Cooperation and collision avoidance for multiple DP ships with disturbances

Mingyu Fu, Yujie Xu
College of automation,
Harbin Engineering University,
Harbin, China

Shimin Wang
College of automation,
Harbin Engineering University,
Harbin, China

***Abstract*—In this paper, we address the problem of cooperation and collision avoidance for multiple DP ships with disturbances. The DP ships are regarded as elliptical agents with nonlinear Lagrangian dynamics. We design control laws that guarantee cooperation as well as collision-free maneuvers. We show, using a two-step proof, that the avoidance part of the control laws guarantees safety of the agents independently of the coordinating part. Then, we establish an ultimate bound on the avoidance term. The obtained theoretical results are illustrated through numerical examples.**

Keywords—Lagrangian dynamics; cooperative collision avoidance; dynamic positioning; Lyapunov function

I. INTRODUCTION

Coordination control of multiple DP ships finds various applications in fields such as FPSO offloading, supporting, and lifting. The DP ships can be regarded as elliptical agents with nonlinear Lagrangian dynamics. There is a long line of research on fully autonomous consensus seeking methods in which a group of agents try to agree on the value of a global objective using only local data (see [1,2] for a recent survey of the subject). Refs. [3,4] discuss reaching consensus on the heading of a group of agents using nearest neighbor rules and switching communication graphs. Finite-time consensus has been recently treated in Sundaram and Hadjicostis [5], and optimal design of the underlying graph has appeared in [6,7]. Consensus under communication delays has been treated in Lee and Spong [8] and synchronization using passivity-based concepts has appeared in [9–11]. Recent results on autonomous formation control have appeared in Olfati-Saber [12] using potential-based methods, in Tanner et al. [13] using hierarchial design, and in Stipanovic ´ et al. [14] using decentralized overlapping control. It is also important to mention some new modeling techniques for multi-agent systems that take into account the control, computation, and communication in one modeling framework [15,16,17].

With the increasing interest in multi-agent systems, the problem of establishing conditions for their safety verification has become important. In the case of the non-cooperative scenario for a two-agent system, the problem of collision avoidance has been studied in Mitchell et al. [18], based on the ideas proposed in Tomlin et al. [19], using the level set methods[20,21] for computing solutions of Hamilton–Jacobi–Isaacs (HJI) partial differential equations (e.g., see [22,23] and references reported therein). Since these methods were related to the viscosity solutions of HJI equations, implying their high computational complexity, an efficient polytopic approximation method for computing guaranteed avoidance strategies for one of the two agents has been proposed in [24,25]. However, one of the open problems is how to efficiently apply these methods to multi-agent scenarios even in the case of very simple dynamic models describing motions of the individual agents. In the context of a multi-agent control and coordination problem where the agents' motion is described by kinematic models, the collision avoidance was addressed using multi-objective and decentralized optimization methods in Inalhan et al. [26],heuristic methods in Kim et al. [27], stochastic optimization for choosing conflict-free optimal maneuvers in Hu et al.[28], navigation functions [29,30,31] in Dimarogonas et al. [32], and attractive/repulsive potentials in [33]. In this paper, we follow the concept of avoidance control that was originally formulated by Leitmann and Skowronski in [34], and later further developed in [35-37]. The work of Leitmann and his collaborators is particularly important since it formulated, for the first time, collision avoidance problem using the Lyapunov analysis approach. By doing so, many advantages of using the Lyapunov approach such as capabilities of including nonlinear dynamics in higher dimensions, uncertainties and robustness issues, time-delays, multiple agents that may be cooperative or non-cooperative, etc., may be explored. A chronological survey with more details on avoidance control is provided in Stipanovic ´ [38]. In the methods mentioned above, the avoidance region is considered as a circular disk which is not accurate for most ships. Then the separation condition between the ellipsoidal agents proposed by K.D. Do[39] becomes very useful.

The work of this paper is development and application from refs. [40].The remainder of this paper is organized as follows: In Section II we provide the dynamics, separation condition, avoidance functions, control laws, communication topology, and augmented closed-loop dynamics. In Section III we give the main result of the paper which is comprised of a preliminary result on collision avoidance under the effect of bounded disturbances. A numerical example is provided in Section IV, and we give some final comments in Section V.

II. PROBLEM STATEMENT

A. Dynamics

First, Consider the nonlinear equations of motion for the i th DP ship with disturbance:

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$$\begin{cases} \dot{\eta}_i = J_i(\eta_i)\nu_i \\ M_i\dot{\nu}_i + C_i(\nu_i)\nu_i + D_i(\nu_i)\nu_i = \tau_i + \omega_i \end{cases} \tag{1}$$

This is the classical DP model developed by Thor I. Fossen[41].To simplify the model, the restoring force is neglected. The external disturbances are described as ω_i.

