An Approach to Using Basic Three-Element Arrays in Tetrahedral-Based USBL Systems

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***Abstract*—This article describes an approach to designing the ultra-short baseline (USBL) system with a four-element three-dimensional receiving USBL array. In particular, the receiving array based on a four-element tetrahedron is considered. The four-element tetrahedral USBL array is considered as a set of the elemental skew three-element USBL arrays. The coordinates of the object are determined by utilizing the basic three-element USBL system of this set with the subsequent averaging of the obtained coordinates. In the article the Cartesian coordinates of the object are determined in the reference coordinate system bounded up with the USBL system's carrier. It is assumed that the receiving tetrahedral array is oriented as an inverted pyramid and mounted rigidly on the lower part of the carrier hull. It is assumed that the receiving USBL array can have an inclination, which is controlled by the measurement of the array's pitch and roll angles. The paper focused on exploiting the latent resources of the tetrahedral receiving USBL array for accurate coordinate determination in conditions when the receiving antenna can have significant inclination and the location of the object in the lower hemisphere is arbitrary.**

***Index Terms*—Ultra-short baseline (USBL) system; tetrahedral array; underwater object; carrier coordinate system; local coordinate system; skew coordinate system; pitch and roll angles.**

I. Introduction

Precise positioning of underwater objects (ROVs, AUVs, manned submersibles, etc.) in sea operating conditions is one of the most important tasks in ocean engineering. Over the years, significant efforts have been made to improve the accuracy of the underwater positioning systems (LBL, SBL and USBL systems). The development and application of advanced signal processing techniques (chirp signals, direct sequence spread-spectrum signals, etc.) enabled a significant improvement of reliability and accuracy of such systems in marine operating conditions (results of some studies are presented in [1],[2],[3] and [4]). Concurrently with these studies there is an ongoing search for the optimum design of the receiving USBL array (see [5] and [6]).

USBL systems determine the location of underwater objects (ROV, AUV and other submersibles) by measuring their range and angular position to a transponder installed on the object. The range is determined by measuring the propagation times of acoustic interrogation and response pulses. The angular position of the object is determined by processing the time delays of the response pulse using the elements of the USBL array. The minimum number of the receiving elements in a USBL array of a USBL system is three (see [7] and [8]). The receiving elements of a three-element USBL array are commonly located in the horizontal plane and form two equal orthogonal receiving bases. To improve the reliability and accuracy of determining the coordinates, the number of elements in the receiving USBL array can be increased. Adding the new receiving elements to an additional plane leads to a three-dimensional receiving USBL array. For example, in [5] and [6], the USBL systems with three-dimensional four-element receiving USBL arrays are investigated, while in [9], a five-element USBL array is studied. The five-element square pyramid USBL array proposed in [9] is viewed as a set of basic three-element orthogonal and non-orthogonal arrays. The coordinates of the object are determined by using the basic three-element USBL system in this set with the subsequent averaging of the obtained coordinates. This approach allowed the author to obtain a noticeable increase in the accuracy of the object's position when using a single measurement (the position of an underwater object is measured once).

In the present article the four-element tetrahedral receiving USBL array is studied. The receiving antenna elements are arranged at the vertices of the inverted tetrahedron. It is assumed that the receiving array is rigidly mounted on the lower part of the carrier hull. The proposed design method is based on the decomposition of the receiving USBL array on the elemental three-element arrays. The coordinates of the object are determined by using each elemental three-element USBL system and then the obtained coordinates are averaged. The specially designed algorithm accomplishes the selection of reliable elemental USBL arrays utilizing the analysis of the latitude angles to the object for each elemental three-element array. The cases of using an incomplete and a complete set of the elemental (three-element) USBL arrays are investigated. In the case of the tetrahedron USBL array the total number of three-element USBL arrays is twelve (three three-element arrays in each face).

One of the goals of this work is the evaluation of the position-determination accuracy of the object that can be obtained with the four-element tetrahedral receiving USBL array. For the purpose of obtaining a valid and exact comparison of the accuracy of the proposed system and a five-

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element USBL system [9] we will assume that the system operating conditions are the same. In particular, during the research it is supposed that the object is located at a distance where the plane wave approximation is valid. The receiving array at the moment of receiving a response pulse can have significant inclination. Also we assume that the propagation medium is homogeneous and the refraction of the acoustic rays is absent. We also assume the absence of the multipath interference. The object's position is determined in the case of a single measurement.

This paper is organized as follows. In section II we introduce the four-element USBL system based on the tetrahedron and the reference coordinate systems for the proposed receiving array. In section III the formulas for the three-element non-orthogonal USBL array are presented. Next, in section IV the pitch and roll of the receiving array are introduced. In section V the formulas for the proposed four-element USBL system are presented. Further, in section VI we describe the algorithm that accomplishes the selection of reliable elemental USBL arrays and averaging of coordinates obtained with the selected three-element arrays. Finally, in section VII we present the results of the simulation, which was based on the proposed algorithm, and evaluate the accuracy of the proposed USBL system.

II. Four-element Tetrahedral USBL Array and Its Coordinate Systems

As was mentioned above, the angular position of the underwater object is determined by using a three-dimensional four-element receiving USBL array. We assume that the USBL array is formed like a regular tetrahedron (or triangle pyramid) and the geometrical centers of receiving elements of the array coincide with the tetrahedron vertices.

