Research on six-degree-of-freedom motion modeling and simulation of single point moored ship

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***Abstract*—According to the kinematics and dynamics, the moored ship maneuvering motion equation with the six degree of freedom (DOF) is derived and established based on the idea of the MMG (Manoeuvring Model Group). The hydrodynamic added mass, potential damping due to wave radiation and viscous damping are mainly considered. The mathematical model of single point mooring system is established based on the catenary theory. Marine environmental disturbance model are established which includes the wind, wave and current. According to the simulation analysis of moored ship, the curves of ship motion trajectory and the mooring line tension are obtained. The simulation result is satisfying. It is helpful for the further research on moored ships, FPSO and permanent platforms.**

Keywords-six DOF motion; kinematics and dynamics modeling; single point moored ship; mooring line tension calculation

I. Introduction

Position mooring system is one of the most important positioning system. Position mooring system (PM) have proved to be a cost-effective alternative. In the oceanic engineering area, the research on motion of the moored ship is meaningful. And at the same time, the choice of anchor chain is decided by the maximum of anchor chain forces to ensure the safety of moored ship. So it is necessary to simulate and analyze the moored ship's motion and anchor chain force.

This paper does some research on the six DOF motion of the moored ship[1]. According to the kinematics and dynamics, the moored ship maneuvering motion equation with the six DOF is derived and established. The six DOF motion of the moored ship and mooring line tension are obtained. The maximum value of the mooring line tension can be used as the reference of the anchor chain security[2,3].

II. Mathematical Model of Moored Ship

Two different reference frames are used in position mooring, the earth-fixed reference frame $X_E Y_E Z_E$ and the body-fixed reference frame $x_b y_b z_b$ shown in Fig. 1. For convenience, the body frame is often chosen at the vessel's center of gravity.

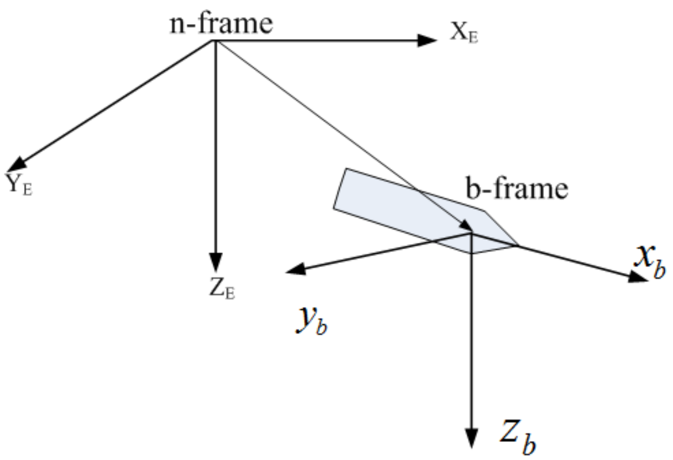

Fig. 1. Reference frame

Dynamics and kinematics equation of six DOF vessel is described as follows[4].

$$M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = \tau_{env} + \tau_h + \tau_m$$
$$\dot{\eta} = J(\eta)\nu$$

Where: τ_{env} is vector of weather disturbance force. τ_h is a vector of hydrodynamic force. τ_m is a vector of generalized mooring forces. The body-fixed surge, sway, heave, roll, pitch and yaw velocities are defined by the vector $\nu = [u, v, w, p, q, r]^T$. The earth-fixed ship position and attitude are defined by the vector $\eta = [x, y, z, \phi, \theta, \psi]^T$. $J(\eta)$ is rotation matrix. M_{RB} is ship inertia matrix.

