

Tracking High-speed Source Based on Moving Source Acoustic Field Model in Shallow Ocean Environment

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Abstract—The model-based approach was usually used to iteratively update the source location and acoustic field parameters to overcome the mismatch problem that plagued the matched-field processing (MFP) method in target tracking and localization in an ocean environment. But there are several drawbacks in the existing work such as low-speed target tracking limitation and large computation burden. A source tracking method based on acoustic field model of a moving source is proposed to tracking high-speed target. And the application of the unscented Kalman filter (UKF) as the state estimator reduces the computational complexity. Computer simulations and experimental data processing under a typical shallow ocean environment are performed, indicating the effectiveness of the proposed method.

Keywords—source tracking; shallow ocean; moving source acoustic field model; UKF; SWellEx-96

I. INTRODUCTION

The matched-field processing (MFP) method was usually used to accomplish the target tracking and localization in an ocean environment [1-4]. But the MFP method was plagued by the mismatch problem mainly because of the uncertainty in ocean environment parameters or target moving [5-7]. The model-based approach was then proposed to iteratively update the source location and acoustic field parameters to overcome the mismatch problem [8-10]. However, there are several drawbacks in the existing work. The acoustic field model used is static model which leads to low-speed target tracking limitation, for example 5m/s or lower speed. The adoption of particle filter (PF) as the estimator bears large computation burden, therefore, it is not conducive to online processing. And the model assumes that the target only moves horizontally, which is usually not the actual situation. A source tracking method based on acoustic field model of a moving source is proposed in this paper to solve the above problems.

The usually used acoustic field model generally considers the condition of fixed sources and fixed receivers. Among the earlier work reported on the problem of solving the wave equation for a source moving in the acoustic waveguide formed by the ocean and its boundaries, notable results are the work of Guthrie et al. [11] and Clark, Flanagan, and Weinberg [12].

Guthrie et al. considered a source moving radially with respect to a stationary receiver and obtained a formal expression for the acoustic field from normal mode theory. Clark et al. formulated the moving source problem in terms of ray theory, each ray path having a different Doppler shift according to its emission angle. These authors also presented the results of some specific numerical computations showing Doppler shifts, Doppler spreading, and other properties of the acoustic field.

Hawker's work [13] contained a generalization of the problem considered by Guthrie et al. to the important nonradial case and derived the acoustic field of a slowly moving point source with constant horizontal velocity in a uniform waveguide. The acoustic field for sources moving with arbitrary but small horizontal velocities, a more general result, was obtained by Lim et al. [14,15]. Hawker's solution, as is the case for most solutions in acoustics, is given in terms of contemporary time, i.e., the time at which the sound reaches the receiver. However, Lim et al. solves the wave equation via a temporal Green's function, and expresses the ensuing acoustic field in terms of retarded time, i.e., the time at which energy was radiated by the source.

II. TARGET TRACKING ALGORITHM BASED ON MOVING SOURCE ACOUSTIC FIELD MODEL

A. Acoustic Field Model of a Moving Source

For the case of a point source moving with arbitrary velocity and fixed center frequency ω_0 , the acoustic field is given in the following form

$$p(r, z, t) = j\sqrt{2\pi}e^{-j\left(\omega_0 t + \frac{\pi}{4}\right)} \sum_n \frac{\phi_n^0(z)\phi_n^0(z_s(t'))}{\sqrt{k_n^0(t')}} e^{jk_n^0 D(t')}, \quad (1)$$

where t' is the retarded time

$$t' = t - D(t')/c_n^0, \quad (2)$$

and superscripts 0 indicate functions (ϕ_n , c_n and k_n) evaluated at frequency ω_0 . $\phi_n(z)$ is the n th modal function varying with depth. c_n and k_n represent the group velocity and horizontal wavenumber of the n th normal mode, respectively. The term z_s is the source depth and D is the horizontal source-receiver distance.

