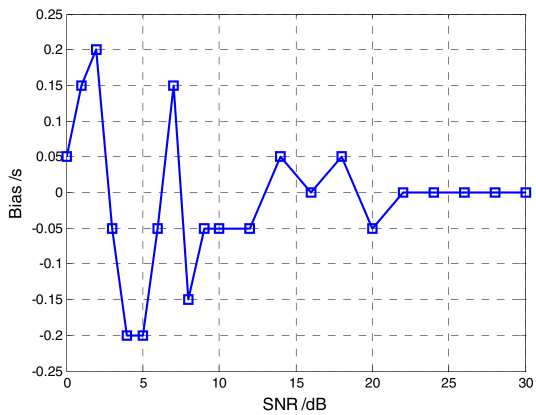


(b) RMSEs of estimated IFR versus SNR

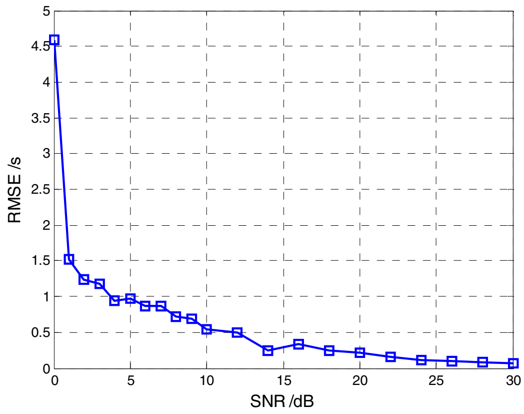
Fig. 4. Errors of estimated IFR versus SNR.

The relations between the errors of estimated IFR and SNR are illustrated in Fig. 4. Fig. 4(a) shows the bias curve of IFR. With the increasing SNR, the bias of estimated IFR is close to a stable value. This value is about 5×10^{-4}Hz/s and its corresponding threshold of SNR is 5dB. This indicates that discrete-time LPD is a biased estimate for IFR with SNR. However, the bias can be reduced with choosing right parameters. Fig. 4(b) shows the analysis of RMSE. It can be seen that the greater the SNR, the less the RMSE of IFR is; and the RMSE is less than -20dB when SNR is above 5dB. These results demonstrate that the IFR of Doppler signal can be accurately estimated by the discrete-time LPD in low SNR.

Fig. 5 illustrates the relations between the errors of $\hat{t}_0$ and SNR. It can be seen from Fig. 5(a) that the bias curve fluctuates slightly around zero, and this indicates that the proposed approach is an unbiased estimate for t_0 with SNR. Fig. 5(b) shows that the greater the SNR, the less the RMSE of $\hat{t}_0$ is; and the RMSE is less than 1s when SNR is above 5dB, i.e., the RMSE of localization error is less than 4.12m. Since the sampling rate f_s is 20Hz in the simulations, the biases and RMSEs of $\hat{t}_0$ can only be integer multiples of 0.05s. So, the minimum scale of localization error is 0.206m, but it can be improved by increasing sampling rate f_s or interpolation.

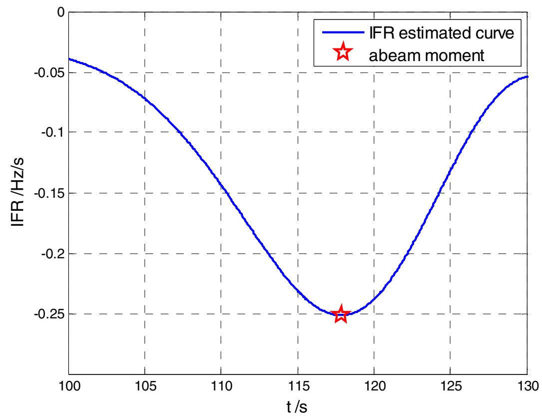


(a) Biases of $\hat{t}_0$ versus SNR



(b) RMSEs of $\hat{t}_0$ versus SNR

Fig. 5. Errors of $\hat{t}_0$ versus SNR.



