

Research of Interacting Multiple Model Particle Filter Based on Passive Multi-sonar

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Abstract—The passive target tracking problem of multi-sonar belongs to nonlinear problem, and the background noise is not gauss noise entirely, so a kind of filtering method which suit for nonlinear and non-gaussian system of is needed. In this paper, the interacting multiple model particle filter which combined by particle filter and interacting multiple model is applied in tracking maneuvering target. Combing the particle filter's ability of disposing nonlinear and non-gauss problem and the IMM's ability of tracking target's arbitrary maneuvering, the difficulty that traditional filter encounters in tracking maneuvering target under the circumstance of nonlinear and non-gaussian can be solved. The interacting multiple model particle filter is simulated in two different model set, the results indicate that, under the case of model matching, this method has good ability in tracking the maneuvering target of nonlinear and non-gaussian system. (Abstract)

Keywords—passive multi-sonar; nonlinear and non-gaussian; maneuvering target; interacting multiple model; particle filter(key words)

I. INTRODUCTION

Under passive circumstance, because of the incomplete observation, multi-sonar are needed to fulfilled the tracking task. The tracking issue of passive multi-sonar belongs to nonlinear problem, the traditional linear filters such as extended kalman filter^[1], unscented kalman filter^[2] can only deal with weakly nonlinear system, and limited by gaussian noise. Particle filter has good effect in processing nonlinear and non-gaussian problem. The focus of particle filter began in 1993, later a lot of the improved algorithm are proposed by scholar.

The mew progress of particle filter include, gaussian particle filter and gaussian sum particle filter which are proposed by Kotecha J Hand Djuric P M^[3,4], resampling excuse from these two methods, gaussian distribution and multiple gaussian distribution and the weighted sum of multiple gaussian distributions are used to approximate filtering and predictor distribution, so the problem of particle degeneration can be resolved. Rao-Blackwellised particle filter^[5], dig potential from dynamic system model, minish the state

dimension that estimated by particle filter to increase the efficiency of particle filter. MoYiwei, et al. introduce evolution and programing algorithm to particle filter, propose Evolutionary particle filter^[6]. Fission bootstrap particle filtering^[7] proposed by Cheng Shuiying and Zhang Jianyun is designed for sample indigence problem, its fission process is a kind of random sampling method, its sampling plan is adjust continuously according to simulation result, so the problem of particle degeneration can be conquered. In the recent years, genetic algorithm^[8], fuzzy algorithm are introduced by Yang Ning and Kamel H respectively to the resample process^[9], so a series of particle filters that based on intelligent optimization idea are produced. Neural network and typical resample algorithm are combined by Zhu Zhiyu^[10] to improve the diversity of particles.

Particle filter has been widely applied to various filed, including locating and tracking . in 2005, kemel particle filter was applied in visual tracking of image sequence by Cheng Chang and Rashid Ansari^[11]. In 2006, Rao-Blackwellised particle filter was applied in locating robotPantelis Elinas^[12]. In 2009, Particle filter structure was applied in robust pedestrian detection and tracking by Michael D. Breitenstein^[13].

For maneuvering target tracking, the interacting multiple model algorithm(IMM) proposed by Blom and Bar-Shalom^[14] is a effective method which has Markov switching coefficient. However, in the procedure of parallel filter, kalman or extended kamlamn filter is used which unsuitable for non-gaussian problem. The interacting multiple model particle filter(IMM-PF) proposed by Boers and Driessen^[15] is a method with multiple model to tracking target's any maneuvering, and apply particle filter in parallel filter to process the nonlinear and non-gaussian problem.

In this paper, IMM-PF is applied to tracking passive maneuvering target with multiple base sonar, and simulated in two model set.

II. PASSIVE MULTI-SONAR TRACKING MODEL

Particle filter algorithm is widely applied in radar filed, literature [16] only apply it to tracking the target ship with linear orbit, almost no body apply it to the maneuvering tracking in passive multi-sonar. Compared with radar, passive sonar facing greater difficulties which are: 1. Serious transmission loss and complex background noise, therefore, lead to the low measurement accuracy of azimuth, that the moving feature of target has larger error. 2. For the target of

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longer distance and lower speed, the observation ability of system is poor, the tracking convergence rate will be slower. This paper applied IMM-PF algorithm, strive to realize the passive target tracking in multi-sonar filed.

