# Track Analysis and Design for Ultra Short Baseline Installation Error Calibration

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***Abstract* — The Ultra Short Baseline (USBL) positioning system calibration performance relates to its data collection track, so a proper track design must be an essential research. This paper firstly studies the calibration performance of the arc track and the line track, which are basic components of complex tracks. Then it is found that contradiction exists between the solvability of the installation angle error and no sound velocity caused error. If solvability is guaranteed, it can be analyzed that sound velocity caused errors have more influence on roll installation angle and less effect on pitch installation angle in arc tracks, while line tracks take the opposite. Therefore, a track combined with the arc and line is designed to make compensation. Simulation and field data application show that the combined track can reduce the effect of the sound velocity error to both roll and pitch installation angel. Compared with the arc track, the combined track calibration results is more robust and precise.**

***Keywords* — *Ultra-short Base Line (USBL); installation error calibration; track design***

## I. INTRODUCTION

Because of the convenience in application and the high accuracy in short distance positioning, Ultra Short Baseline (USBL) positioning system is widely used in ocean observation, offshore oil engineering and other relative fields [4] Generally, USBL transducer array is installed in through-hull or over-the-side style by deployment structures. Installation will generate angle bias between the array and the ship coordinates, which decreases the positioning accuracy [1]-[4]. Former research shows that 1° heading installation angle generates 1.7% slant range positioning error [1]-[4]. So USBL must take installation error calibration procedure to fix system bias before offering positioning service. During the procedure, a ship sails round the underwater transponder with a predesigned track to collect calibration data. The quality and error of the calibration results relate to the track deeply. So tracks should be properly designed to reduce the calibration error. Based on this requirement, USBL calibration track was analyzed and designed in this paper.

First of all, the calibration performances of the arc and line track, which are basic components of complex tracks, are studied. Then, by contrast between the performances, a track combined with the both is designed to make compensation and reduce calibration error. Finally, the improvement in calibration robustness and positioning accuracy is proved by simulation and field data applications.

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## II. USBL CALIBRATION THEORY

In USBL calibration data processing, the transponder position and the effective sound velocity between the transponder and the array are firstly calibrated [4][5]. Its observation equation is based on Inverted Long Baseline (ILBL) principle:

$$\left\| \mathbf{X}_{Transponder}^{Ground} - \mathbf{X}_{Array}^{Ground} \right\| = C \cdot T \qquad (1)$$

Where $\mathbf{X}_{Array}^{Ground}$ is the geographical position of array measured by GPS. $T$ represents the time delay between the transponder and the array measured by USBL. $\mathbf{X}_{Transponder}^{Ground}$ is the geographical position of transponder and $C$ is the effective sound velocity, which are parameters to be calibrated. The essence of (1) is ranging-only method treating the track as measurement baseline. If the baseline is chose improperly, (1) will be insolvable. Thus, the track does make significant sense to calibration.

Taking the calculated transponder position and effective sound velocity as known parameters, USBL installation angle can be calibrated by USBL positioning equation [4][5]:

$$\mathbf{X}_{Transponder}^{Ground} - \mathbf{X}_{Array}^{Ground} = \mathbf{R}_{Ship}^{Ground} \mathbf{R}_{Array}^{Ship} \mathbf{X}_{Transponder}^{Array} \qquad (2)$$

Where $\mathbf{R}_{Ship}^{Ground}$ is the rotation array between geographical coordinate and the ship coordinate calculated by gyro data. $\mathbf{X}_{Transponder}^{Array}$ is the transponder position in array coordinate measured by USBL. $\mathbf{R}_{Array}^{Ship}$ is the rotation array between ship coordinate and the array coordinate, which needs to be calibrated. $\mathbf{X}_{Array}^{Ground}$, the parameters represents ship track, exists in (2), that means the track also relates to the angle calibration.

## III. CALIBRATION PERFORMANCE OF THE ARC TRACK AND THE LINE TRACK

Ship tracks can be decompounded into different. arc and line tracks. So study on the character of these basic tracks is helpful with the calibration track design.