For a rigid body the system inertia matrix is positive definite and constant, that is

$$M_i = M_i^T > 0,\ \dot{M}_i = 0 \tag{2}$$

For a rigid body moving though an ideal fluid the Coriolis and centripetal matrix $C_i(\nu_i)$ can always be parameterized such that it is skew-symmetric, that is

$$C_i(\nu_i) = -C_i^T(\nu_i), \tag{3}$$

The equations of motion (1) when transformed to NED frame take the following form:

$$M_i^*(\eta_i)\ddot{\eta}_i + C_i^*(\nu_i,\eta_i)\dot{\eta}_i + D_i^*(\nu_i,\eta_i)\dot{\eta}_i = \tau_i^* + \omega_i^* \tag{4}$$

The transformation is as follows:

$$\begin{aligned} &M_i^*(\eta_i) = J_i^{-T}(\eta_i)M_iJ_i^{-1}(\eta_i), \\ &C_i^*(\nu_i,\eta_i) = J_i^{-T}(\eta_i)[C_i(\nu_i) - M_iJ_i^{-1}(\eta_i)\dot{J}_i(\eta_i)]J_i^{-1}(\eta_i), \\ &D_i^*(\nu_i,\eta_i) = J_i^{-T}(\eta_i)D_i(\nu_i)J_i^{-1}(\eta_i), \\ &\tau_i^* + \omega_i^* = J_i^{-T}(\eta_i)(\tau_i + \omega_i) \end{aligned} \tag{5}$$

From (2) and (3), we can easily get the following properties:

$$\begin{aligned} &P1.\quad M_i^*(\eta_i) = M_i^*(\eta_i)^T > 0 \\ &P2.\quad s^T(\dot{M}_i^*(\eta_i) - 2C_i^*(\nu_i,\eta_i))s = 0 \\ &P3.\quad D_i^*(\nu_i,\eta_i) > 0 \end{aligned}$$

B. Separation condition between two elliptical disks

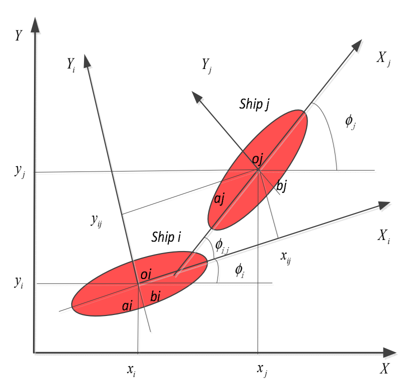

Fig. 1. Two ellipsoids and their coordinates

This section presents a condition for separation of two ellipses that is applicable for collision avoidance in the coordination control design later. As such, we consider two ellipses i and j shown Fig.1. In this figure, OXY is the earth-fixed frame, $O_iX_iY_i$ is the body-fixed frame attached to ellipse i, $q_i = [x_i, y_i]^T$ denotes the position of the center O_i, and ψ_i denotes the orientation (yaw angles) of the ellipse i. These notations are similar for the ellipse j.

Lemma. Consider two elliptical disks i and j, which have semi-axes (a_i, b_i) and (a_j, b_j), and heading angles ψ_i and ψ_j, and are centered at (x_i, y_i) and (x_j, y_j), respectively. And the transformed distance between two elliptical disk is:

$$\Delta_{ij} = \left\| Q_{ij}\bar{q}_{ij} \right\| - 1 \tag{6}$$

Where

$$\begin{aligned} &Q_{ij} = I_{2\times2} - (I_{2\times2} + \kappa_{ij}T_j)^{-1}, \\ &\bar{q}_{ij} = P_iq_{ij}, \end{aligned} \tag{7}$$

With $I_{2\times2}$ being a 2×2 identity matrix. The vector q_{ij} denotes the relative position vector between the ellipsis i and j. The matrix P_i represents the so-called negative inverse of the rotational matrix of the ellipse i by the angle ψ_i around its axes. The vector q_{ij} and the matrix P_i are given by

$$\begin{aligned} &q_{ij} = q_i - q_j, \\ &P_i = -A_i^{-1}R^{-1}(\psi_i), \\ &A_i = diag(a_i,\ b_i) \end{aligned} \tag{8}$$