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & I_x & -I_{xy} & -I_{zx} \\ mz_g & 0 & -mx_g & -I_{xy} & I_y & -I_{yz} \\ -my_g & mx_g & 0 & -I_{zx} & -I_{yz} & I_z \end{bmatrix}$$

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$C_{RB}(\nu)$ is Coriolis and centripetal force matrix.

$$C_{RB}(\nu)=$$
$$\left[\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -m(y_g q+z_g r) & m(y_g p+w) & m(y_g p+w) \\ m(x_g q-w) & -m(z_g r+x_g p) & -m(z_g r+x_g p) \\ m(x_g r+v) & m(y_g r-u) & -m(x_g p+y_g q) \end{matrix}\right.$$
$$\left.\begin{matrix} m(y_g p+z_g r) & -m(y_g p-w) & -m(x_g r+v) \\ -m(y_g p+w) & m(z_g r+x_g p) & -m(y_g r-u) \\ -m(z_g p-v) & -m(z_g q+u) & m(x_g p+y_g q) \\ 0 & -I_{yz}q-I_{xz}p+I_z r & I_{yz}r+I_{xy}p-I_y q \\ I_{yz}q+I_{xz}p-I_z r & 0 & -I_{xz}r-I_{xy}q+I_x p \\ -I_{yz}r-I_{xy}p+I_y q & I_{xz}r+I_{xy}q-I_x p & 0 \end{matrix}\right]$$

A. Ship hydrodynamics

The research of rigid-body hydrodynamic force and moment is very important to establish ship model. The hydrodynamic added mass, potential damping due to wave radiation and viscous damping are mainly considered.

Hydrodynamic forces and moments are as follows[5,6]:

$$\begin{aligned} X_H &= -\lambda_{11}\dot{u}+\lambda_{22}vr+X_N \\ Y_H &= -\lambda_{22}\dot{v}-\lambda_{11}ur+Y_N \\ Z_H &= -\lambda_{33}\dot{w}-Z_w w-Z_{\dot{q}}\dot{q}-Z_q q-Z_\theta\theta-Z_z z \\ K_H &= -\lambda_{44}\dot{p}-2L_p p-\Delta\cdot GM\cdot\sin\varphi-Y_H\cdot z_H \\ M_H &= -\lambda_{55}\dot{q}-M_q q-M_\theta\theta-M_{\dot{w}}\dot{w}-M_w w \\ N_H &= -\lambda_{66}\dot{r}+N_N \end{aligned}$$

The model to estimate the viscous hydrodynamic forces and moments[6] is used in this paper:

$$\begin{aligned} X_N &= X_{uu}u^2+X_{vv}v^2+X_{vr}vr+X_{rr}r^2 \\ Y_N &= Y_v v+Y_r r+Y_{v|v|}v\,|\,v\,|+Y_{v|r|}v\,|\,r\,|+Y_{r|r|}r\,|\,r\,| \\ N_N &= N_v v+N_r r+N_{r|r|}r\,|\,r\,|+N_{vvr}v^2 r+N_{vrr}vr^2 \end{aligned}$$

B. Mathematical model of the mooring system

As shown in Fig. 2, the vessel is constrained by the anchor chain tension T. Mooring line tension T can be separated into two parts, the horizontal tension T_H and vertical tension T_V. The horizontal tension affects of the rigid-body surge, sway and yaw motion. And the vertical tension affects the rigid-body heave and pitch motion.

According to the catenary theory, the model that Shoji Kuniaki derived in [7] is employed to calculate the horizontal tension. When the horizontal tension and vertical tension of anchor chain are obtained, the forces and moments acting on the rigid-body can be derived.