The basis of target tracking including discrete motion state model and observation model. A class of system with linear state model and nonlinear observation is studied in this paper.

A. Discrete State Equation

Target move model can be represent by discrete state equation as following Equation:

$$X(k+1) = F(k)X(k) + \Gamma(k)W(k) \quad (1)$$

In equation (1): $X(k+1)$ is system's state vector, $F(k)$ is state shift matrix, $\Gamma(k)$ is state noise input matrix, $W(k)$ is gauss white noise with zero mean, it's covariance is $Q(k)$. For different model, state components are different. Centralize all models, X can be represe-nted as:

$$X = [x \ y \ v_x \ v_y \ a \ w]^T \quad (2)$$

In equation (2), x , y is the coordinate of target, v_x and v_y are speed, a is acceleration, w is turn rate.

The familiar target move models mostly have speed unchangeable (CV) model, acceleration unchangeable (CA) model and swerve speed unchangeable (CT) model. Considering planar instance, these three models^[1] are narrated as follows:

□ CV model

This model supposes acceleration of target is zero, the state vector can be simplified as $X(k) = [x \ y \ v_x \ v_y]^T$, matrix F and Γ are represented as:

$$F = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \Gamma = \begin{bmatrix} T^2/2 & 0 & T & 0 \\ 0 & T^2/2 & 0 & T \end{bmatrix}^T$$

□ CT model

This model is swerve speed unchangeable model, state vector can be simplified as $X(k) = [x \ y \ v_x \ v_y]^T$, the meaning of the former four quantities are same as CV model, the last one represent swerve speed, matrix F and Γ are represented as :

$$F = \begin{bmatrix} 1 & 0 & \frac{\sin wT}{w} & \frac{\cos wT}{w} \\ 0 & 1 & \frac{1 - \cos wT}{w} & \frac{\sin wT}{w} \\ 0 & 0 & \cos wT & -\sin wT \\ 0 & 0 & \sin wT & \cos wT \end{bmatrix}, \Gamma = \begin{bmatrix} T^2/2 & 0 & T & 0 \\ 0 & T^2/2 & 0 & T \\ 0 & 0 & 0 & 0 \end{bmatrix}^T$$

T represent time interval, w represent maneuvering

parameter, unit is rad/s.

□ CA model

This model suppose acceleration unchangeable, state vector can be simplified as $X(k) = [x \ y \ v_x \ v_y \ a_x \ a_y]^T$, matrix F and Γ are represented as:

$$F = \begin{bmatrix} 1 & 0 & \frac{\sin wT}{w} & \frac{\cos wT}{w} \\ 0 & 1 & \frac{1 - \cos wT}{w} & \frac{\sin wT}{w} \\ 0 & 0 & \cos wT & -\sin wT \\ 0 & 0 & \sin wT & \cos wT \end{bmatrix}, \Gamma = \begin{bmatrix} T^2/2 & 0 & T & 0 \\ 0 & T^2/2 & 0 & T \\ 0 & 0 & 0 & 0 \end{bmatrix}^T$$

B. Discrete Observe Equation

The observe model of target can be represented as follows:

$$Z(k+1) = h(X(k+1)) + V(k+1) \quad (3)$$

In Equation (3): Z is observation vector, $h(X(k+1))$ is nonlinear uncton, $V(k+1)$ is gauss white noise with zero mean, here consider non-gauss noise.

Passive tracking received the azimuth information, the observation equation is:

$$h(X(k+1)) = \left[tg^{-1}\left(\frac{y-y_1}{x-x_1}\right) \dots tg^{-1}\left(\frac{y-y_i}{x-x_i}\right) \dots tg^{-1}\left(\frac{y-y_{N_r}}{x-x_{N_r}}\right) \right]^T \quad (4)$$

Observation noise modeling, assuming observation error as a stationary random process, meet ergodic, it can be composed of two noise with different variance, and these two noise occur randomly in accordance with certain probability.

The observation model can be approximate established as^[18]:

$$V = aV_1 + (1-a)V_2 \quad (5)$$

In equation (5): V_1 and V_2 are gaussian random variable with zero mean, σ_1^2 and σ_2^2 variance. a is also random variables, and obey 0-1 distribution.