The matrix $R(\bullet)$ represents the three-dimensional rotational matrix with respect to the vector $\bullet$. The matrix T_j is the system matrix of the ellipse j when the ellipse i and j are transformed to the circular –elliptical coordinates detailed. It is given by

$$T_j = \begin{bmatrix} T_{j11} & T_{j12} \\ T_{j12} & T_{j22} \end{bmatrix}, \tag{9}$$

Where

$$T_{j11} = \varepsilon_{11}^2 + \varepsilon_{21}^2,\quad T_{j12} = \varepsilon_{11}\varepsilon_{12} + \varepsilon_{21}\varepsilon_{22},\quad T_{j22} = \varepsilon_{12}^2 + \varepsilon_{22}^2$$

With ε_{mn} for $m = 1, 2$ and $n = 1, 2$ being the element (m, n) of the matrix $(A_i^{-1}R(\psi_{ij})A_j)^{-1}$, with

$$\begin{aligned} &\psi_{ij} = \psi_i - \psi_j, \\ &A_j = diag(\mathrm{a}_j, b_j), \end{aligned} \tag{10}$$

The variable κ_{ij} is the largest root (the rightmost root) of the shortest distance equation

$$\bar{q}_{ij}^T(I_{2\times2} + \kappa_{ij}T_j)^{-T}T_j(I_{2\times2} + \kappa_{ij}T_j)^{-1}\bar{q}_{ij} - 1 = 0, \tag{11}$$

Where $(I_{2\times2}+\kappa_{ij}T_j)^{-T}$ denotes the transpose of the $(I_{2\times2}+\kappa_{ij}T_j)^{-1}$.

The two ellipsis are externally separated, if $\Delta_{ij}>0$.

Remark The transformed distance Δ_{ij} is a smooth function of q_{ij}, ψ_j and ψ_{ij}. Alternatively, Δ_{ij} is a smooth function of $\bar{q}_{ij}$ and ψ_{ij}.

Proof. See ref[42].

C. Avoidance functions

Our goal is to be able to guarantee that the DP ships arrive at a common position, which is taken to be the contour profile of the ships, without collisions. In order to do so, there is a need to specify a common goal for the ships, sensing capacity of the DP ships, and collision avoidance capacity of the ships.

For the overall system, we define the avoidance region by defining the avoidance sets for each pair of ships as

$$\Omega_{ij}=\left\{x:x\in R^n,\Delta_{ij}\le r\right\} \tag{12}$$

And the overall system sensing or detection region by defining pairwise detection region

$$D_{ij}=\left\{x:x\in R^n,\left\|x_i-x_j\right\|\le R\right\}, \tag{13}$$

Where, $r<R$, so we have $\Omega_{ij}\subset D_{ij}$.

Then, the overall avoidance region is given by

$$\Omega=\bigcup_{i,j\in N,j>i}\Omega_{ij} \tag{14}$$

And the overall detection region is defined by

$$D=\bigcup_{i,j\in N,j>i}D_{ij} \tag{15}$$

DEFINITION 1. The dynamic system $\dot{x}=f[x,u(x)]$ avoids Ω, $\Omega\subset R^n$, if and only if for each solution $x(t,x_0)$, $t\in T=[0,+\infty]$, $x_0\notin\Omega$ implies $x(t,x_0)\notin\Omega$ for all $t\in T$.

For each pair of ships, we define the following modified avoidance functions

$$V_{ij}^a=V_{ij}^a(x_i,x_j)=\left(\min\left\{0,\ \frac{\beta_{ij}-b_{ij}}{\alpha_{ij}-a_{ij}}\right\}\right)^2 \tag{16}$$

Where,

$$\begin{aligned}&\alpha_{ij}=\Delta_{ij}^2,a_{ij}=r^2,\\&\beta_{ij}=\left\|x_i-x_j\right\|^2,b_{ij}=R^2,\end{aligned} \tag{17}$$

Compute partial derivative of the avoidance function, we can get

$$\frac{\partial V_{ij}}{\partial x_i^T}=2\frac{\beta_{ij}-b_{ij}}{(\alpha_{ij}-a_{ij})^3}\left[(\alpha_{ij}-a_{ij})\frac{\partial\beta_{ij}}{\partial x_i^T}-(\beta_{ij}-b_{ij})\frac{\partial\alpha_{ij}}{\partial x_i^T}\right] \tag{18}$$

Note that the function $V_{ij}(\bullet,\bullet)$ is continuously differentiable outside of the avoidance region Ω_{ij} and zero outside of the sensing region D_{ij}.