Let $\Sigma=(0,x,y,z)$ be the sea-surface associated left-handed Cartesian coordinate system with the origin in point O (see Fig. 1). We assume that the origin of the coordinate system is located on the face of the tetrahedron with elements 1, 2 and 3. The (x-y) plane will be parallel to the sea surface (it is supposed that the sea surface is not perturbed) and the z-axis will go downward. Let $\Sigma_{carrier}= (0,x_{carrier},y_{carrier},z_{carrier})$ be the carrier coordinate system with the origin also in point O. It is supposed that the carrier coordinate system $\Sigma_{carrier}$ coincides with the $\Sigma=(0,x,y,z)$ when the carrier does not have pitch and roll. Also it is supposed that the $y_{carrier}$-axis coincides with the carrier's longitudinal axis L-L' (the positive direction coincides with the direction of the straight arrowed line), the $x_{carrier}$-axis coincides with the carrier's lateral axis B-B'. In the case of the pitch and roll, the carrier coordinate system $\Sigma_{carrier}$ will be inclined. The definitive coordinates of the object (transponder) are determined in the $\Sigma=(0,x,y,z)$ coordinate system.

The position of the proposed four-element USBL array in the carrier coordinate system $\Sigma_{carrier}$ is shown in Fig. 1. We will consider one of the faces of the tetrahedron as a base and will assume that it lies in the carrier horizontal plane. The apex of the introduced pyramid is located below (the pyramid is upturned).

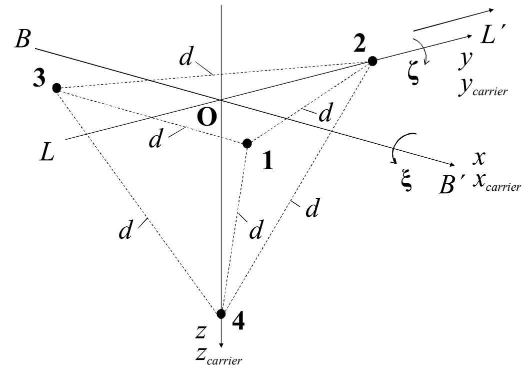

Fig. 1. Location of the tetrahedral USBL array in the introduced coordinate systems.

The numeration of the receiving elements of the USBL array is shown in Fig. 1. The proposed USBL array has one triangle horizontal face formed by receiving elements 1, 2 and 3, and three differently oriented triangle faces: the face with elements 1, 2 and 4; the face with elements 2, 3 and 4; the face with elements 3, 1 and 4.

The horizontal face can be considered as a set of the three elemental three-element USBL arrays: $USBL_{123}$, $USBL_{231}$ and $USBL_{312}$ (all elemental USBL arrays are non-orthogonal). Each triangular face also contains the set of the three non-orthogonal (skew) three-element USBL arrays. These non-orthogonal three-element USBL arrays are: $USBL_{142}$, $USBL_{421}$, $USBL_{214}$, $USBL_{243}$, $USBL_{432}$, $USBL_{324}$, $USBL_{341}$, $USBL_{413}$ and $USBL_{134}$. The first three arrays lie in the face with elements 1, 2 and 4; the next three arrays lie in the face with elements 2, 3 and 4; and the last three arrays lie in the face with elements 3, 1 and 4.

For each elemental three-element USBL array we introduce a local skew coordinate frame. The origin of each local coordinate system coincides with the common receiving element of the corresponding three-element USBL array. The coordinate axes coincide with the receiving bases. If the four-element receiving array has an inclination (has pitch and roll), the three-element arrays and corresponding coordinate systems also will be inclined.

After determining the coordinates of the object in the local coordinate frame, the reduction of the local object's coordinates to the Cartesian $\Sigma=(0,x,y,z)$ coordinate system is performed. This reduction is realized by means of a series of rotations of the local coordinate systems around the corresponding axes. For convenience of calculations before the beginning of reduction, the transformation of the local skew coordinate systems to the local Cartesian coordinate systems is performed.

The Z-coordinate sign ambiguity problem is resolved by applying a special procedure that involves the analysis of the time delays on the bases with different spatial orientation. The problem of the imprecise measure of the Z-coordinate for the cases when the object is located close to the plane of the three-element array is resolved by means of eliminating these basic three-element arrays from the averaging.

III. Basic Three-Element Non-Orthogonal USBL System

The formulas of the Cartesian coordinates of the object in the case of using the three-element non-orthogonal USBL array have been derived in [9]. So we will consider the procedure of obtaining the local Cartesian coordinates in the corresponding local coordinate systems of elemental three-element USBL arrays very briefly. In order to have a correspondence with the formulas obtained in [9] we will consider the three-element array $USBL_{123}$.

The $USBL_{123}$ array lies in the horizontal plane and includes the receiving elements 1, 2 and 3. Let $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ be the local non-orthogonal coordinate system for the considered $USBL_{123}$ system. It is assumed that the origin of the Σ_{123} coordinate system coincides with the location of the receiving element 2, the x_{123}-coordinate axis coincides with the base 3-2, the y_{123}-coordinate axis coincides with the base 1-2 and z_{123}-coordinate axis goes downwards (and z_{123}-coordinate axis is orthogonal to the x_{123}-coordinate axis and to the y_{123}-coordinate axis). The angle between the x_{123}-coordinate axis and the y_{123}-coordinate axis is designated as ν (for the considered USBL array ν=60°).

We will assume that the underwater object is located in point P and the incline distance R to the object is known. The geometry of the receiving three-element $USBL_{123}$ array, introduced coordinate system $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ and the point P are shown in Fig. 2.