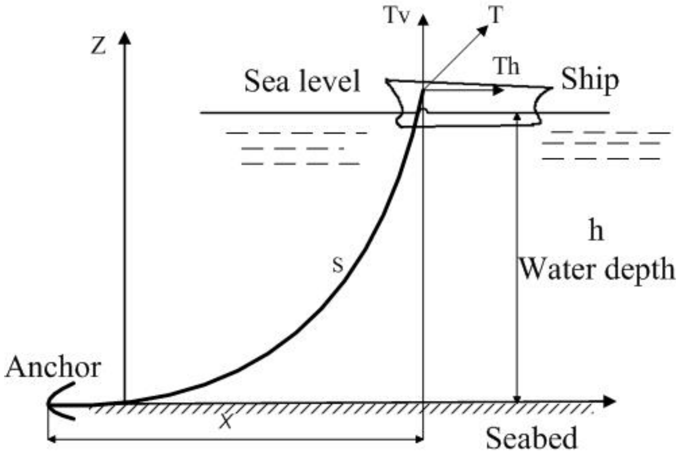

Fig. 2. Anchors and anchor chain force diagram

The equation of the mooring line horizontal tension acting on the rigid-body is as follows.

$$\begin{aligned} &T_H{}^6+EA(5-2x/s)T_H{}^5+(EA)^2\left[10-8x/s+(x/s)^2\right] \\ &\times T_H{}^4+(EA)^3\left[10-12x/s+3(x/s)^2\right]T_H{}^3 \\ &+(EA)^4(1-x/s)[5-3x/s]T_H{}^2 \\ &+(EA)^5(1-x/s)^2T_H-2\omega Z^3(EA)^5/9s^2=0 \end{aligned}$$

The vertical tension of mooring line acting on the rigid-body is as follows.

$$T_V=\sqrt{\frac{2\omega ZT_H}{1+T_H/EA}}$$

Where, T_H, T_V denote the level tension of the anchor chain, and the vertical tension, E is the chain's elastic modulus, A is the cross-sectional area of the anchor chain, Z is the height between hawse and the seabed, s is the length of anchor chain, x is the horizontal distance between hawse and anchor position, ω is the weight per unit length in water of anchor chain.

The model of the forces and moments acting on the rigid-body is as follows:

$$\begin{aligned} X_T &= T_H\cos(\psi-\theta_R) \\ Y_T &= T_H\sin(\psi-\theta_R) \\ Z_T &= T_V \\ N_T &= Y_T\times l \end{aligned}$$

Where, θ_R is the azimuth angle of the anchor chain, and l is the horizontal distance from the center of gravity of the hull to the hawse.

C. Environmental forces and moments

It is inevitably that the movement of moored ship on the sea is affected by the marine environment. Marine environmental disturbance include the wind, wave and current. In this paper, the wind and waves are mainly considered.

D. Environmental forces and moments

It is inevitably that the movement of moored ship on the sea is affected by the marine environment. Marine environmental disturbance include the wind, wave and current. In this paper, the wind and waves are mainly considered.

1) Mathematical model of wind: It assumes the role of wind as a constant wind. Acting on the hull, the wind forces and moments model are as follows[8].

$$X_{wind} = 0.5\rho_a A_f U_R{}^2 C_{wx}(\alpha_R)$$
$$Y_{wind} = 0.5\rho_a A_s U_R{}^2 C_{wy}(\alpha_R)$$
$$N_{wind} = 0.5\rho_a A_s L_{OA} U_R{}^2 C_{wn}(\alpha_R)$$
$$K_{wind} = Y_{wind} H_{LM}$$

Where, ρ_a is air density; L_{OA} is the total length of the vessel; A_f , A_s are the wind area front and the wind area side; α_R is the relative angle; H_{LM} is relative height.

2) Mathematical model of wave: The wave disturbance force is the most complex kind of disturbing force. The modified Pierson-Moskowitz spectrum which is recommended in 12th ITTC is used in this paper. First and second order wave forces are all considered.

The wave spectrum is as follows.

$$S(\omega) = \frac{A}{\omega^5}\exp\left(-\frac{B}{\omega^4}\right)$$

Where, $A = \dfrac{173 h_{1/3}{}^2}{T_1^4}$, $B = \dfrac{691}{T_1^4}$, $h_{1/3}$ is significant wave height, and T_1 is the peak period.