D. Control laws

Since the ships communicate with each other, every agent i has a group of neighbors N_i defined as $\mathrm{N}_i=\{j\in\{1,\cdots,N\}:i\sim j\},i\in\{1,\cdots,N\}$
Where $i\sim j$ indicates that ship i communicates with ship j. We assume that the communication is undirected. We design linear state feedback control laws such that if ship i communicate with ship j, then the input τ_i and τ_j contain an element of the form $-k(\eta_i-\eta_j)$ and $-k(\eta_j-\eta_i)$, respectively, where $k>0$ is a scalar gain. Moreover, we assume that the master system has a local control term of the form $-k\eta_1$. The latter control term is needed in order to guarantee convergence of the whole group to a set point that is provided to the master ship, which acts like an anchor for the whole group. Therefore, the control law for the master ship (hereafter taken to have index i = 1) is given by

$$\tau_1=\underbrace{-b\dot{\eta}_1-k\sum\nolimits_{\forall j\ne1}(\eta_1-\eta_j)}_{coordination}-\underbrace{\begin{cases}\rho\dfrac{\dot{\eta}_1}{\|\dot{\eta}_1\|}, & \dot{\eta}_1\ne0\\ 0, & \dot{\eta}_1=0\end{cases}}_{disturbance\ mitigation}-\underbrace{\sum\nolimits_{\forall j\ne1}\frac{\partial v_{1j}^T}{\partial x_j}}_{collision\ avoidance} \tag{19}$$

and the control law for other agents is given by

$$\tau_i=\underbrace{-b\dot{\eta}_i-k\sum\nolimits_{\forall j\ne i}(\eta_i-\eta_j)}_{coordination}-\underbrace{\begin{cases}\rho\dfrac{\dot{\eta}_i}{\|\dot{\eta}_i\|}, & \dot{\eta}_i\ne0\\ 0, & \dot{\eta}_i=0\end{cases}}_{disturbance\ mitigation}-\underbrace{\sum\nolimits_{\forall j\ne i}\frac{\partial v_{ij}^T}{\partial x_j}}_{collision\ avoidance} \tag{20}$$

$\forall i\in\{2,\ldots,N\}$, where $b>0$ and ρ were defined earlier. Finally, we assume that the communication graph is connected. Note that there is no loss of generality by considering scalar gains b and k instead of symmetric positive definite matrices $B=B^T>0$ and $K=K^T>0$, as all the results can be rewritten using the maximal and minimal eigenvalues of these matrices.

A. Augmented closed-loop system

By substituting the control law into the dynamic equations, We can rewrite the closed-loop system in an augmented form as follows:

$$M^*(\eta)\ddot{\eta}+C^*(\nu,\eta)\dot{\eta}+D^*(\nu,\eta)\dot{\eta}=-b\dot{\eta}-kS^TS\eta-\rho N(\dot{\eta})-L(\eta)+W \tag{21}$$

where $\eta^T = \left[\eta_1^T, \ldots, \eta_N^T\right] \in R^{nN}$ is the state vector, $W^T = \left[\omega_1^T, \ldots, \omega_N^T\right]$ is the disturbance vector, $N^T(\dot\eta) = \left[\begin{cases} \dfrac{\dot\eta_1^T}{\|\dot\eta_1\|}, \dot\eta_1 \neq 0 \\ 0^T, \quad \dot\eta_1 = 0 \end{cases} \cdots \begin{cases} \dfrac{\dot\eta_N^T}{\|\dot\eta_N\|}, \dot\eta_N \neq 0 \\ 0^T, \quad \dot\eta_N = 0 \end{cases} \right]^T$ is a componentwise normalized velocity vector, and $L(\eta) = \left[\sum_{j \in N_1} \dfrac{\partial v_{1j}^a}{\partial x_j}, \ldots, \sum_{j \in N_N} \dfrac{\partial v_{Nj}^a}{\partial x_j} \right]^T$ is the collision avoidance control part. The matrix S is the connection matrix and is defined as follows: Let E contain η_1 as an element as well as the error vector of the form $\eta_i - \eta_j$, if $i \sim j$ are neighbors. Then S is defined such that $E = S\eta$, with the first element in E being η_1. Since we assume that the communication graph is connected, then S has full column rank and as such $S^T S$ is a symmetric positive definite matrix. This full rank condition is achieved because of the term $-k\eta_1$ in the master's control law, without which the matrix $S^T S$ would have rank $(N-1)n$ and corresponds to the Laplacian of the coordination graph.