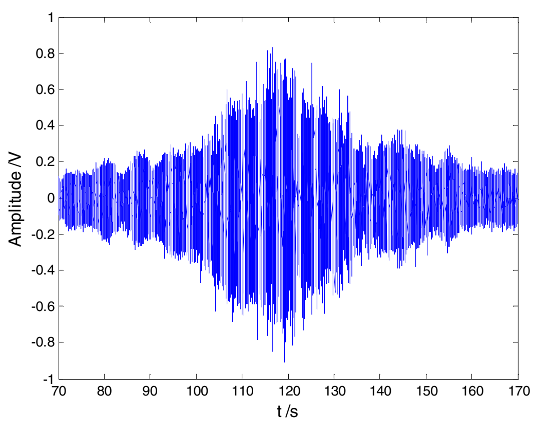
(b) The optimizing curve of estimated IFR and the estimated $\hat{t}_0$

Fig. 6. Analysis of the data with the resting frequency being 220Hz.

In the above simulations, the resting frequency of the Doppler signal f_0 is 200Hz and the sampling rate f_s is only 20Hz. It is very clear that the discrete-time LPD can estimate the IFR of Doppler signal without satisfying the Nyquist sampling theorem. The above results of simulations verify the conclusion of Part B in Section III.

V. Data Analysis of Sea Experiment

The experiment has been done in shallow sea area of China. The received hydrophone was set out by the moored measurement ship A, and the depth of the measurement hydrophone was 10m below the waterline. As the radiated source, the low-frequency emitting transducer was carried on the catamaran B, where the emitting part of the hydrolocator was used as the reference source. These two acoustic sources were separately fixed at the bottom of two adjustable holders on the central axis of the catamaran B. The distance D between them was 19.2m and the depth was 6m below the waterline. To avoid the harmonic peaks of the catamaran B, the frequencies of emitted signals were chosen to be 55Hz, 105Hz, 220Hz and 500Hz. The emitting ship traveled in straight line under 5kn working conditions and the relative speed between the ship and the measurement hydrophone was obtained with the information of the hydrolocator.



(a) Doppler signal received by the measurement hydrophone

Fig. 6 depicts the analysis of the data with resting frequency 220Hz. The time-domain waveform of the received Doppler signal is shown in Fig. 6(a). Fig. 6(b) shows the fitting curve of estimated IFR, and the estimated $\hat{t}_0$ is 117.7s after searching the minimum of the estimated IFR.

The abeam moments under other working conditions can be also obtained by search the minimum of the estimated IFR. With the knowledge of the reference time t_r, relative speed v and the distance D, the localization errors of the emitting source are calculated by the equation $\Delta L = \left| v \left| \hat{t}_0 - t_r \right| - D \right|$. The parameters in the experiment and the analysis results are shown in TABLE I. It can be seen that the localization errors are close to each other, and all of them are less than 3m. These results are consistent with the simulation analysis.

TABLE I. Results of the Proposed Method under Various Working Conditions in Sea Experiment

No.	f_0 / Hz	v / m/s	t_r / s	$\hat{t}_0$ / s	ΔL / m
1	55	2.9	109.8	104.2	2.96
2	105	2.5	142.6	135.9	2.45
3	220	2.9	110.4	117.7	1.97
4	500	3.6	89.1	83.2	2.04

VI. Conclusions

As a new time-frequency rate distribution, LPD has good concentration in time-frequency rate plan for high-order polynomial phase in low SNR condition. The discrete-time form of LPD can be effectively realized in an iteration procedure. This paper uses the discrete-time LPD to estimate IFR of the Doppler signal produced by the relative motion between the tone noise sources and the receiver. Then, according to the minimum of the estimated IFR, we localize the tone noise sources on the underwater moving object. The results of simulation and data analysis show that the proposed method has high accuracy of localization.