The angular position of the object in the coordinate system $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ is defined by the direction cosines of the angles α, β and γ (α, β and γ are the angles between the direction to point P and the coordinate axes x_{123}, y_{123} and z_{123} respectively). The angles α_1 and β_1 define the angular position of the object in the horizontal plane (point P_h defines the projection of the point P on the horizontal plane).

The covariant coordinates X_{123_covar}, Y_{123_covar} and Z_{123_covar} of the object in the introduced coordinate system $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ can be expressed as follows: $X_{123\_covar} = R\cos\alpha$, $Y_{123\_covar} = R\cos\beta$, $Z_{123\_covar} = R\cos\gamma$. These coordinates also can be represented as a row vector $\boldsymbol{p}_{123\_covar} =[X_{123\_covar}, Y_{123\_covar}, Z_{123\_covar}]^T$.

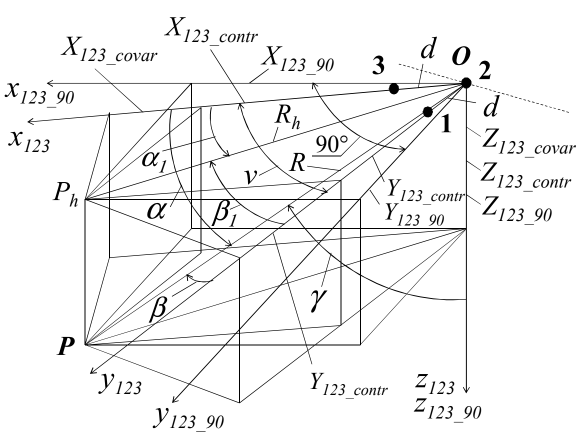

Fig. 2. Geometry of the skew three-element USBL array and the underwater object (point P).

Let d be the length of the edge of the tetrahedral USBL array. With this assumption d also will be the size of the receiving bases 1-2 and 3-2 of the three-element $USBL_{123}$ array. The time delays τ_{12} and τ_{32} of the signals on the outputs of the receiving elements of the base 1-2 and the base 3-2 (it is supposed that $R>>d$) can be expressed in the following way: $\tau_{12}=(d/c)\cos\beta$ and $\tau_{32}=(d/c)\cos\alpha$, where c is the speed of sound in water. If we define d/c as τ_d we can write the direction cosines $\cos\alpha$ and $\cos\beta$ as follows:

$$\cos\alpha=\tau_{32}/\tau_d\,; \quad \cos\beta=\tau_{12}/\tau_d\,. \tag{1}$$

The value of the third direction cosine $\cos\gamma$ is determined by the values of time delays τ_{12} and τ_{32} (by the values of the $\cos\alpha$ and $\cos\beta$) and by the value of the angle ν.

The formula for the direction cosine $\cos\gamma$ can be derived by utilization of the expression for the angles α_1 and β_1 obtained in [9]. This formula can be written as follows:

$$\cos\gamma=\sqrt{\sin^2\beta-\frac{\left(\cos\alpha-\cos\beta\cos\nu\right)^2}{\sin^2\nu}}\,. \tag{2}$$

The contravariant coordinates X_{123_contr}, Y_{123_contr} and Z_{123_contr} obtained in the coordinate system Σ_{123} can be represented as a row vector $\boldsymbol{p}_{123\_contr} =[X_{123\_contr}, Y_{123\_contr}, Z_{123\_contr}]^T$).

The components of the vector $\boldsymbol{p}_{123\_contr}$ are defined by the expressions [9]:

$$X_{123\_contr}=R\cos\alpha\left[1-\frac{\cos\nu}{\sin^2\nu}\left(\frac{\cos\beta}{\cos\alpha}-\cos\nu\right)\right];$$

$$Y_{123\_contr}=R\cos\beta\left[1-\frac{\cos\nu}{\sin^2\nu}\left(\frac{\cos\alpha}{\cos\beta}-\cos\nu\right)\right];$$

$$Z_{123\_contr}=R\sqrt{\sin^2\beta-\frac{\left(\cos\alpha-\cos\beta\cos\nu\right)^2}{\sin^2\nu}}\,. \tag{3}$$

The covariant coordinates X_{123_covar} and Y_{123_covar} and contravariant coordinates X_{123_contr}, Y_{123_contr} of the object are shown in Fig. 3.

After determining the coordinates of the object in the skew coordinate system $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$, the transformation to local orthogonal coordinates and then the reduction of the obtained coordinates to the carrier (Cartesian) coordinate system are performed.

Let $\Sigma_{123\_90}=(0,x_{123\_90},y_{123\_90},z_{123\_90})$ be the local orthogonal coordinate system for the considered $USBL_{123}$ array. This coordinate system is obtained by increasing the angle between coordinate axes x_{123} and y_{123} of coordinate system $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ up to 90°. The arrangement of the coordinate system Σ_{123_90} relative to the previously introduced skew coordinate system Σ_{123_90} is shown in Fig. 2. The coordinate plane x_{123}-y_{123} coincides with the plane x_{123_90}-y_{123_90}.