III. MAIN RESULTS

The main result of this paper is twofold: Coordination and collision avoidance. By giving **Lemma 2**, we show that it is possible to have a cooperative scheme that guarantees safety of the DP ships. However, in general this scheme may conflict with the main objective of each agent. We rule out this possibility by showing that once collision-free maneuvers are guaranteed, then the effect of the internal forces that provide this collision-free guarantee is bounded. Using this latter fact, we show in **Theorem 1**, that the overall behavior of the agents can be characterized by an ultimate bound on how far they can be away from each other.

Theorem 1. *Consider N ships with dynamics(1), the control law(7) and (6), and an initial configuration* $\eta(0) \notin \Omega$, *where* Ω *is defined in (9). Then the set* Ω *is avoidable in the sense of DEFINITION 1.*

Proof. Considering the following Lyapunov-type function

$$V_{col} = \frac{1}{2}\dot\eta^T M^* \dot\eta + \frac{1}{2} k \eta^T S^T S \eta + \frac{1}{2}\sum_{i=1}^{N}\sum_{j \neq i} V_{ij}^a \tag{22}$$

The derivative of V_{col} along the trajectories of the augmented system is given by

$$\dot V_{col} = \dot\eta^T\left(-C^*\dot\eta - D^*\dot\eta - b\dot\eta - \rho N(\dot\eta) - kS^T S\eta - L(\eta) + W\right) + \frac{1}{2}\dot\eta^T \dot M^* \dot\eta + k\eta^T S^T S\dot\eta + \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\left(\frac{\partial V_{ij}^a}{\partial \eta_i}\dot\eta_i + \frac{\partial V_{ij}^a}{\partial \eta_j}\dot\eta_j\right) \tag{23}$$

Using the properties $P1$ and $P3$, we obtain that (10) can be upper bounded as follows:

$$\dot V_{col} \le -b\|\dot\eta\|^2 + \dot\eta^T\left(W - \rho N(\dot\eta)\right) - \dot\eta^T L(\eta) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\left(\frac{\partial V_{ij}^a}{\partial \eta_i}\dot\eta_i + \frac{\partial V_{ij}^a}{\partial \eta_j}\dot\eta_j\right) \tag{24}$$

It is not difficult to show, by change of indices and the use of avoidance function, that

$$-\dot\eta^T L(\eta) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\left(\frac{\partial V_{ij}^a}{\partial \eta_i}\dot\eta_i + \frac{\partial V_{ij}^a}{\partial \eta_j}\dot\eta_j\right) = 0. \tag{25}$$

Moreover, by the assumption (2) and the way we defined ρ, we have that

$$\dot\eta^T\left(W - \rho N(\dot\eta)\right) \le \sum_{i}^{N} \|\dot\eta_i\|\left(\|\omega_i\| - \rho\right) \le 0 \tag{26}$$

Therefore, V_{col} is bounded as

$$\dot V_{col} \le -b\|\dot\eta\|^2 \le 0 \tag{27}$$

Since $\dot V_{col} \le 0$, then the function V_{col} is non-increasing in the sensing region D ,Also, notice that the values of the function V_{col} are finite for finite values of its argument η that are outside of the region Ω, that is, when $\eta \in \Omega^c$ where $\Omega^c = R^{nN} \setminus \Omega$ denotes the set complement of Ω. Due to the continuity of solutions of the system (8), the assumption that the initial condition satisfies $\eta(0) \in \Omega^C$, and that the following conditions hold:

$$\lim_{\Delta_{ij} \to r+} V_{ij}^a\left(\eta_i, \eta_j\right) = +\infty, \quad \forall i, j \in \{1, \ldots, N\} \tag{28}$$

We conclude that the partial state trajectory $\eta\left(t, \eta(0), \dot\eta(0)\right)$ will never enter Ω. Note that in (28), $\Delta_{ij} \to r+$ denotes convergence to r from above.

The result in Theorem 1 says nothing about the whereabouts of the various agents, the only conclusion we can deduce is that collisions are guaranteed not to occur. In the Lemma 2, we will develop a bound on the effect of the avoidance terms on the performance of the ships.