Decomposing the source velocity into the horizontal velocity v_r towards the receiver and the vertical velocity v_z downwards, ignoring the higher-order terms in the amplitude factor, and a more concise expression of the acoustic field can be derived

$$p(r, z, t) = j\sqrt{2\pi}e^{-j\left(\omega_0 t + \frac{\pi}{4}\right)} \sum_n \frac{\phi_n^0(z) \phi_n^0(z_s(t))}{\sqrt{k_n^0 D(t)}} e^{jk_n^0 D(t)(1+v_r/c_n^0)}. \quad (3)$$

Assuming the change in source position is small from time t_1 to t_2 , and the source depth and distance are z_s^0 and D^0 at time t_1 , then the frequency-domain form is more convenient to handle

$$\begin{aligned} p(r, z, \omega) &= \int_{t_1}^{t_2} p(r, z, t) e^{-j\omega t} dt \\ &= j\sqrt{2\pi}e^{-j\frac{\pi}{4}} \sum_n \frac{\phi_n^0(z) \phi_n^0(z_s^0)}{\sqrt{k_n^0 D^0}} e^{jk_n^0 D^0(1+v_r/c_n^0)} \\ &\quad \times \frac{e^{j t_2 [k_n^0 v_r (1+v_r/c_n^0) - \omega - \omega_0]} - e^{j t_1 [k_n^0 v_r (1+v_r/c_n^0) - \omega - \omega_0]}}{j [k_n^0 v_r (1+v_r/c_n^0) - \omega - \omega_0]} \end{aligned} \quad (4)$$

B. State Space Model

Define the state vector as $\mathbf{x} = [r, z, v_r, v_z, a_r, a_z]^T$, with the state variables indicating horizontal source distance, source depth, horizontal velocity, vertical velocity, horizontal acceleration and vertical acceleration, respectively. The continuous state space model can be represented as follow

$$\begin{aligned} \frac{d\mathbf{x}(t)}{dt} &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{w}(t), \quad (5) \\ &= \mathbf{F}\mathbf{x}(t) + \mathbf{L}\mathbf{w}(t) \end{aligned}$$

where $\mathbf{F}$ and $\mathbf{L}$ are constant matrices. $\mathbf{x}(t)$ represents the state of the target at time t , and $\mathbf{w}(t)$ is the process noise assumed to be white and with the power spectral density

$$\mathbf{Q}_c = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}, \quad (6)$$

where q_1 and q_2 represent the disturbance values in horizontal and vertical acceleration, respectively.

For the continuous wiener process acceleration model shown in (5), the discrete state transition matrix and the covariance matrix of the process noise can be derived through the following equation [16,17]

$$\begin{aligned} \mathbf{A}_{k-1} &= \exp(\mathbf{F}\Delta t) \\ \mathbf{Q}_{k-1} &= \int_0^{\Delta t} \exp(\mathbf{F}(\Delta t - \tau)) \mathbf{L} \mathbf{Q}_c \mathbf{L}^T \exp(\mathbf{F}(\Delta t - \tau))^T d\tau, \quad (7) \end{aligned}$$

with $\Delta t = t_k - t_{k-1}$ denoting the time interval.

The final form of the state transition matrix and process noise covariance matrix can be obtained through derivation and are presented below

$$\mathbf{A}_{k-1} = \mathbf{A} = \begin{bmatrix} 1 & 0 & \Delta t & 0 & \frac{1}{2}\Delta t^2 & 0 \\ 0 & 1 & 0 & \Delta t & 0 & \frac{1}{2}\Delta t^2 \\ 0 & 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (8)$$