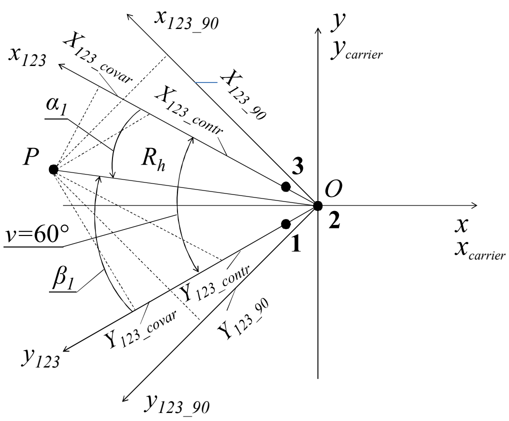

Fig. 3. Covariant and contravariant coordinates of the object (point *P*) in the horizontal plane

The angle between the axis x_{123} and the axis x_{123_90} (as well between the axis y_{123} and the axis y_{123_90}) is (90°-ν)/2. The axis z_{123} coincides with the axis z_{123_90}. The coordinates X_{123_90} and Y_{123_90} of the object in the Σ_{123_90} coordinate system with utilization of the formulas from [9] will be defined as follows:

$$X_{123\_90} = X_{123\_contr}\cos\left[360° - \frac{90° - \nu}{2}\right] + Y_{123\_contr}\cos\left[360° - \frac{90° + \nu}{2}\right],$$

$$Y_{123\_90} = X_{123\_contr}\cos\left[\frac{90° + \nu}{2}\right] + Y_{123\_contr}\cos\left[\frac{90° - \nu}{2}\right]. \qquad (4)$$

The third coordinate will be the same as in the skew coordinate system ($Z_{123\_90}=Z_{123\_covar}=Z_{123\_contr}$). The coordinates X_{123_90}, Y_{123_90} and Z_{123_90} can be represented as a row vector $\boldsymbol{p}_{123\_90}=[X_{123\_90}, Y_{123\_90}, Z_{123\_90}]^T$. To obtain the object's coordinates in the carrier coordinate system $\Sigma_{carrier}$ as well as in the $\Sigma=(0,x,y,z)$, it is necessary to accomplish simple rotation of the coordinate system $\Sigma_{123\_90}= (0,x_{123\_90},y_{123\_90},z_{123\_90})$ around the vertical axis (axis z_{123_90}).

IV. Pitch and Roll of USBL Array

In order to estimate the influence of the incline of the receiving antenna on the system's operation we will use an approach that was used in [9]. This approach involves a preliminary transformation of the local non-orthogonal coordinate systems of each three-element USBL array to the local orthogonal coordinate systems. After that, the various rotations of each coordinate system must be made to reduce the local object's coordinates to the sea-surface coordinate system. All local coordinate systems are treated separately.

We will consider only the case of the $USBL_{123}$ system. For the other elemental three-element USBL arrays only the final calculation formulas will be presented.

Let angles ξ and ζ be pitch and roll angles of the receiving USBL antenna (in Fig. 1 these angles show the rotations relative to the carrier lateral *B-B′* axis and the carrier longitudinal *L-L′* axis). The local skew Σ_{123} and local orthogonal Σ_{123_90} coordinate systems for the $USBL_{123}$ array are shown in Fig. 2.

If the pitch and roll of the receiving antenna take place, we will have the next transformations of the coordinate system Σ_{123}: after the turn on the pitch angle ξ the coordinate system $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ transforms to the coordinate system $\Sigma^{\xi}_{123}=(0,x^{\xi}_{123},y^{\xi}_{123},z^{\xi}_{123})$ and after the turn on the roll angle ζ transforms to coordinate system $\Sigma^{\xi,\zeta}_{123}=(0,x^{\xi,\zeta}_{123}, y^{\xi,\zeta}_{123}, z^{\xi,\zeta}_{123})$.

In the general case the receiving array has some pitch and roll inclination. So, the object's coordinates will be determined in the $\Sigma^{\xi,\zeta}_{123}$ coordinate system. These coordinates can be represented as a vector $\boldsymbol{p}^{\xi,\zeta}_{123}=[X^{\xi,\zeta}_{123}, Y^{\xi,\zeta}_{123}, Z^{\xi,\zeta}_{123}]^T$. Before performing the rotations of the local coordinate system we have to calculate the object's coordinates in the orthogonal coordinate $\Sigma^{\xi,\zeta}_{123\_90}=(0,x^{\xi,\zeta}_{123\_90}, y^{\xi,\zeta}_{123\_90}, z^{\xi,\zeta}_{123\_90})$. In this stage Eq. 1, 2, 3 and 4 are utilized. The position of the object in the coordinate system $\Sigma^{\xi,\zeta}_{123\_90}$ can be represented by means of the vector $\boldsymbol{p}^{\xi,\zeta}_{123\_90}=[X^{\xi,\zeta}_{123\_90}, Y^{\xi,\zeta}_{123\_90}, Z^{\xi,\zeta}_{123\_90}]^T$. In the case of the orthogonal coordinate systems, the rotation of the coordinate system can be made by using well known formulas for rotation of Cartesian coordinate systems [10].

Let $\boldsymbol{p}_{USBL\_123}=[X_{USBL\_123}, Y_{USBL\_123}, Z_{USBL\_123}]^T$ be the vector that represents the reduced object's coordinates obtained by means of the elemental skew three-element $USBL_{123}$ system and reduced to the coordinate system $\Sigma=(0,x,y,z)$. This reduction in the case of the $USBL_{123}$ array can be made by means of a chain of transformations of the coordinate vector $\boldsymbol{p}^{\xi,\zeta}_{123\_90}=[X^{\xi,\zeta}_{123\_90}, Y^{\xi,\zeta}_{123\_90}, Z^{\xi,\zeta}_{123\_90}]^T$. Also this reduction of the coordinates can be considered a chain of rotations of the local orthogonal coordinate system of the $USBL_{123}$ array. This procedure has been described in detail in [11]. The final equation for the vector $\boldsymbol{p}_{USBL\_123}$ can be expressed as follows:

$$\boldsymbol{p}_{USBL\_123} = \boldsymbol{C}^{-1}_{123}\boldsymbol{A}^{-1}_{123}\boldsymbol{B}^{-1}_{123}\boldsymbol{p}^{\xi,\zeta}_{123\_90}. \qquad (5)$$