Lemma 2. In the control law above, The effect of the collision avoidance terms on the overall system can be viewed as a disturbance and can be upper bounded as

$$\sup_{t \ge 0}\left\|L\left(\eta(t)\right)\right\| \le N^{\frac{3}{2}} g \tag{29}$$

Proof. Under the conditions of Theorem 1, we have that $V_{col} \ge 0$ and $\dot V_{col} \le 0$, which imply that $V_{col}(t) \le V_{col}(0), \forall t \ge 0$. As such, we have that

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i} V_{ij}^a \le V_{col}(t) \le V_{col}(0), \forall t \ge 0. \tag{30}$$

Moreover, we have that

$$\max_{i\in\{1,\ldots,N\},j\neq i}\sup_{t\geq 0} V_{ij}^{a} \leq 2V_{col}(0).$$

Due to the continuity and monotonicity of V_{ij}^{a}, there exists a constant $\underline{r}\in(r,R)$, with $\underline{r}>r$, such that

$$\inf_{t\geq 0}\Delta_{ij}\geq \underline{r}, \forall i\in\{1,\ldots,N\}, j\neq i.$$

Using this last fact, we can show that

$$\sup_{t\geq 0}\left\|\frac{\partial V_{ij}^{a}}{\partial \eta}\right\| = \sup_{t\geq 0}\left\|2\frac{\beta_{ij}-b_{ij}}{(\alpha_{ij}-a_{ij})^{3}}\left[(\alpha_{ij}-a_{ij})\frac{\partial \beta_{ij}}{\partial \eta_i^{T}}-(\beta_{ij}-b_{ij})\frac{\partial \alpha_{ij}}{\partial \eta_i^{T}}\right]\right\| \leq \left\|2\frac{\beta_{ij}-R}{(\underline{r}^{2}-r^{2})^{3}}\left[(\underline{r}-r)\frac{\partial \beta_{ij}}{\partial \eta_i^{T}}-(\beta_{ij}-R)\frac{\partial \alpha_{ij}}{\partial \eta_i^{T}}\right]\right\| \leq g$$

Where g is a bounded positive constant. Therefore, the effect of the collision avoidance terms on the overall system can be viewed as a disturbance and can be upper bounded as

$$\sup_{t\geq 0}\left\|L(\eta(t))\right\| \leq \sqrt{N}\max_{i\in\{1,\ldots,N\}}\sum_{j\neq i}\sup_{t\geq 0}\left\|\frac{\partial V_{ij}^{a}}{\partial \eta_i}\right\| \leq N^{\frac{3}{2}}g \quad (31)$$

IV. SIMULATION RESULTS

In this section, we provide a numerical simulation to illustrate the effectiveness of the proposed coordination control design stated above. Consider a group of N agents with the same dynamics given by the following equation:

$$\ddot{x}_i = \tau_i + d_i, \forall i\in\{1,\ldots,4\}$$

where d_1 has a fixed value $d_1=\begin{bmatrix}5\\5\end{bmatrix}$, which is used to drive the whole formation to a given set-point. The remaining disturbance inputs d_2, d_3 and d_4 are chosen to be uniformly distributed in the range $[-3, 3]$. We take the communication graph to be $1\sim 2\sim 3\sim 4$, and design the control laws according to (19) and (20) with $b=k=10$ and $\rho_i=10, \forall i\in\{1,2,3,4\}$ This choice of ρ_i satisfies the requirement . The sensing radii are chosen to be $R=30$ for the all ships, but the avoidance regions are different from each other. The elliptical regions are chosen as $(2\sqrt{5},\sqrt{10}),(\sqrt{10},2\sqrt{5}),(2\sqrt{5},\sqrt{10}),(2\sqrt{5},2\sqrt{5})$ from 1 to 4, respectively.

The agents are chosen to lie initially at the corners of a square shape of length 100 units, as shown in Fig. 2 (top-left). The large circles indicate the sensing region of every ship, whereas the solid disks indicate the avoidance regions of each agent. The evolution of all the agents over time is shown in Fig. 2.

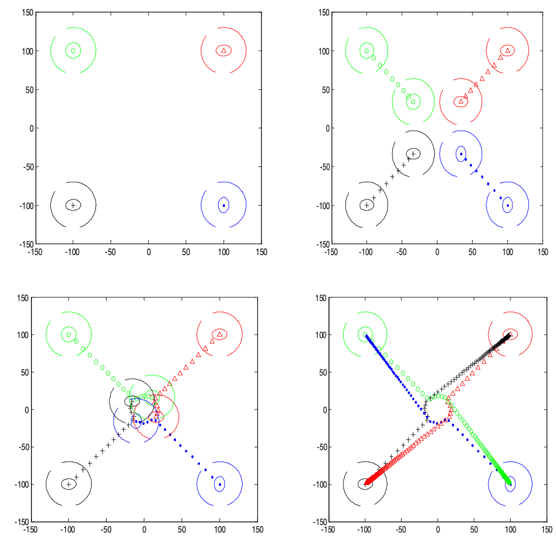

Fig. 2. Snapshots of the agents' trajectories at t_1= 0 s(top-left), t_1= 80 s(top-right), t_1= 120 s(down-left), t_1= 300 s(down-right)

V. CONCLUSION AND FUTURE WORK

In this paper we presented a unified framework that guarantees the safety of multiple DP ships while performing a specific coordination task, through the use of cooperative collision avoidance schemes. It is shown that this scheme works in the presence of unknown but bounded disturbances with a known upper bound. An ultimate bound is calculated how far agents can be away from their desired goals due to the avoidance control effect. Further research is needed to address the issue of including delays in the sensing between the agents, as this would make this approach even more applicable to realistic scenarios.