and

$$\begin{aligned} \mathbf{Q}_{k-1} &= \mathbf{Q} \\ &= \begin{bmatrix} \frac{1}{20}\Delta t^5 q_1 & 0 & \frac{1}{8}\Delta t^4 q_1 & 0 & \frac{1}{6}\Delta t^3 q_1 & 0 \\ 0 & \frac{1}{20}\Delta t^5 q_2 & 0 & \frac{1}{8}\Delta t^4 q_2 & 0 & \frac{1}{6}\Delta t^3 q_2 \\ \frac{1}{8}\Delta t^4 q_1 & 0 & \frac{1}{6}\Delta t^3 q_1 & 0 & \frac{1}{2}\Delta t^2 q_1 & 0 \\ 0 & \frac{1}{8}\Delta t^4 q_2 & 0 & \frac{1}{6}\Delta t^3 q_2 & 0 & \frac{1}{2}\Delta t^2 q_2 \\ \frac{1}{6}\Delta t^3 q_1 & 0 & \frac{1}{2}\Delta t^2 q_1 & 0 & \Delta t q_1 & 0 \\ 0 & \frac{1}{6}\Delta t^3 q_2 & 0 & \frac{1}{2}\Delta t^2 q_2 & 0 & \Delta t q_2 \end{bmatrix}. \quad (9) \end{aligned}$$

Therefore the discrete form of the state equation is

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{w}_{k-1}, \quad (10)$$

where $\mathbf{w}_{k-1}$ is the process noise with the covariance matrix of $\mathbf{Q}$.

The measurement equation is

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k = s_k \mathbf{d}(\mathbf{x}_k) + \mathbf{v}_k, \quad (11)$$

where s_k denotes the unknown signal amplitude and phase. $\mathbf{d}(\bullet)$ is the acoustic field propagation function given by (4). The process noise $\mathbf{w}$ and measurement noise $\mathbf{v}$ satisfy the following conditions

$$\begin{aligned} E\{\mathbf{w}_k \mathbf{w}_k^T\} &= \mathbf{Q} \delta_{ki}, \\ E\{\mathbf{v}_k \mathbf{v}_k^T\} &= \mathbf{R} \delta_{ki}, \\ E\{\mathbf{v}_k \mathbf{w}_i^T\} &= \mathbf{0}, \quad \forall i, k. \end{aligned} \quad (12)$$

The real center frequency of the received signal is unknown due to the movement of the target. We expand the measurement equation around the center frequency f_0 as follow to solve this problem

$$\begin{bmatrix} \mathbf{y}_k(f_1) \\ \vdots \\ \mathbf{y}_k(f_{n_f}) \end{bmatrix} = \begin{bmatrix} s_k(f_1) \mathbf{d}(\mathbf{x}_k, f_1) \\ \vdots \\ s_k(f_{n_f}) \mathbf{d}(\mathbf{x}_k, f_{n_f}) \end{bmatrix} + \begin{bmatrix} \mathbf{v}_k(f_1) \\ \vdots \\ \mathbf{v}_k(f_{n_f}) \end{bmatrix}, \quad (13)$$

where n_f is normally odd and $f_{(n_f+1)/2} = f_0$.

In practice, the complex amplitude of the signal is usually unknown, and it can be replaced by its maximum likelihood estimation (MLE) $\hat{s}_k$ when the measurement data is available [18]. The likelihood function of the k th step is

$$L(\mathbf{x}_k) = \prod_{i=1}^{n_f} \frac{1}{(\pi v_i)^{n_h}} \exp \left[-\frac{|\mathbf{y}_k(f_i) - s_k(f_i) \mathbf{d}(\mathbf{x}_k, f_i)|^2}{v_i} \right], \quad (14)$$

and n_h is the number of hydrophones, v_i is the variance of the noise at frequency f_i . With $\partial L / \partial s_k = 0$, the MLE of s_k can be obtained

$$\hat{s}_k(f_i) = \frac{\mathbf{d}(\mathbf{x}_k, f_i)^H \mathbf{y}_k(f_i)}{|\mathbf{d}(\mathbf{x}_k, f_i)|^2}. \quad (15)$$

With the state equation (10) and the measurement equation (13) derived, the unscented Kalman filter (UKF) can be applied as the state estimator to reduce the computational complexity.