In Eq. 5 the matrix $\boldsymbol{B}^{-1}_{123}$ transforms vector $\boldsymbol{p}^{\xi,\zeta}_{123\_90}$ to vector $\boldsymbol{p}^{\xi}_{123\_90}$; matrix $\boldsymbol{A}^{-1}_{123}$ transforms vector $\boldsymbol{p}^{\xi}_{123\_90}$ to vector $\boldsymbol{p}_{123\_90}$; matrix $\boldsymbol{C}^{-1}_{123}$ transforms vector $\boldsymbol{p}_{123\_90}$ to vector $\boldsymbol{p}_{USBL\_123}$.

In the same way we can introduce a chain of vectors and transform matrices for the other three-element USBL arrays and obtain the analogous formulas for the vectors that represent the coordinates of the object in the $\Sigma=(0,x,y,z)$ coordinate system.

V. Tetrahedral-Based USBL System

Each face of the tetrahedral USBL array includes three receiving elements and can be considered as an independent basic three-element USBL array. Meanwhile, it should be noted that the tetrahedral face is also an equilateral triangle and any receiving element (any of the three receiving elements) can be selected as a common element of the receiving bases. This gives us the opportunity to form three identical independent three-element arrays by utilizing one tetrahedral face. In the case of the horizontal tetrahedral face with receiving elements 1, 2 and 3, the basic three-element systems are the following: $USBL_{123}$, $USBL_{231}$ and $USBL_{312}$.

Decomposition of each face of the tetrahedron on the three-element USBL arrays allows us to have in total 12 three-element arrays (3horizontal+9inclined) and correspondingly to have in total 12 basic three-element USBL systems.

As it was noted above we have three basic USBL arrays located in the horizontal plane and as a result three basic USBL systems: $USBL_{123}$, $USBL_{231}$ and $USBL_{312}$. The other nine three-element USBL arrays lie in the inclined planes (faces) of the tetrahedron. Corresponding elemental USBL systems are as follows: $USBL_{142}$, $USBL_{421}$, $USBL_{214}$, $USBL_{243}$, $USBL_{432}$, $USBL_{324}$, $USBL_{341}$, $USBL_{413}$ and $USBL_{134}$.

Firstly, the coordinates of the object are determined by each elemental three-element USBL system. The general procedure of the reduction of the coordinates of the basic USBL system to the carrier's coordinate system was discussed in the previous section. This procedure was worked out for the receiving USBL array located in the horizontal plane.

Now we consider some of the particularities that will take place in the case of the other three-element USBL systems.

The receiving $USBL_{231}$ and $USBL_{312}$ arrays (just like $USBL_{123}$ array) lie in the horizontal face of the tetrahedron. The difference between these two arrays and the $USBL_{123}$ array is in their orientation in the horizontal plane. If the receiving antenna has some inclination the values of the angles between longitudinal and lateral axes for these three-element arrays will be different. Thereafter the transform matrixes must be worked out for the $USBL_{123}$, $USBL_{231}$ and $USBL_{312}$ systems separately.

The procedure of the reduction of the coordinates obtained by each basic three-element USBL system to the carrier's coordinate system can be represented as a chain of transformation. In the first stage the non-orthogonal coordinates transform to the orthogonal by using the Eq. 1, 2, 3 and 4. After that several rotations of the corresponding orthogonal coordinate systems must be performed. In the general case the pitch and roll inclinations of the receiving antenna take place and vectors that represent the orthogonal coordinates obtained from the horizontal three-element arrays must be transformed three times.

The formulas for the coordinate vectors for the horizontal three-element USBL systems (corresponding three-element arrays lie in the horizontal face of the tetrahedron) can be expressed as follows:

$$\boldsymbol{p}_{USBL\_ijk}=\boldsymbol{C}^{-1}_{ijk}\boldsymbol{A}^{-1}_{ijk}\boldsymbol{B}^{-1}_{ijk}\boldsymbol{p}^{\xi,\zeta}_{ijk\_90}\,, \quad (6)$$

where index ijk has the following values: $ijk=123$ for the $USBL_{123}$ system; $ijk=231$ for the $USBL_{231}$ system; and $ijk=312$ for the $USBL_{312}$ system. Matrix $\boldsymbol{B}^{-1}_{ijk}$ transforms vector $\boldsymbol{p}^{\xi,\zeta}_{ijk}$ to vector $\boldsymbol{p}^{\xi}_{ijk}$ (back rotation by the ζ angle); matrix $\boldsymbol{A}^{-1}_{ijk}$ transforms vector $\boldsymbol{p}^{\xi}_{ijk}$ to vector $\boldsymbol{p}_{ijk}$ (back rotation by the ξ angle); matrix $\boldsymbol{C}^{-1}_{ijk}$ transforms vector $\boldsymbol{p}_{ijk}$ to vector $\boldsymbol{p}_{USBL\_ijk}$.

In the case of the basic three-element arrays that lie in the inclined faces of the tetrahedron the reduction of the coordinates to the carrier's coordinate system has additional steps. Coordinates obtained with inclined basic three-element USBL array firstly must be transformed to the orthogonal (by using the Eq. 1, 2, 3 and 4). Thereafter, the consequent rotations of the local orthogonal coordinate systems are performed with utilization of the corresponding transformation matrixes. If the receiving antenna has pitch and roll inclination the vectors that represent the orthogonal coordinates obtained with the elemental inclined three-element arrays must be transformed five times.