Irregular waves can be seen as the superposition of regular waves.

$$\zeta(t) = \lim_{n\to\infty}\sum_{i=1}^{n}\cos\left(k_i x - \omega_i t + \varepsilon_i\right)\sqrt{2\Delta\omega S(\omega_i)}$$

Where, $S(\omega_i)$ is wave power spectrum; ε_i is random phase; k_i is wave number; $\Delta\omega$ is the difference between two adjacent frequency.

a) First-order wave force model:

The shape of the vessel itself is complicated, in the calculation of the forces caused by wave, and the vessel can be simplified to cuboid. And ignore smaller force, we can acquire irregular wave mathematical model of interference forces and moments on the hull as follows[9,10]

$$X_w = -\sum_{i=1}^{n}\frac{4\rho g^3}{\omega_i^4\sin\chi}E_i\left(1\text{-}e^{-\frac{\omega_i^2 d}{g}}\right)\sin\frac{\omega_i^2 L\cos\chi}{2g}\sin\frac{\omega_i^2 B\sin\chi}{2g}$$
$$\sin\left(\left(\omega_i - \frac{\omega_i^2 V\cos\chi}{g}\right)t - \varepsilon_i\right)$$

$$Y_w = \sum_{i=1}^{n}\frac{4\rho g^3}{\omega_i^4\cos\chi}E_i\left(1\text{-}e^{-\frac{\omega_i^2 d}{g}}\right)\sin\frac{\omega_i^2 L\cos\chi}{2g}\sin\frac{\omega_i^2 B\sin\chi}{2g}$$
$$\sin\left(\left(\omega_i - \frac{\omega_i^2 V\cos\chi}{g}\right)t - \varepsilon_i\right)$$

$$Z_w = -\sum_{i=1}^{n}\frac{4\rho g^3}{\omega_i^4\sin\chi\cos\chi}E_i\left(1\text{-}e^{-\frac{\omega_i^2 d}{g}}\right)\sin\frac{\omega_i^2 L\cos\chi}{2g}$$
$$\sin\frac{\omega_i^2 B\sin\chi}{2g}\cos\left(\left(\omega_i - \frac{\omega_i^2 V\cos\chi}{g}\right)t - \varepsilon_i\right)$$

$$L_w = \sum_{i=1}^{n}\frac{4\rho g^3}{\omega_i^4\cos\chi}E_i\left(1\text{-}e^{-\frac{\omega_i^2 d}{g}}\right)\sin\frac{\omega_i^2 L\cos\chi}{2g}\sin\frac{\omega_i^2 B\sin\chi}{2g}$$
$$\sin\left(\left(\omega_i - \frac{\omega_i^2 V\cos\chi}{g}\right)t - \varepsilon_i\right)z_b - \sum_{i=1}^{n}\frac{2\rho g^2}{\omega_i^2\cos\chi}E_i\left(1\text{-}e^{-\frac{\omega_i^2 d}{g}}\right)$$
$$\sin\left(\frac{\omega_i^2 L\cos\chi}{2g}\right)\sin\left(\left(\omega_i - \frac{\omega_i^2 V\cos\chi}{g}\right)t - \varepsilon_i\right)$$
$$\left(\frac{2g^2\sin\left(\frac{\omega_i^2 B\sin\chi}{2g}\right)}{\omega_i^4\sin^2\chi} - \frac{Bg\cos\frac{\omega_i^2 B\sin\chi}{2g}}{\omega_i^2\sin\chi}\right)$$

$$M_w = \sum_{i=1}^{n}\frac{2\rho g^2}{\omega_i^2\sin\chi}E_i\left(1\text{-}e^{-\frac{\omega_i^2 d}{g}}\right)\sin\frac{\omega_i^2 B\sin\chi}{2g}$$
$$\sin\left(\left(\omega_i - \frac{\omega_i^2 V\cos\chi}{g}\right)t - \varepsilon_i\right)$$
$$\left(\frac{2g^2\sin\left(\frac{\omega_i^2 L\cos\chi}{2g}\right)}{\omega_i^4\cos^2\chi} - \frac{Lg\cos\frac{\omega_i^2 B\cos\chi}{2g}}{\omega_i^2\cos\chi}\right)$$