III. SIMULATION AND EXPERIMENTAL DATA ANALYSIS

A. Computer Simulation

Considering a typical shallow ocean environment with a vertical line array (VLA) measurement system, we assume a flat bottom, range independent, three-layered ocean environment with a depth of 80 meters and the VLA contains 15 hydrophones spaced 5 meters apart ranging from 5 meters

to 75 meters in the water column. The detailed ocean environment is shown in Fig. 1.

The point source is narrowband at frequency 100Hz. The average SNR at the hydrophone is from -10dB to -13dB because the source is moving away. The process noise power in the UKF iteration are $q_1 = 1 \times 10^{-3}$ and $q_2 = 1 \times 10^{-5}$. The initial value of the error covariance matrix of the state estimate is set to be $10^{-3} \times \text{diag}([1, 0.5, 0.5, 0.5, 0.1, 0.1])$. The target trajectory after 300 UKF iterations is shown in Fig. 2. The true and estimated velocities and accelerations are shown in Fig. 3 and Fig. 4. It can be seen that the position and the horizontal motion properties of the target are estimated precisely, and the vertical motion properties are a little less accurate. This is because the vertical motion properties are not much related to the acoustic pressure measurements as shown in the forward model (4), and therefore they cannot be effectively corrected according to the measurements. However, the effect is negligible on the overall tracking performance.

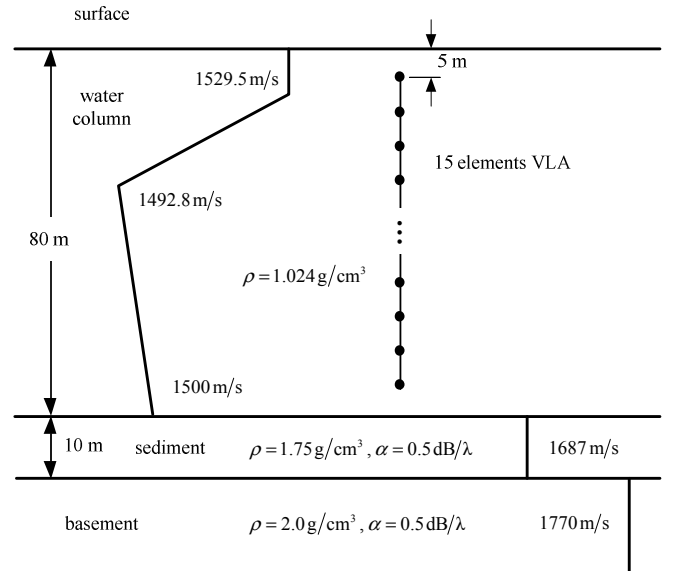


Fig. 1. Typical shallow ocean environment with parameter specifications.

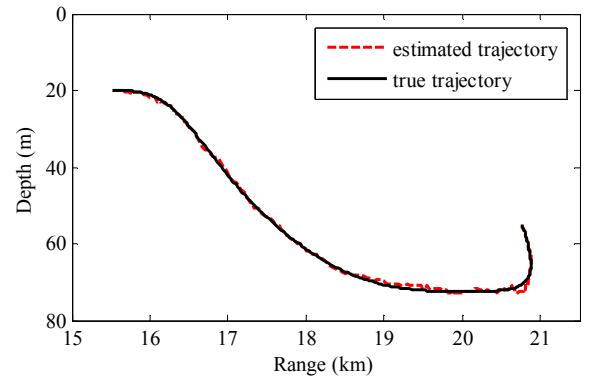


Fig. 2. Estimating the true source trajectory using simulated data.