As before, we will assume that the pitch and roll inclinations of the receiving antenna take place. The skew coordinates of the object after their transformation to the orthogonal coordinates can be represented as a vector $\boldsymbol{p}^{\xi,\zeta}_{ijk\_90}=[X^{\xi,\zeta}_{ijk},\ Y^{\xi,\zeta}_{ijk},\ Z^{\xi,\zeta}_{ijk}]^T$, where index ijk is utilized to define the USBL system located on one of the inclined tetrahedral faces.

The formulas of the coordinate vectors $\boldsymbol{p}_{USBL\_ijk}$ (coordinate vectors $\boldsymbol{p}^{\xi,\zeta}_{ijk\_90}$ are reduced to the carrier's coordinate system) can be expressed as follows:

$$\boldsymbol{p}_{USBL\_ijk}=\boldsymbol{C}^{-1}_{ijk\_90}\boldsymbol{D}^{-1}_{ijk\_90}\boldsymbol{F}^{-1}_{ijk\_90}\boldsymbol{A}^{-1}_{ijk\_90}\boldsymbol{B}^{-1}_{ijk\_90}\boldsymbol{p}^{\xi,\zeta}_{ijk\_90}\,, \quad (7)$$

where index ijk has the following values: $ijk=142,\ 421,\ 214,\ 243,\ 432,\ 324,\ 341,\ 413,\ 134$ for the corresponding $USBL_{142}$, $USBL_{421}$, $USBL_{214}$, $USBL_{243}$, $USBL_{432}$, $USBL_{324}$, $USBL_{341}$, $USBL_{413}$ and $USBL_{134}$ systems. Matrix $\boldsymbol{B}^{-1}_{ijk}$ transforms vector $\boldsymbol{p}^{\xi,\zeta}_{ijk\_90}$ to vector $\boldsymbol{p}^{\xi}_{ijk\_90}$; matrix $\boldsymbol{A}^{-1}_{ijk}$ transforms vector $\boldsymbol{p}^{\xi}_{ijk\_90}$ to vector $\boldsymbol{p}_{ijk\_90}$; matrix $\boldsymbol{F}^{-1}_{ijk}$ transforms vector $\boldsymbol{p}_{ijk\_90}$ to vector $\boldsymbol{p}_{ijk\_90\_vert}$ (vector $\boldsymbol{p}_{ijk\_90\_vert}$ is obtained by reduction of the local coordinate system Σ_{ijk_90} to the vertical plane); matrix $\boldsymbol{D}^{-1}_{ijk}$ transforms vector $\boldsymbol{p}_{ijk\_90\_vert}$ to vector $\boldsymbol{p}_{ijk\_90\_horiz}$ (vector $\boldsymbol{p}_{ijk\_90\_horiz}$ is obtained by reduction of the local coordinate system $\Sigma_{ijk\_90\_vert}$ to the horizontal plane); matrix $\boldsymbol{C}^{-1}_{ijk}$ transforms vector $\boldsymbol{p}_{ijk\_90\_horiz}$ to vector $\boldsymbol{p}_{USBL\_ijk}$.

The last step of the determination of the object coordinates is the calculation of the arithmetic means of the coordinates obtained with the Eq. 6 and 7. The averaging of the coordinates is done by means of the special algorithm.

VI. Algorithm Description

Designing an algorithm for determining the position of the underwater object using the decomposition of the receiving antenna on the basic three-element arrays was discussed in [9]. As it was announced above, one of the goals of this investigation is to check the possibility of obtaining the accuracy of considered (four-element) USBL system comparable with the accuracy of the five-element USBL system described in [9]. For this reason we will utilize a similar calculation algorithm that was designed and applied in [9].

For the presented four-element USBL system some changes in the algorithm have been made. In particular, in the modified algorithm we have no orthogonal basic arrays, no vertical oriented basic arrays. The number of the elemental three-element arrays of the considered receiving antenna is noticeably smaller.

It is supposed that during the measurements the following data are obtained: the propagation time of the interrogation and reply pulses; pitch and roll angles ξ and ζ of the receiving USBL array; and time delays for all receiving bases of the proposed four-element USBL array. It is assumed that the transponder pulse is sinusoidal. To provide the positive values of time delays the second-indexed output signals of the receiving array are inverted.

We describe the revised algorithm briefly. In detail we present only the procedure of the calculation of the coordinates for the elemental $USBL_{123}$ system (see Fig.1).

Slope distance to the object is calculated using the values of propagation time of the interrogation and reply pulses. The angular position of the object is calculated by utilizing the values of the time delays τ_{12} and τ_{32} and values of the angles ξ and ζ. Firstly the covariant coordinates of the object (vector $\boldsymbol{p}^{\xi,\zeta}_{123\_covar} = [X^{\xi,\zeta}_{123\_covar}, Y^{\xi,\zeta}_{123\_covar}, Z^{\xi,\zeta}_{123\_covar}]^T$) are calculated. After that the contravariant coordinates (vector $\boldsymbol{p}^{\xi,\zeta}_{123\_contr} = [X^{\xi,\zeta}_{123\_contr}, Y^{\xi,\zeta}_{123\_contr}, Z^{\xi,\zeta}_{123\_contr}]^T$) are calculated. Then the Cartesian coordinates of the object (vector $\boldsymbol{p}^{\xi,\zeta}_{123\_90} = [X^{\xi,\zeta}_{123\_90}, Y^{\xi,\zeta}_{123\_90}, Z^{\xi,\zeta}_{123\_90}]^T$ are calculated.