APPENDIX

The FT of $\mathrm{LPD}_s^{L^2}(t,\Omega)$ can be written as

$$\begin{aligned}
&\underset{\Omega\to\omega}{\mathrm{FT}}\left\{\mathrm{LPD}_s^{L^2}(t,\Omega)\right\} \\
&= L\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[s^{L^2}(t+\tau)s^{L^2}(t-\tau)\right]\cdot \\
&\quad \exp\left(-\mathrm{j}\Omega L^2\tau^2\right)d\tau\exp\left(-\mathrm{j}\omega\Omega\right)d\Omega \\
&= L\int_{-\infty}^{+\infty}\left[s^{L^2}(t+\tau)s^{L^2}(t-\tau)\right]\int_{-\infty}^{+\infty}\exp\left(-\mathrm{j}\Omega\left(L^2\tau^2+\omega\right)\right)d\Omega d\tau \\
&= 2\pi L s^{L^2}(t+\tau)s^{L^2}(t-\tau)\Big|_{L^2\tau^2+\omega=0}
\end{aligned} \tag{A1}$$

It takes advantage of the integration equation $2\pi\delta(x)=\int_{-\infty}^{+\infty}\exp(\mathrm{j}xy)dy$ and the property of Dirac function $\int_{-\infty}^{+\infty}f(x)\delta(x)dx=f(0)$. In the same way, by using the above two equations, the FT of $\mathrm{LPD}_s^{L^2/2}(t,2\Omega)\otimes_\Omega\mathrm{LPD}_s^{L^2/2}(t,2\Omega)$ is expressed as

$$\begin{aligned}
&\mathrm{FT}\left\{\mathrm{LPD}_s^{L^2/2}(t,2\Omega)\otimes_\Omega\mathrm{LPD}_s^{L^2/2}(t,2\Omega)\right\} \\
&= \mathrm{FT}\left\{\int_{-\infty}^{+\infty}\left[s^{L^2/2}\left(t+\sqrt{2}\tau/L\right)s^{L^2/2}\left(t-\sqrt{2}\tau/L\right)\right]\cdot\right. \\
&\quad \left.\exp\left(-\mathrm{j}2\Omega\tau^2\right)d\tau\right\}^2 \\
&= \frac{L^2}{2}\mathrm{FT}\left\{\int_{-\infty}^{+\infty}\left[s^{L^2/2}(t+\tau')s^{L^2/2}(t-\tau')\right]\exp\left(-\mathrm{j}\frac{2\Omega L^2\tau'^2}{2}\right)d\tau'\right\} \\
&= \frac{L^2}{2}\left\{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[s^{L^2/2}(t+\tau)s^{L^2/2}(t-\tau)\right]\cdot\right. \\
&\quad \left.\exp\left(-\mathrm{j}\Omega L^2\tau^2\right)d\tau\exp\left(-\mathrm{j}\omega\Omega\right)d\Omega\right\}^2 \\
&= \frac{L^2}{2}\left\{\int_{-\infty}^{+\infty}\left[s^{L^2/2}(t+\tau)s^{L^2/2}(t-\tau)\right]\cdot\right. \\
&\quad \left.\int_{-\infty}^{+\infty}\exp\left(-\mathrm{j}\Omega\left(L^2\tau^2+\omega\right)\right)d\Omega d\tau\right\}^2 \\
&= 2\pi^2L^2\left[s^{L^2/2}(t+\tau)s^{L^2/2}(t-\tau)\Big|_{L^2\tau^2+\omega=0}\right]^2 \\
&= 2\pi^2L^2 s^{L^2}(t+\tau)s^{L^2}(t-\tau)\Big|_{L^2\tau^2+\omega=0}
\end{aligned} \tag{A2}$$

So it is easily known that

$$\mathrm{FT}\left\{\mathrm{LPD}_s^{L^2}(t,\Omega)\right\}=\frac{1}{\pi L}\mathrm{FT}\left\{\mathrm{LPD}_s^{L^2/2}(t,2\Omega)\otimes_\Omega\mathrm{LPD}_s^{L^2/2}(t,2\Omega)\right\} \tag{A3}$$

That is

$$\mathrm{LPD}_s^{L^2}(t,\Omega)=\frac{1}{\pi L}\mathrm{LPD}_s^{L^2/2}(t,2\Omega)\otimes_\Omega\mathrm{LPD}_s^{L^2/2}(t,2\Omega) \tag{A4}$$

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