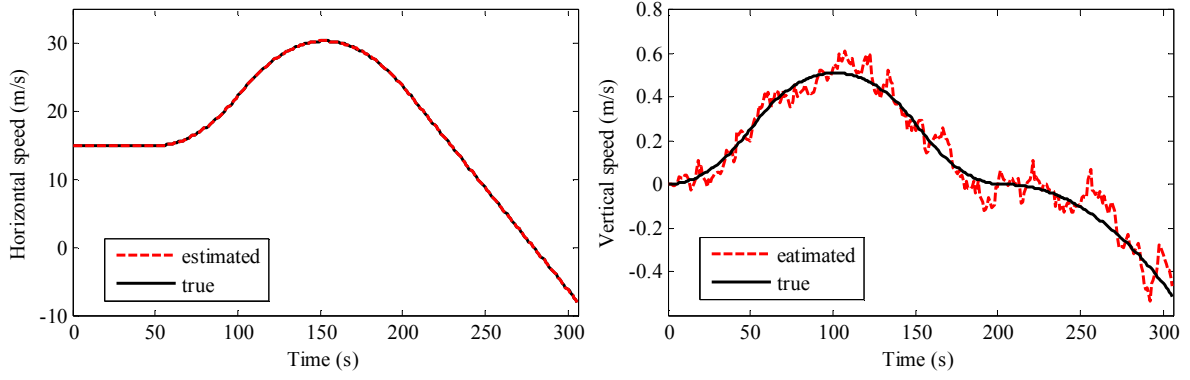


Fig. 3. The true and estimated velocities of the source.

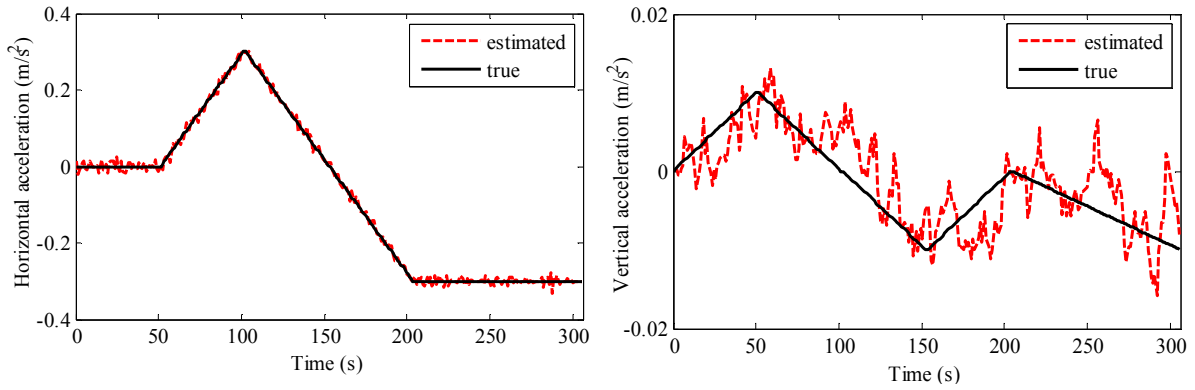


Fig. 4. The true and estimated accelerations of the source.

The forward model runs are the most computationally intensive section in a tracing problem. The tracking performance of the UKF is similar to the 200-particle PF [9]. The UKF estimator needs 13 forward model runs per iteration step and a total of 3900 runs in this case, while the 200-particle PF needs a total of 60000 runs. It is obvious that using UKF as the state estimator greatly reduces the amount of calculation.

B. Experimental Data Processing

The data collected from the SWellEx-96 experiment is used to evaluate the performance of the proposed method. The SWellEx-96 experiment was conducted between May 10 and 18, 1996, approximately 12km from the tip of Point Loma near San Diego, California [19]. This site is a shallow-water channel approximately 200 meters in depth with a relatively flat bottom and a downward refracting sound-speed profile.

The S5 Event consists of two sources towed along an isobath and is devoid of loud interferers. The 109-Hz frequency data transmitted by the shallow source towed at a depth of about 9 meters is processed. The receiver is a VLA consists of 21 hydrophones. The environmental parameters and the array configuration are shown in Fig. 5.