$$N_w = -\sum_{i=1}^{n}\frac{2\rho g^2}{\omega_i^2}E_i\left(1\text{-}e^{-\frac{\omega_i^2 d}{g}}\right)\sin\frac{\omega_i^2 B\sin\chi}{2g}$$
$$\cos\left(\left(\omega_i - \frac{\omega_i^2 V\cos\chi}{g}\right)t - \varepsilon_i\right)$$
$$\left(\frac{2g^2\sin\left(\frac{\omega_i^2 L\cos\chi}{2g}\right)}{\omega_i^4\cos^2\chi} - \frac{Lg\cos\frac{\omega_i^2 B\cos\chi}{2g}}{\omega_i^2\cos\chi}\right)$$

Where, χ is relative wave direction angle; z_b is the vertical distance from the hull load waterline to the center of gravity; E_i is amplitude of each discrete wave calculated as follows.

$$E_i = \sqrt{2\Delta\omega S_\zeta(\omega_i)}$$

b) Second-order wave force model:

Wave drift force will change the course and track of the vessel, especially has effect on the movement of the vessel's position. Irregular wave drift force also can be seen as the superposition of regular waves of various frequencies. The mathematical model is described as follows.

$$X_{wd} = \rho g L \cos\chi \sum_{i=1}^{n} C_{Xwd}(\lambda_i)\cdot S(\omega_i)\cdot\Delta\omega$$

$$Y_{wd} = \rho g L \sin\chi \sum_{i=1}^{n} C_{Ywd}(\lambda_i)\cdot S(\omega_i)\cdot\Delta\omega$$

$$N_{wd} = \rho g L^2 \sin\chi \sum_{i=1}^{n} C_{Nwd}(\lambda_i)\cdot S(\omega_i)\cdot\Delta\omega$$

Where, $\lambda_i = \dfrac{2\pi\omega_i^2}{g}$; $C_{Xwd}, C_{Ywd}, C_{Nwd}$ are empirical coefficient.

III. SIMULATION

The particulars of anchor chain are given in Table I, and the moored ship parameters are given in Table II.

TABLE I. PARTICULARS OF ANCHOR CHAIN

Designation	Unit	Quantity
Speke anchor	-	-
Water depth	m	100.0
Anchor chain length	m	220
Anchor weight in water	ton	6
Dry weight of chain	Kg/m	100
Diameter	cm	9.33
E·A	kg	107

TABLE II. MAIN PARAMETERS OF MOORED SHIP

Designation	Symbol	Unit	Quantity
Length	LOA	m	210.0
Breadth	B	m	26.2
Displacement	△	ton	18800
Draft	T	m	6.65
Center of gravity	KG	m	10.56
Block coefficient	Cb	-	0.86
Wind area front	Af	m^2	3001.5
Wind area side	Ab	m^2	700.4

Computer simulation are performed with wind direction; wave direction is 0 degree; wind speed is 10.3 m/s; the significant wave height is 1.5m; period of wave is 6.8s. According to the simulation analysis of moored ship, the curves of ship motion trajectory and the mooring line tension are obtained.

From the curves of Fig. 3, the trajectory of the ship in the disturbance of wind, wave and current is eventually stabilized at a "∞" shape in the case of position mooring. The yaw angle changes cyclically in the range of $\pm 30^{\circ}$. The 6DOF motion of the moored ship are obtained.

As shown in Fig. 4 - Fig. 6, the moored ship will produce obvious pitch and heave motion with the disturbance of marine environment.