Initially the sign of the $Z^{\xi,\zeta}_{123\_covar}$-coordinate ($Z^{\xi,\zeta}_{123\_contr}$-coordinate and $Z^{\xi,\zeta}_{123\_90}$-coordinate) is assumed to be positive. In order to check the correctness of the hypothesis about the sign of the $Z^{\xi,\zeta}_{123\_covar}$-coordinate ($Z^{\xi,\zeta}_{123\_contr}$-coordinate and $Z^{\xi,\zeta}_{123\_90}$-coordinate) the special procedure is applied. This part of the algorithm includes some additional calculations using a basic three-element array located in the other face of the tetrahedron. By selecting as such system the $USBL_{431}$ system, we can describe the special calculation procedure as follows. With the assumed sign for the $Z^{\xi,\zeta}_{123}$-coordinate the coordinates of the object (vector $\boldsymbol{p}_{USBL\_123} = [X_{USBL\_123}, Y_{USBL\_123}, Z_{USBL\_123}]^T$) in the $\Sigma_{carrier}$ coordinate system are calculated by applying Eq. 6. After the calculation of the object coordinates by means of the $USBL_{123}$ system, the values of the time delays (τ'_{34} and τ'_{14}) for the $USBL_{341}$ are calculated. After that the modules of the differences between the measured (τ_{34} and τ_{14}) and calculated (τ'_{34} and τ'_{14}) values of the time delays are calculated: $\delta 1 = |\tau'_{34} - \tau_{34}|$ and $\delta 2 = |\tau'_{14} - \tau_{14}|$. Later the value $\Delta 1 = (\delta 1^2 + \delta 2^2)^{0.5}$ is calculated. The same procedure is repeated with the opposite sign of $Z^{\xi,\zeta}_{123}$-coordinate with calculation of the corresponding vector $\boldsymbol{p}_{USBL\_123}$ and the value $\Delta 2$. If $\Delta 1 > \Delta 2$ the last sign is assumed as correct. Otherwise the initial sign value of $Z^{\xi,\zeta}_{123}$-coordinate is assumed as correct.

The same procedure applies to the other elemental USBL systems with the three-element USBL arrays located in the same plane as the $USBL_{123}$ array. In order to resolve the problem of Z-coordinate ambiguity the $USBL_{341}$ system is utilized.

The same procedure applied to the other basic USBL systems with the elemental three-element USBL arrays located in the other three differently oriented planes.

The part of the algorithm which is responsible for good accuracy of the Z-coordinate determination is resolved by means of analysis of the value of the latitude angle to the object. The value of the latitude angle for each elemental USBL system defines the decision to utilize or not utilize this particular system in further calculations. It can be shown that for reliable calculation of the *Z*-coordinate the modulus of the object's latitude angle must be more than 10° [12]. For example, if the value of the corresponding polar angle of the $USBL_{123}$ system is found outside [80°, 100°] diapason, the obtained coordinates of the $USBL_{123}$ system are utilized in averaging. If the value of the polar angle of the $USBL_{123}$ system lies inside of the [80°, 100°] diapason then the obtained coordinates are not taken into account. This procedure must be applied to all elemental three-element USBL systems.

The last step of the algorithm implies the calculation of the arithmetic means of the Cartesian coordinates of the object in the $\Sigma=(0,x,y,z)$ coordinate system (or the calculation of the coordinate vector $\boldsymbol{p}_{USBL} = [X_{USBL}, Y_{USBL}, Z_{USBL}]^T$) by utilizing the reduced coordinates of the all elemental USBL systems that have remained in consideration.

VII. Simulation Results for the Tetrahedral-Based USBL System

The algorithm discussed above has been implemented as a computer program and examined for various relative locations of the measuring system and object (object is located in the lower hemisphere, the maximum horizontal distance was assumed to be 100m).

The simulation parameters were as follows. The size of the receiving bases *d* (edges of the tetrahedron) was taken to be 0.056m. The transponder response pulse was assumed to be a sinusoidal. The pulse frequency has been chosen equal to 11KHz. It was assumed that the measurement of the time delays is provided by utilizing the binary counters and the signal reception conditions allow us to measure the time delays without errors. It was also assumed that the accuracy of measurement of the time delays is limited by the frequency of the time delay counter (25MHz). The speed of sound in the water *c* is constant and assumed to be 1500m/s. The values of pitch and roll angles ξ and ζ were assumed to be in the range from –40° to +40°. During the simulation it was assumed that the distance to the object, and the pitch and roll angles were being measured precisely.

Let *X*, *Y*, *Z* be the true values of the coordinates of the object in the coordinate system $\Sigma=(0,x,y,z)$. Let *R* be the true incline distance to the object. Let X_{USBL}, Y_{USBL}, Z_{USBL} be the values of the coordinates obtained by applying the developed algorithm. The values of the true errors of the determination of the coordinates are: $\Delta X_{USBL} = X_{USBL} - X$; $\Delta Y_{USBL} = Y_{USBL} - Y$, $\Delta Z_{USBL} = Z_{USBL} - Z$. The values of the relative true errors of the coordinates are: $\Delta X_{USBL}/R$; $\Delta Y_{USBL}/R$; $\Delta Z_{USBL}/R$.

Firstly we consider the USBL system based on the tetrahedron array with the set of six three-element USBL arrays (with three basic USBL arrays in the horizontal plane and three inclined basic USBL arrays with the common receiving element 4).