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REFERENCES

[1] R. Olfati-Saber, J.A. Fax, R.M. Murray, Consensus and cooperation in networked multi-agent systems, Proceedings of the IEEE 95 (1) (2007) 215–233.

[2] R.M. Murray, Recent research in cooperative control of multi-vehicle systems, ASME Journal of Dynamic Systems, Measurement, and Control 129(2007) 571–583.

[3] A. Jadbabaie, J. Lin, A.S. Morse, Coordination of groups of mobile autonomous agents using nearest neighbor rules, IEEE Transactions on AutomaticControl 48 (2003) 988–1001. June.

[4] D.P. Bertsikas, J.N. Tsitsiklis, Comments on ''coordination of groups of mobile autonomous agents using nearest neighbor rules'', IEEE Transactions onAutomatic Control 52 (5) (2007) 968–969.

[5] S. Sundaram, C.N. Hadjicostis, Finite-time distributed consensus in graphs with time-invariant topologies, in: Proceedings of the American Control Conference, July 2007, pp. 711–716.

[6] L. Xiao, S. Boyd, Fast linear iterations for distributed averaging 53 (1) (2004) 65–78.

[7] L. Xiao, S. Boyd, S. Lall, A scheme for robust distributed sensor fusion based on average consensus, in: Proceedings of the Fourth International Symposium on Information Processing in Sensor Networks (IPSN), 2005, pp. 63–70.

[8] D.J. Lee, M.W. Spong, Agreement with non-uniform information delays, in: Proceedings of the American Control Conference, June 2006, pp. 756–761.

[9] N. Chopra, M.W. Spong, Synchronization of networked passive systems with applications to bilateral teleoperation, in: Society of Instrumentation and Control Engineering of Japan Annual Conference, Okayama, Japan, August 2005..

[10] N. Chopra,M.W. Spong, Advances in Robot Control: From Everyday Physics to Human-LikeMovements, Chapter Passivity-Based Control ofMulti-Agent Systems, Springer-Verlag, Berlin, 2006, pp. 107–134..

[11] N. Chopra, M.W. Spong, Output synchronization of nonlinear systems with time delay in communication, in: Proceedings of the IEEE Conference on Decision and Control, December 2006, pp. 4986–4992.

[12] R. Olfati-Saber, Flocking for multi-agent dynamic systems: algorithms and theory, IEEE Transactions On Automatic Control 51 (2006) 401–420.

[13] H.G. Tanner, G.J. Pappas, V. Kumar, Leader-to-formation stability, IEEE Transactions on Robotics and Automation 20 (3) (2004) 433–455.

[14] D.M. Stipanovic ´ , G. Inalhan, R. Teo, C.J. Tomlin, Decentralized overlapping control of a formation of unmanned aerial vehicles, Automatica 40 (2004)1285–1296.

[15] A. Ganguli, S. Susca, S. Martínez, F. Bullo, J. Cortés, On collective motion in sensor networks: sample problems and distributed algorithms, in Proceedings of the IEEE Conference on Decision and Control, Seville, Spain, 2005, pp. 4239–4244.

[16] S. Martínez, F. Bullo, J. Cortés, E. Frazzoli, On synchronous robotic networks – Part I: models, tasks and complexity, IEEE Transactions on Automatic Control 52 (2007) 2199–2213.

[17] S. Martínez, F. Bullo, J. Cortés, E. Frazzoli, On synchronous robotic networks – Part II: time complexity of rendezvous and deployment algorithms, IEEE Transactions on Automatic Control 52 (2007) 2214–2226.

[18] I. Mitchell, A.M. Bayen, C.J. Tomlin, A time-dependent Hamilton–Jacobi formulation of reachable sets for continuous dynamic games, IEEE Transactions on Automatic Control 50 (2005) 947–957.

[19] C.J. Tomlin, J. Lygeros, S. Sastry, A game theoretic approach to controller design for hybrid systems, Proceedings of the IEEE 88 (2000) 949–970.