Twenty-minute data processing results are shown in Fig. 6.

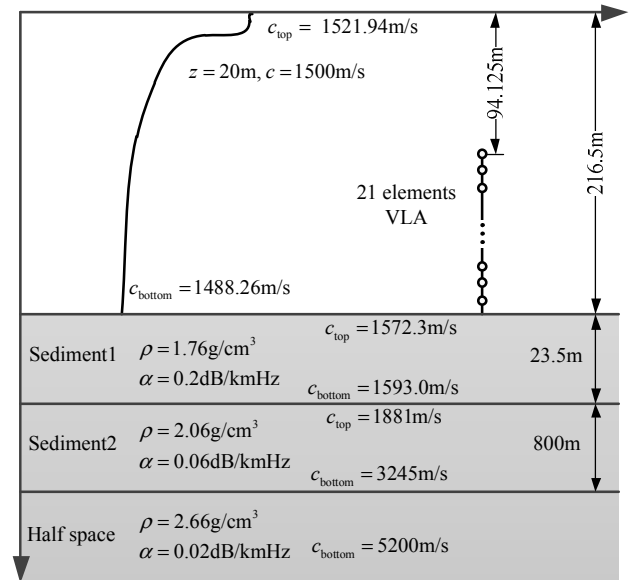


Fig. 5. Environmental configuration of the SWellEx-96 experiment.

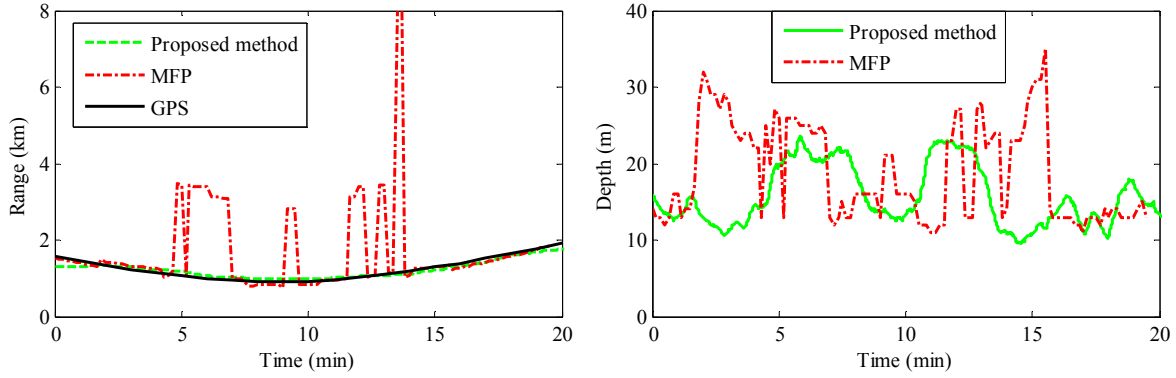


Fig. 6. Range and depth tracking results of MFP and proposed method using SWellEx-96 experimental data.

In the twenty-minute data of S5 Event, a total of 2400 time-domain snapshots are generated for each sensor. Each snapshot consists of 1500 points, occupying 1 second, with 50% overlap between successive snapshots. An 8192-point FFT is performed on each snapshot to generate the frequency-domain data vectors. The tracking performance of the MFP method is also given in Fig. 6 for comparison. It can be seen that the MFP method fails sometime while the proposed method can effectively estimate the position of the target, showing that the proposed method has a better tracking performance.

IV. CONCLUSIONS

This paper proposed a source tracking method based on moving source acoustic field model and expanded measurement equation. The method uses the UKF as the state estimator to avoid the extra computational burden of the PF. Computer simulations under a typical shallow ocean environment are performed, indicating the effectiveness of the proposed method. Experimental data processing results also show the effectiveness of the proposed method.

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