We consider the case when the object is located at the horizontal distance (R_h) 100 meters and the relative depth (*Z*) is 10 meters. The receiving array is inclined and the pitch and roll angles have the following values: $\xi = -5°$; $\zeta = 5°$. During the simulation the azimuth angle φ to the object is changing from 0° to 360° with the step of 1°.

The graphs of the variation of the relative true errors of the object's coordinates are represented in Fig. 4. In the same figure the graph of the number (*N*) of the basic three-element USBL systems used in the averaging is shown.

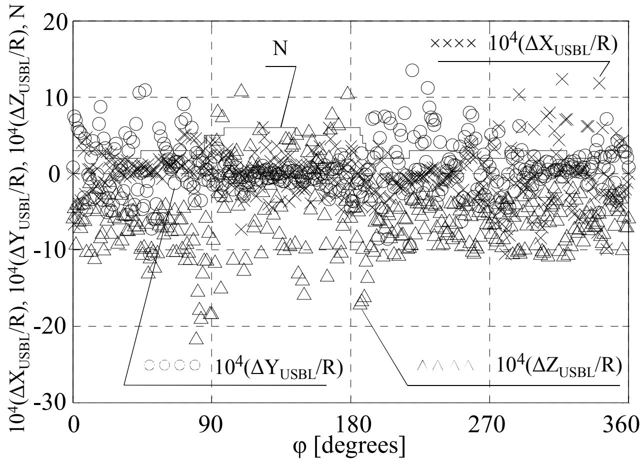

Fig. 4. The variation of the relative errors of the object's coordinates obtained from the USBL system with the use of the six three-element USBL arrays, *N* is the number of the basic USBL systems that were used for calculation of the coordinates means; R_h=100m; Z=10 m; ξ= -5°; ζ= 5°.

The graphs of the relative errors show that in all diapason of the changing of the angle φ the proposed array design and elaborated algorithm provide very good accuracy (relative true errors are less than 0.22% of the slant distance to the object). The number of the basic three-element USBL systems (see graph *N*) utilized in the averaging varies from 2 to 6. The relative errors of the determination of the horizontal coordinates (coordinates X_{USBL} and Y_{USBL}) are less than 0.15%.

Now we will consider the USBL system with the complete set of three-element USBL arrays. In Fig. 5 the graphs of the true relative errors of the determination of the object's coordinates of the proposed USBL system are presented. In the same figure the graph of the number (*N*) of the basic USBL systems utilized in the averaging is represented. In Fig. 5 we can observe that the relative errors of the determination of the object's coordinates are less than 0.12%.

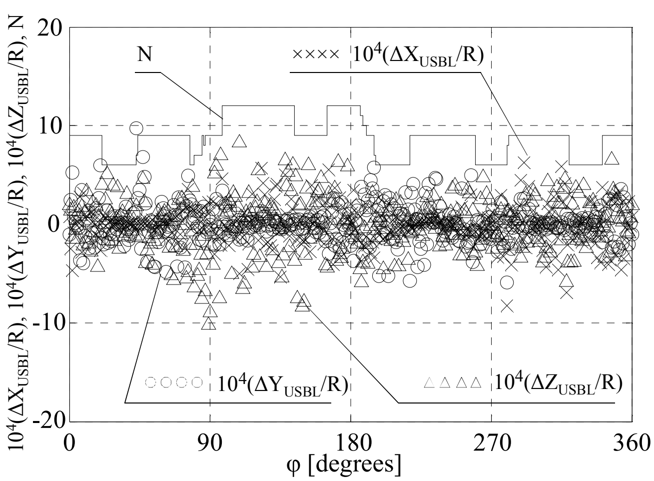

Fig. 5. The variation of the relative errors of the object's coordinates obtained from the USBL system with the use of the complete set of three-element USBL arrays, *N* is the number of the basic USBL systems that were used for calculation of the coordinates means; R_h=100m; Z=10 m; ξ= -5°; ζ= 5°.

For the considering case the number of the utilizing elemental USBL systems (graph *N*) is varying from 6 to 12. The relative errors of the determination of the horizontal coordinates (coordinates X_{USBL} and Y_{USBL}) are less than 0.10%.

Comparison of the relative errors in Fig. 4 and Fig. 5 demonstrates that the utilization of the complete set of the three-element arrays leads to the obvious improvement of the position-determination accuracy. The increase of the number of utilizing basic three-element USBL arrays (the number of the basic three-element array in use is doubled) gives us the improvement in accuracy of about 1.6 times.

VIII. CONCLUSION

This article describes an approach to designing a USBL system, which treats a tetrahedral USBL array as a set of elemental non-orthogonal three-element arrays. The article focuses on exploiting the latent resources of a three-dimensional receiving USBL array for accurate determination of the object's coordinates. The coordinates of the object are determined by using the basic three-element USBL systems with the subsequent averaging of the obtained coordinates. Processing of the local coordinates obtained by using basic three-element arrays is performed by means of the special algorithm. The Z-coordinate sign ambiguity problem for an elemental three-element array is resolved by using a basic three-element array located in the other tetrahedral face. The selection of reliable basic USBL arrays is based on the analysis of the latitude angles to the object for each of the three-element arrays. The algorithm was tested for a number of mutual positions of the USBL system and the object and in a wide range of pitch and roll angles of the receiving antenna. The proposed design approach allowed us to obtain an accuracy comparable with the accuracy of the five-element square-pyramid-based USBL system [9].

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