The probability density $f(v)$ of observation noise variable

V can be represented as:

$$f(v) = pf_1(v) + (1-p)f_2(v) \quad (6)$$

In equation (6): $f_1(v)$ and $f_2(v)$ are the probability density of V_1 and V_2 . p is the occur probability density of V_1 , it can be counted by observations of one orbit. Suppose the time

summation that statistics observations less than $\sigma_1 + \sigma_2 / 2$ is T_1 , the whole observation time is T , so $p = T_1 / T$.

III. IMM-PF ALGORITHM^[10]

A. PF algorithm

In 2000, Doucet^[19] gives the universal description of particle filter. It is statistical filter based on monte carlo method and Bayesian estimation. It solving integral operation in Bayesian estimation by monte carlo method according to big number theory. The basic thinking is: firstly, according to initial mean and covariance of system state vector, a set of random style-books are produced which called particles, secondly, according to the system state equation to forecast particle state in next time, then according to the observation adjust the particle's weight and position continually, use the information after adjusting to approach the posterior probability distribution of the state.

For dynamic system, suppose in k time the posterior probability distribution $p(x_k / z_k)$ of target state can be described by N samples $\{x_k^i, \omega_k^i\}$ which abstracted from shift priori density function $p(x_k / x_{k-1})$. Weight ω_k^i can be normalized as $\sum \omega_k^i = 1$, so k time, the posterior probability distribution of the target can be weighted as:

$$p(x_k / z_k) \approx \sum_{i=1}^N \omega_k^i \cdot \delta(x_k - x_k^i) \quad (7)$$

In equation (7): the select of weight by importance sampling, the particle set can be obtained by importance density function $q(x_k / x_{k-1}, z_k)$, weight is:

$$\omega_k^i \propto \omega_{k-1}^i \frac{p(z_k / x_k^i) \cdot p(x_k^i / x_{k-1}^i)}{q(x_k^i / x_{k-1}^i, z_k)} \quad (8)$$

priori probability density is chosen as importance function of standard particle filter, that is:

$$q(x_k^i / x_{k-1}^i, z_k) = p(x_k^i / x_{k-1}^i) \quad (9)$$

So importance weight can be simplified as:

$$\omega_k^i \propto \omega_{k-1}^i \cdot p(z_k / x_k^i) \quad (10)$$

The procedure of standard particle filter can be generalized as follows:

- 1) Initialization. Produce particles $\{x_0^i\}_{i=1}^N$ from priori probability density.
- 2) The weights of all particles are $1/N$
- 3) Forecast. In time k , according to state shift equation, forecast the particle and value x_k^i and observation value

$h(x_k^i)$.

- 4) Updating. According to (8) update the value of weight.

$$w_k^i = w_{k-1}^i p(z_k / x_k^i) = w_{k-1}^i p_{e_k}(z_k - h(x_k^i)) \quad (11)$$

In equation (11), z_k 、 $h(x_k^i)$ are N_r dimensions column vector, and normalize the weight. N_r is the number of sonar.

- 5) Resample. Get the new particle set $\{x_k^i, i = 0, 1, 2, \dots, N\}$.

The minimum mean square estimation of the state in time k is:

$$\bar{x} \approx \sum_{i=1}^N \omega_k^i \cdot x_k^i \quad (12)$$

- 6) Time $k = k + 1$, turn to the step (2).

B. IMM algorithm

The theory diagram of IMM is shown as figure 1:

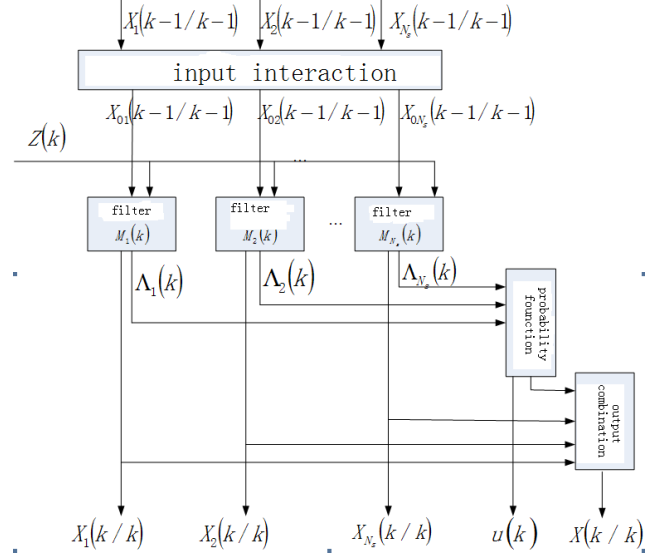


Fig.1 the principle diagram of IMM

The procedure of IMM^[20] algorithm as follows:

- 1) Calculate the mixing probability
- 2) Input interactive
- 3) Model filtering
- 4) Model probability update
- 5) Calculate the state and covariance

The detail description procedure see IIM-PF.

C. IMM-PF algorithm

Apply IMM algorithm in the parallel process of PF can get the IMM-PF algorithm researched in this paper. The detail procedure of IMM-PF can be narrated as follows:

- 1) Input interactive^[20]: suppose there are N_s models,

calculate

$$\overline{c_{l_1}} = \sum_{l_2=1}^{N_s} p_{l_2 l_1} \cdot u_{k-1, l_2} \quad (13)$$

$$u_{k-1/k-1, l_2 / l_1} = \frac{1}{\overline{c_{l_1}}} p_{l_2 l_1} \cdot u_{k-1, l_2} \quad (14)$$

$$x_{k-1/k-1, l_1}^0 = \sum_{l_2=1}^{N_s} x_{k-1/k-1, l_1} \cdot u_{k-1/k-1, l_2 / l_1} \quad (15)$$

$$P_{k-1/k-1, l_1}^0 = \sum_{l_2=1}^{N_s} u_{k-1/k-1, l_2 / l_1} \cdot \{P_{k-1/k-1, l_2} + [x_{k-1/k-1, l_2} - x_{k-1/k-1, l_1}^0] [x_{k-1/k-1, l_2} - x_{k-1/k-1, l_1}^0]^T\} \quad (16)$$

2) Parallel filter: the samples meet the relationship :

$x_{k-1/k-1, l_1}^i \sim N(x_{k-1/k-1, l_1}^0, P_{k-1/k-1, l_1}^0), i=1, \dots, N$, According state shift equation to predict particles, get new particle set $x_{k-1/k-1, l_1}^i, z_{k-1/k-1, l_1}^i$ can be obtained after substituting it into observation equation, calculate residual error:

$$\delta_{k, l_1}^i = z_k - z_{k-1/k-1, l_1}^i \quad (17)$$

Update weight and normalized;

3) Model probability update:

Predict the observation mean:

$$S_{k, l_1} = R + \sum_{i=1}^N w_{k, l_1}^i \cdot (z_{k-1/k-1, l_1}^i - \overline{z_{k, l_1}}) \cdot (z_{k-1/k-1, l_1}^i - \overline{z_{k, l_1}})^T \quad (18)$$

Innovation meet equation (19):

$$v_{k, l_1}^i = z_k - \overline{z_{k, l_1}} \quad (19)$$

Likelihood function:

$$\Lambda_{k, l_1} \sim N(v_{k, l_1}; 0, S_{k, l_1}) \quad (20)$$

Model probability:

$$u_{k, l_1} = \frac{1}{c} \Lambda_{k, l_1} \cdot \overline{c_{l_1}} \quad (21)$$

In equation (21) c meet the equation (22):

$$c = \sum_{i=1}^N \Lambda_{k, l_1} \cdot \overline{c_{l_1}} \quad (22)$$

4) Resampling: according importance weight to extract new

particle set $\{x_{k/k-1, l_1}^i, i=1, \dots, N_s\}$ with resampling method, and reallocate weights $w_{k, l_1}^j = \frac{1}{N}$;

5) Estimate state and covariance:

$$x_{k/k, l_1} = \sum_{j=1}^N w_{k, l_1}^j x_{k/k, l_1}^j \quad (23)$$

$$P_{k/k, l_1} = \sum_{j=1}^N w_{k, l_1}^j [x_{k/k, l_1}^j - x_{k/k, l_1}][x_{k/k, l_1}^j - x_{k/k, l_1}]^T \quad (24)$$