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### A. Arc Track Calibration Performances

In the arc track, a ship sails round the transponder, which is the center of the arc, with a uniform circular motion, as Fig. 1(a) shows. Taking any observation point in the track for example, (1) can be expanded as:

$$\left[\left(x_{Transponder}^{Ground}-x_{Array}^{Ground}\right)^2+\left(y_{Transponder}^{Ground}-y_{Array}^{Ground}\right)^2\right]/C^2 +\left(z_{Transponder}^{Ground}-z_{Array}^{Ground}\right)^2/C^2=T^2 \quad (3)$$

According to the effective sound velocity theory [6]-[8], $C$ is the confirmed by transponder depth, array depth and the horizontal distance between them. For arc track, all the parameters above are all the same, so $C$ is a constant in this track. Furthermore, the depth difference between array and transponder is also a constant. Thus, $\left(z_{Transponder}^{Ground}-z_{Array}^{Ground}\right)/C$ is same for each observation point, which means (3) is not a full rank equation. So transponder depth and the effective sound velocity cannot be calculated simultaneously. Assume that horizontal distance between transponder and array is a constant, $z_{Transponder}^{Ground}$ and the geometric angle between transponder and array will increase when $C$ decreases and vice versa. There is only one effect sound velocity corresponding to the real transponder depth and the real geometric angle. If transponder's depth is known, (3) has a unique answer. And the solved $C$ can convert measured incidence angles into geometric angles between the transponder and the array plane.

Divided by the geometric distance between transponder and array in both sides, (2) can be switched to an angle transform equation:

$$\cos\mathbf{\Theta}_{Transponder-Array}^{Ground}=\mathbf{R}_{Ship}^{Ground}\mathbf{R}_{Array}^{Ship}\cos\mathbf{\theta}_{Transponder-Array}^{Array} \quad (4)$$

Where $\mathbf{\theta}_{Transponder-Array}^{Array}$ represents the angle vector between the transponder and the array plane. $\mathbf{\Theta}_{Transponder-Array}^{Ground}$ represents the geometric angle vector between the transponder and the surface. Based on the analysis above, $\mathbf{\theta}_{Transponder-Array}^{Array}$ can be converted to geometric angle by $C$ in any observation points of the arc track. Therefore sound velocity doesn't cause module error in arc tracks.

Because of the central symmetry of arc tracks, all the observation data is almost the same, which cause the singularity of coefficient matrix in the installation angle calibration equation. So arc tracks cannot calibrate installation angles.

### B. Line Track Calibration Performances

In the line track, a ship sails through the transponder in a settled direction with uniform line motion, as Fig. 1(b) shows. Thus, (1) can be expanded as:

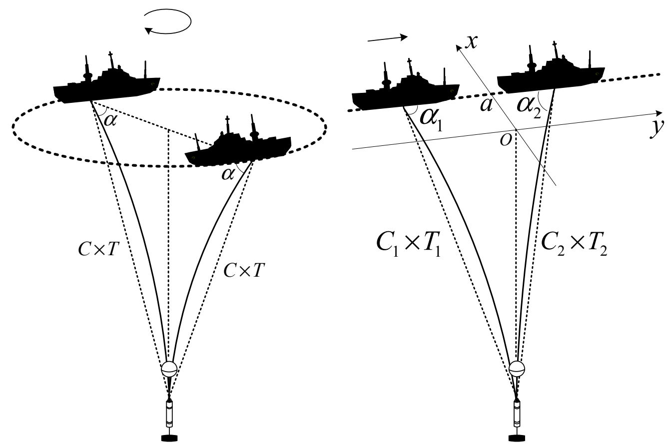


Fig. 1. The arc and line track

$$\left(x_{Transponder}^{Ground}-a\right)^2+\left(y_{Transponder}^{Ground}-y_{Array}^{Ground}\right)^2 +\left(z_{Transponder}^{Ground}-z_{Array}^{Ground}\right)^2=C^2T^2 \quad (5)$$

Here component in depth and the component in $x$ axis satisfy a round equation, so $x_{Transponder}^{Ground}$ and $z_{Transponder}^{Ground}$ cannot be calculated simultaneously.

Relative distance between observation points and the transponder vary in line track, so the coefficient matrix in (4) is no more singular. The installation angle can be calibrated now. However, because of the variety of the horizontal distance between the transponder and the array, the effective sound velocity is different in each observation point. Assume $z_{Transponder}^{Ground}$ is known and then $C$ calculated by (5) is a Minimum Mean-square Error (MMSE) estimated result. For each observation point, there is residual in sound velocity, which means measured incidence angles cannot be fixed into geometric angles between the transponder and the array plane. So sound velocity caused module error exits in line tracks, which generates the installation angle calibration bias.

## IV. USBL CALIBRATION TRACK DESIGN

The analysis in Heading 3 indicates that no sound velocity caused error exists in the arc track, but installation angles cannot be calibrated; the line track can calculate the angle while its accuracy is decreased by sound velocity error. It can be concluded from the contrast of the two tracks that contradiction exists between the solvability of the installation error angle calculation equation and no sound velocity caused angle error, in which the former must be a chief one. So if no ray tracing theory is involved, sound velocity caused error must affect the accuracy of installation angle calibration, which influence can only be reduced but cannot be removed.