[20] S. Osher, J.A. Sethian, Fronts propagating with curvature-dependent speed: algorithms based on Hamilton–Jacobi formulations, Journal of Computational Physics 79 (1988) 12–49.

[21] J.A. Sethian, Level set methods and fast marching methods: evolving interfaces in computational geometry, fluid mechanics, computer vision, and material science, second ed., Cambridge University Press, 2002.

[22] M.G. Crandall, P.-L. Lions, Viscosity solutions of Hamilton–Jacobi equations, Transactions of American Mathematical Society 277 (1983) 1–42.

[23] L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, vol. 19, American Mathematical Society, Providence, RI, 1998.

[24] I. Hwang, D.M. Stipanovic ´ , C.J. Tomlin, Polytopic approximations of reachable sets applied to linear dynamic games and to a class of nonlinear systems,in E.H. Abed (Ed.), Advances in Control, Communication Networks, and Transportation Systems: In Honor of Pravin Varaiya, Systems and Control Foundations and Applications, Birkhäuser, Boston, MA, 2005, pp. 1–20.

[25] D.M. Stipanovic ´ , I. Hwang, C.J. Tomlin, Computation of an over-approximation of the backward reachable set using subsystem level set functions, Dynamics of Continuous, Discrete and Impulsive Systems, Series A: Mathematical Analysis 11 (2004) 399–411.

[26] G.Inalhan, D.M. Stipanovic ´ , C.J. Tomlin, Decentralized optimization, with application to multiple aircraft coordination, in: Proceedings of the 2002 IEEE Conference on Decision and Control, Las Vegas, Nevada, 2002, pp. 1147–1155.

[27] Y. Kim, M. Mesbahi, F.Y. Hadaegh, Multiple-spacecraft reconfiguration through collision avoidance, bouncing, and stalemate, Journal of Optimization Theory and Applications 122 (2004) 323–343.

[28] J. Hu, M. Prandini, S. Sastry, Optimal coordinated motions of multiple agents moving on a plane, SIAM Journal of Control and Optimization 42 (2003)637–668.

[29] D.E. Koditschek, E. Rimon, Robot navigation functions on manifolds with boundary, Advances in Applied Mathematics 11 (1990) 412–442.

[30] E. Rimon, D.E. Koditschek, The construction of analytic diffeomorphisms for exact robot navigation on star worlds, Transactions of the American Mathematical Society 327 (1991) 71–116.

[31] E. Rimon, D.E. Koditschek, Exact robot navigation using artificial potential functions, IEEE Transactions on Robotics and Automation 8 (1992) 501–518.

[32] D.V. Dimarogonas, S.G. Loizou, K.J. Kyriakopoulos, M.M. Zavlanos, A feedback stabilization and collision avoidance scheme for multiple independent non-point agents, Automatica 42 (2006) 229–243.

[33] R. Olfati-Saber, Flocking for multi-agent dynamic systems: algorithms and theory, IEEE Transactions On Automatic Control 51 (2006) 401–420.

[34] G. Leitmann, J. Skowronski, Avoidance control, Journal of Optimization Theory and Applications 23 (1977) 581–591.

[35] G. Leitmann, Guaranteed avoidance strategies, Journal of Optimization Theory and Applications 32 (1980) 569–576.

[36] G. Leitmann, J. Skowronski, Avoidance control, Journal of Optimization Theory and Applications 23 (1977) 581–591.

[37] G. Leitmann, J. Skowronski, A note on avoidance control, Optimal Control Applications and Methods 4 (1983) 335–342.

[38] D.M. Stipanovic ´ , A survey and some new results in avoidance control, in: J. Rodellar, E. Reithmeier (Eds.), Proceedings of the 15th International Workshop on Dynamics and Control, Tossa de Mar, Spain, 2009, pp. 166–173.

[39] K.D. Do, Coordination control of multiple ellipsoidal agents with collision avoidance and limited sensing ranges, Systems & Control Letters 61 (2012) 247–257.

[40] Peter F. Hokayema, Dušan M. Stipanovic, Mark W. Spong, Coordination and collision avoidance for Lagrangian systems with disturbances, Applied Mathematics and Computation 217 (2010) 1085–1094.

[41] Fossen, Thor I, Handbook of Marine Craft Hydrodynamics and Motion control, John Wiley & Sons,Ltd (2011).

[42] K.D. Do, Formation control of multiple elliptical agents with limited sensing ranges, Automatica 48 (2012) 1330–1338.