6) Output interactive: according the estimation results of each filter and the posterior probability of each model to calculate the final state $x_{k/k}$:

$$x_{k/k} = \sum_{l_1=1}^{N_s} x_{k/k, l_1} u_{k, l_1} \quad (25)$$

IV. SIMULATION VERIFICATION

A. Accuracy evaluation index

Using the following index: using the RMS error graph of single simulation as qualitative analysis; using the RMS error(Root-Mean-square-Error) graph of multiple simulation as quantitative analysis. The RMS of single simulation can calculate by equation (26):

$$RMSE = \sqrt{\frac{1}{N} \sum_{k=1}^N (x_k - \hat{x}_k)^2} \quad (26)$$

In equation (26): x_k is the real stat value, $\hat{x}_k$ is the estimate value, N is time-step.

The $RMSE$ mean of J monte carlo simulation:

$$\mu = \frac{1}{J} \sum_{i=1}^J RMSE_i \quad (27)$$

The $RMSE$ mean-square deviation of J monte carlo simulation:

$$\sigma = \sqrt{\frac{1}{J} \sum_{i=1}^J (RMSE_i - \mu)^2} \quad (28)$$

B. simulation analysis

Take three observations for example, each observation is a uniform linear array, observation noise meet equation (4), it is composed by two gaussian noise with 1° and 0.8° standard deviation, after a statistical result get $p=0.81$. suppose target state vector is $X_k \triangleq [x_k, y_k, v_{k,x}, v_{k,y}]$, and the three observation meet equilateral triangle structure, if the coordinate of the first two observations are $(-3000, -800)$ and

(-2000, -800), the abscissa of third observation x_3 is -2500, so

$y_3 = \sqrt{(x_2 - x_1)^2 - (x_3 - x_1)^2} - |y_1|$, particle number is 500. The tracking orbit is set as: the target initial coordinate is (0,0), sample time interval $T=1s$, target initial speed $v=20m/s$, the first 20 sample intervals do uniform linear motion, then turn left, turn rate is $7.84^\circ/s$, continue to the 45th sample, and then continue to do uniform linear motion, continue to the 60th sample, then turn right, and turn rate is $7.84^\circ/s$, continue to the 90th sample, at last 10 sample intervals continue to do uniform linear motion.

Two model set used in this paper are: the first one, uniform linear model and two turn model with different rate. The second one, uniform linear model and two uniformly accelerated linear model with different state noise.

The Markov probability transition matrix of model 1 as follows:

$$p = \begin{bmatrix} 0.98 & 0.01 & 0.01 \\ 0.2 & 0.795 & 0.005 \\ 0.2 & 0.005 & 0.795 \end{bmatrix}$$

The initial probability of three models are 0.98、0.01、0.01, the variance of state noise are $10m/s^2$, suppose the initial covariance is $P(0) = diag(10^{-6} \times [400, 400, 100, 100])$.

The Markov probability transition matrix of model 2 as follows:

$$p = \begin{bmatrix} 0.7 & 0 & 0.3 \\ 0 & 0.8 & 0.2 \\ 0.2 & 0.25 & 0.55 \end{bmatrix}$$

The initial probability of three models are 0.7、0、0.3. the state noise variance of the first two model are $10m/s^2$, the state noise variance of the third model is $15m/s^2$. suppose the initial covariance is

$$P(0) = diag(10^{-6} \times [400, 400, 100, 100, 5, 5])$$

Figure 2 is the orbit tracking result in model 1, figure 3 is the probability of each model, figure 4 is tracking error. Figure 5 is the orbit tracking result in model 2, figure 6 is the probability of each model, figure 7 is tracking error. Table 1 and table 2 are the RMSE mean and covariance after 50 monte carlo simulation.