The study of calibration error generated by sound velocity is as follows:

According to USBL principle [1]-[3], the influence of sound velocity to measured incidence angle can be expressed as (6) :

$$\mathrm{d}\theta = \frac{\cot\theta}{C}\mathrm{d}C \tag{6}$$

Where $\theta$ is the incidence angle measured by USBL. By (6), it is known that the angle error is a monotone decreasing function of $\theta$, which changes with the angle relative to USBL array. Let $\theta_y$ is the angle between the transponder and ship's $y$ axis and $\theta_x$ is the angle between the transponder and ship's $x$ axis. Assume the error of $\theta_y$ measurement is $\Delta\theta_y$, then the measured angle between the transponder and the array's $y$ axis is the same as which measured with a $\Delta\theta_y$ installation angle in roll. Therefore, calibration error in roll is introduced by measurement error in $\theta_y$. Similarly, if $\Delta\theta_x$ is the error of $\theta_x$, the measured angle between the transponder and the $x$ axis of array is the same as that measured with a $\Delta\theta_x$ installation angle error in pitch. So calibration error in roll can be generated by measurement error in $\theta_x$.

For the arc track, $y$ axis of array is always towards the horizontal position of the transponder, and $x$ axis of array is nearly perpendicular to the line between the array and the transponder. Hence, $\theta_y$ is almost $0^\circ$ while $\theta_x$ is almost $90^\circ$. So sound velocity error affect $\theta_y$ much more than $\theta_x$, which means in the arc track calibration error in roll is bigger than that in pitch.

For the line track, the angle between transponder and any axis of the array varies during the sailing. Sound velocity error will influent both pitch and roll installation angle. Especially, if the ship sails towards the transponder, namely the line track goes through the horizontal position of the transponder, $\theta_x$ is close to $0^\circ$ while $\theta_y$ changes while the ship moves. So the calibration error character of this line track is opposite to the arc track.

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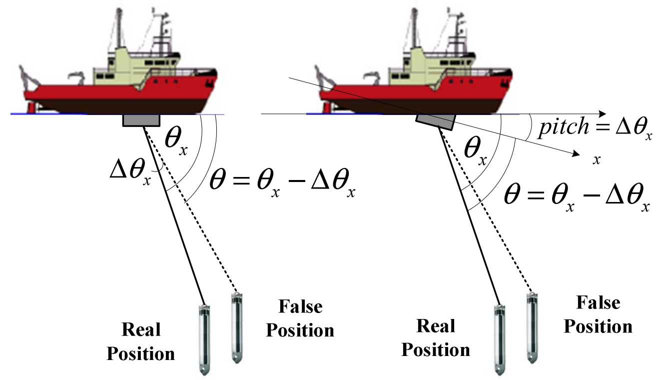



(a) Positioning with angle measured error $\Delta\theta_x$

(b) Positioning with pitch error angle $\Delta\theta_x$

Fig. 2. Angle measurement error causes calibration error

Compared with the calibration performances of tracks mentioned above, it is found that disadvantage of the two track can compensate each other. Therefore the arc and the line track are considered to be combined together to improve the calibration accuracy. Calibration algorithm is based on MMSE theory, which will ensure the minimum mean-square of the residual in every observation point. Under this theory, the calibration results of the combined track lie between results calibrated by two tracks separately. Taking roll installation error angle for example, let the calibration result of the line track is $\gamma$, and the arc track's is $\gamma + \Delta\gamma$. Then the combined track will present a calibration result as $\hat{\gamma} \in (\gamma, \gamma + \Delta\gamma)$. It can be deduced that the combined track calibration result affected by sound velocity errors is less than that calibrated respectively by two tracks in both pitch and roll installation angle.

To prove the performance of the combined track, simulation was executed as follows: Let USBL array sink into the water with 1m depth and the transponder is 45m under the water. The sound velocity profile used in simulation shows as Fig. 3.

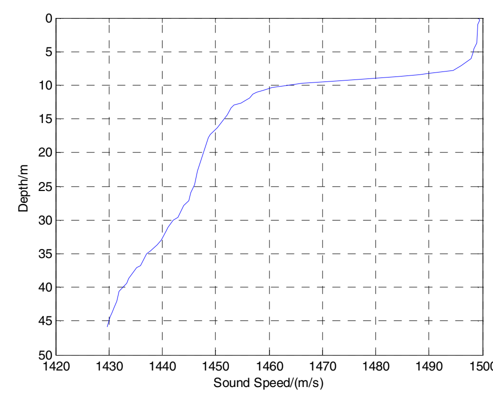



Fig. 3. Sound velocity profile used in simulation

Three kinds of tracks were tested in simulation: (a) an oval track with 50m long semi-axis and 49m short semi-axis; (b) a 100m line track; (c) tracks combined with (a) and (b). In the simulation, the transponder is under the track center and the algorithm is based on MMSE. The calibration accuracy of the three tracks is listed in Table I.