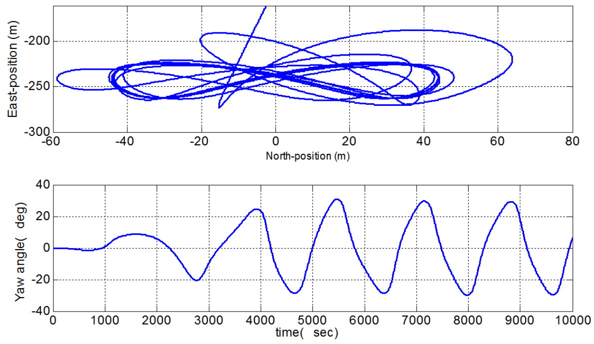

Fig. 3. The trajectory and yaw angle of the mooring ship

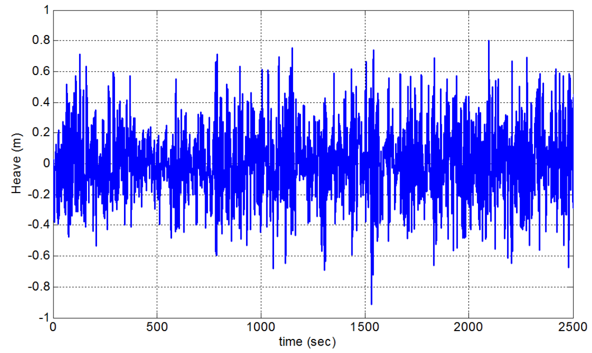

Fig. 4. The vertical position of the mooring ship

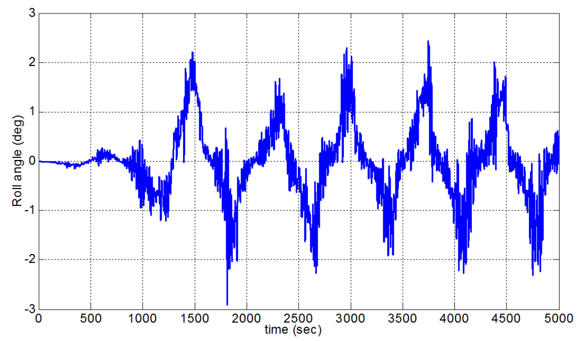

Fig. 5. The roll angle variation of the mooring ship

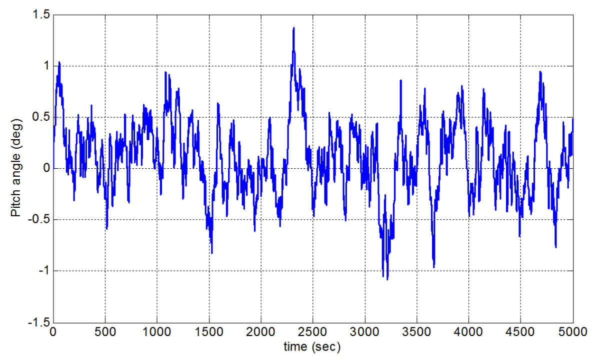

Fig. 6. The pitch angle variation of the mooring ship

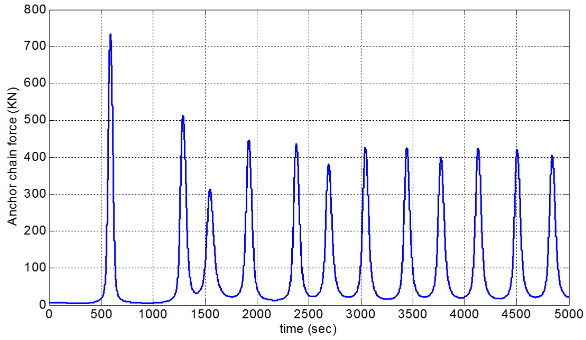

Fig. 7. The simulation result of anchor chain force

The simulation result of anchor chain tension is shown in Fig. 7. Mooring line force varies cyclically.

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