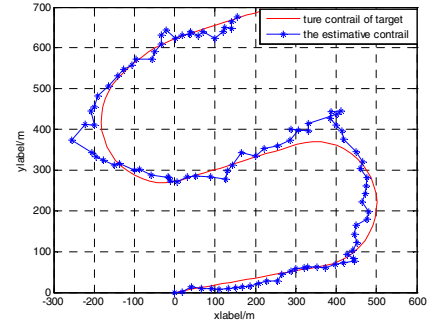


Fig.2 The tracking result of IMM-PF in model 1

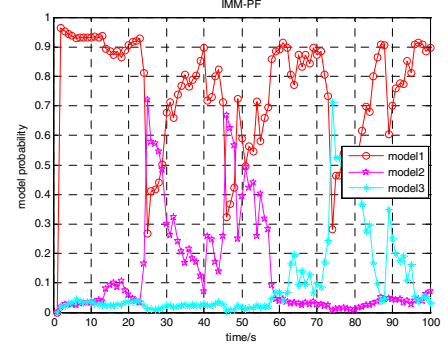


Fig.3 model probability of IMM-PF in model 1

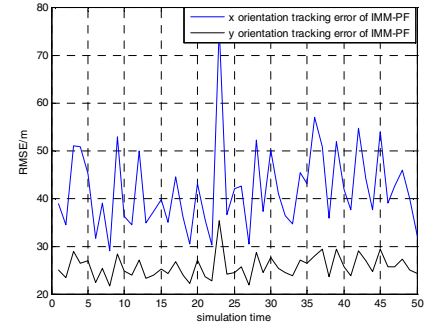


Fig.4 RMSE error in model 1

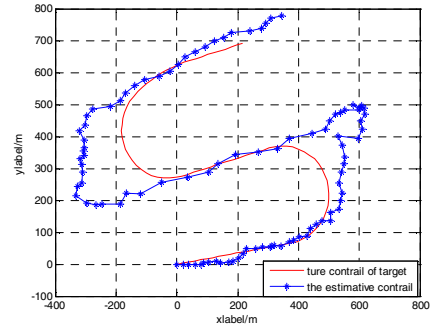


Fig.5 The tracking result of IMM-PF in model 2

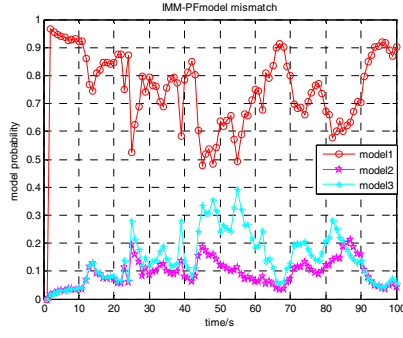


Fig.6 model probability of IMM-PF in model 2

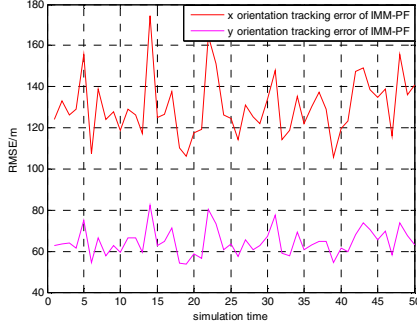


Fig.7 RMSE error in model 2

Tab.1 RMSE error comparison of x direction after 50 times monte carlo simulation

Model	RMSE	
	Mean	Covariance
Model 1	41.8384	8.6859
Model 2	130.1717	14.4531

Tab.2 RMSE error comparison of y direction after 50 times monte carlo simulation

Model	RMSE	
	Mean	Covariance
Model 1	25.6885	2.4929
Model 2	64.4090	6.5659

The following conclusions can be made from the upper simulation results: the first one, in model 1, although under the circumstance that the target with slower speed, far away from the observation and the system nonlinear and non-gaussian, IMM-PF can still estimate target with higher accuracy, this shows that the method in sonar filed has broad application prospects. The second one, tracking accuracy will be decreased in model 2 which the model set mismatch with the target's orbit.

V. CONCLUSIONS

This paper apply IMM-PF to track maneuvering target with multi-sonar, and simulate the algorithm in two model sets. The simulation results show that, when the model set

match with the target's motion model, IMM-PF can track maneuvering target with higher accuracy, but when the model set mismatch with the target's motion model, tracking accuracy will be decreased. So when the target is passive cooperation objective, according the target's orbit to set model. But when the target is non-cooperation target, choose models as much as impossible in the design of model set to cover all target's motion models, thus the tracking task can be fulfilled with higher accuracy.

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