TABLE I USBL CALIBRATION ERROR

| Track | Calibration error in roll/° | Calibration error in pitch/° |
|---|---|---|
| Oval Track | 0.44 | 0 |
| Line Track | 0 | 0.27 |
| Combined Track | 0.16 | 0.13 |

Simulation results shows that the combined track calibration error in roll smaller than that of the oval track, the same as the calibration error in pitch compared with the line track. It proves that the combined track will reduce the calibration error caused by sound speed. Besides, the transponder depth and another parameter can be simultaneously calculated, so the combined track is more suitable for USBL calibration. Combined track shows no central symmetry, which will cause singularity in the equation, thus calibration robustness will be promoted.

To sum up, USBL calibration track is designed as tracks combined with line and round, as Fig. 4 shows. It is composed of (a) a round track whose center is the transponder horizontal position and (b) two line tracks, which is perpendicular to each other, lapped over the diameter. Involving two line tracks is to increase the symmetry and the uniformity of the error distribution.

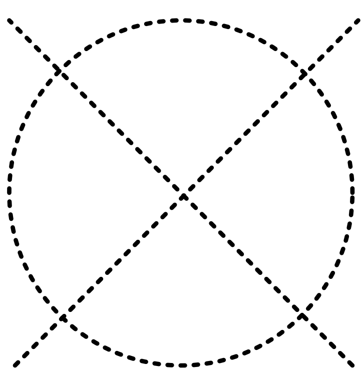


Fig. 4. Designed USBL calibration track

## V. APPLICATION TO THE FIELD EXPERIMENT DATA

October 2012, we did USBL installation error calibration experiment in Songhua Lake, Jilin Province, China. The tested facilities included a self-made USBL system, an acoustic transponder, an IxSea OCTANS gyro and a Leica RTK-GPS. The USBL array was installed in over-the-side style with 2m depth. OCTANS was installed near the gravity center of the ship. GPS was fastened above the array with a pole. Transponder was anchored to 50m depth. The sound speed profile during experiment shows in Fig. 5.

In the experiment, the ship firstly took four 90m radius arc tracks, then line tracks in two different orientations, shows in Fig. 6. The transponder was just under the center of the track.

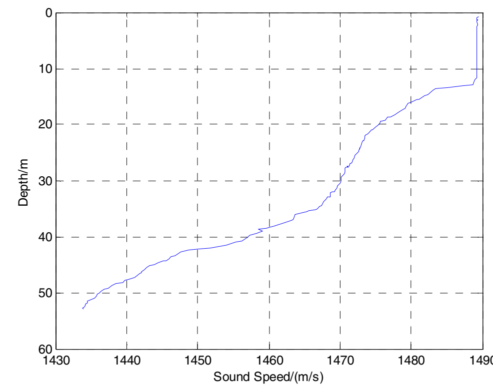



Fig 5 Sound velocity profile during experiment

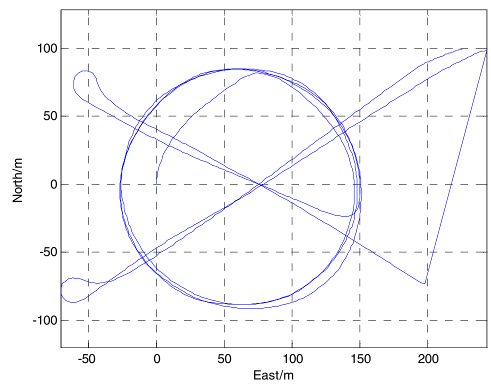



Fig 6 Calibration track in experiment

Interception and combination were taken before calibration data processing. Tracks in Fig. 6 were intercepted into three kinds: four arc tracks, two line tracks with 45° orientation and two line tracks with 135° orientation. Only one track was selected in each of the three kinds, and finally got four different combined tracks. We processed data of four combined tracks, four round tracks and line tracks. The calculation results are listed in Table 2. Because equation singularity occurred in line tracks, the results are not included in the table.

By comparison in Table II, it is found that repetition is excellent for the combined tracks' results. Its standard deviation is smaller than that of the arc tracks. Moreover the calibration results are different between two kinds of tracks. To check which one is correct, we used these results to fix the USBL position scatters. If the position scatter is better in density and precision, the calibration results are more close to the real installation error.

TABLE II CALIBRATION RESULTS OF DIFFERENT TRACKS

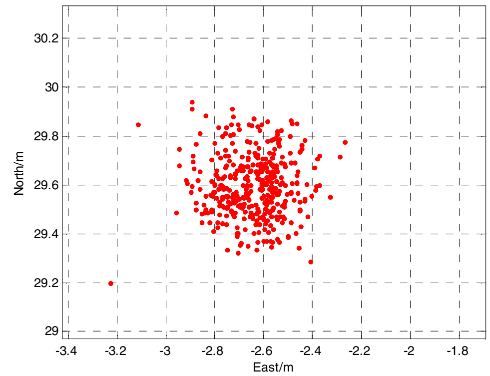
| /° | Combined Track | | | Arc Track | | |
|---|---|---|---|---|---|---|
| | $\alpha$ | $\beta$ | $\gamma$ | $\alpha$ | $\beta$ | $\gamma$ |
| 1 | 1.07 | -0.39 | 2.52 | 0.14 | 0.15 | 2.72 |
| 2 | 1.08 | -0.40 | 2.54 | 0.16 | 0.14 | 2.71 |
| 3 | 1.09 | -0.40 | 2.54 | 0.19 | 0.21 | 2.72 |
| 4 | 1.09 | -0.42 | 2.42 | 0.22 | 0.21 | 2.81 |
| Std | 0.01 | 0.01 | 0.06 | 0.04 | 0.04 | 0.05 |

Fixed USBL positioning scatter is shown in Fig. 7. Fig. 7 (a) is the scatter fixed by the combined track's results. The scatter is even and dense with a precision of 0.4% slant range. Fig. 7 (b) is the one fixed by the arc track's results. The scatter is abnormally hollow. Experiment results mean that arc track's calibration results cannot fix USBL installation error completely. So the performance of combined tracks can be well proved.

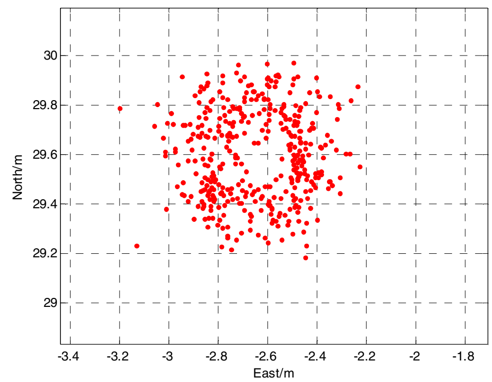
## VI. DISCUSSION AND SUMMARY

This paper mainly designs and analyzes the calibration track of USBL positioning system. Firstly, the character of simple tracks, arc and line tracks, is studied. (1) The transponder's depth and the effective sound velocity cannot be calculated simultaneously in the arc track. If transponder's depth is known, the calculated effective sound velocity can fix the measured direction angle to the geometric angle between the transponder and USBL array. Moreover, the coefficient matrix of the installation angle error calibration equation is singular in the arc track. (2) For the line track, although installation angle error calibration equation is solvable, the calculated effective sound velocity cannot fix angle error caused by equivalent sound velocity variety in observation points, which generates calibration error in the installation error angle calculation. The comparison of the two tracks tells that contradiction exists between the solvability of the installation error angle calculation equation and no sound velocity caused angle error. To satisfy the former, the angle error will occur in both tracks. By theory analysis, it indicates that sound velocity caused angle error has more influence on roll installation angle and less effect on pitch installation angle in the arc track, while the line track that goes through the transponder's horizontal position takes the opposite. Therefore, a track composed by arc and line track is designed as the final one for USBL calibration, which decreases the calibration error in both pitch and roll error angles caused by sound velocity by using MMSE algorithm. The performance of the combined track was proved by field data application.

Generally, calibration track design is a rather complex research. Because there are too many kinds of tracks to check and amount of ways that simple tracks can combine. Although the shape of the track is designed in this paper, the length of the radius and lines haven't been decided yet. Therefore, future works must contain dimension design to fit for different water depth, tracks' direction effect study, tracks' bias influence evaluation, and so forth. Moreover, if ray tracing principle is introduced, the error caused by sound speed can be decreased [4]. Then track design should focus on other errors introduced by acoustic measurement, gyro and GPS. It means calibration tracks might be changed with different processing algorithms. So this paper is just a beginning and we look forward to more progress and cooperation in this field.




(a) Fixed by combined track calibration results




(b) Fixed by arc track calibration results

Fig 7 Comparison of USBL positioning scatter fixed by different track calibration results

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