Online Path Planning Based on MILP for Unmanned Surface Vehicles

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***Abstract*—This paper presents an algorithm for online path planning of USVs to navigate safely in dynamic, sophisticated environments of oceans. The proposed algorithm is based on Mixed Integer Linear Programming (MILP) which integrated with Velocity Obstacle (VO) approach. MILP is an optimization method under multiple constraint conditions of objective function maximization or minimization. The constraints of environment and maneuverability all can be considered and appended to the constraints conditions expressed in the form of inequality. The objective function and constraint conditions are required in linear by MILP, however, the motion of USV and its path planning are nonlinear. So the principle problem is to transform the nonlinear problem into the linear problem. On the other hand, VO makes a linear prediction, which is well appended into the constraint conditions. MILP has advantages in astringency, optimization, real-time and VO also has the advantages in real-time. The combination of MILP and VO utilized the advantages of rapidity of computations, which is well suited for embedded system of robotic applications. The algorithm is demonstrated via simulation with more safety and acclimation in sophisticated environments of oceans.**

Keywords—USV; Online path planning; MILP; Velocity Obstacle

I. INTRODUCTION

In recent years, there is a great advance in the capabilities of Unmanned Surface Vehicles (USVs). As USVs are operated in dynamic environments, they must have an online path planer which can avoid the moving obstacles and reach the target point. Online path planning is not a new problem for ground mobile robots, but it is a hard one for USVs. Because when the USVs are navigating in marine, they are usually much speedy so that they can't turn around urgently and stop in the turn. For the maneuverability of USVs, a special algorithm which predicts the emergence of future encounters and makes a decision in advance quickly should be well suited. Moreover the calculations should be small and simple for the embedded computer which carried by the USV.

Many studies have been done by some institutions in recent years. Interval programming is used to represent the navigation rules [1]. SSC San Diego who divided their obstacle avoidance system into two parts, in which the online path planning part is called deliberative obstacle avoidance [2][3]. JPL has integrated the velocity obstacle approach with the COLREGs for online obstacle avoidance [4], but it should combine a global path planner for generating a path in a mazelike environment while avoiding moving hazards. Even though the global path planner can conquer the flaw of local minimum, but the computational load can become very high and the USV can't react in time. Mixed Integer Linear Programming recently is used in online motion planning [5][6].

In this paper, the Autonomous Collision Avoidance System (ACAS) is used. This paper considers enough encounter situations which come from obstacle avoidance of ship. Judgment and decision are made for each situation. The decisions are expressed together with VO [7] in a very natural way. Through improvement of Velocity Obstacle which integration with encounter situations decisions, USVs are more suited for sophisticated environments of oceans. The algorithm also obeys COLREGS.

The rest of the paper is organized as follows. Section II simply introduces the structure of Autonomous Collision Avoidance System. Section III briefly reviews Velocity Obstacles, Mixed Integer Linear Programming and describes the intelligent decision making planner which include the linearization modeling of constraints base on VO and the linearization modeling of maneuverability constraint and objective function. Section IV shows the simulation results.

II. STRUCTURE OF ACAS

The structure of the Autonomous Collision Avoidance System of the USV is as shown in figure 1.The first part of the ACAS is the sensor part which contains Stereo, GPS, IMU and information preprocess module. This part mainly collects information of the USV. The second part of the ACAS is the hazard avoidance module which contains four sections. The function of the module is to avoid obstacles and plan its trajectory in time. The encounter situation judgment part and the risk judgment of collision part are described in another paper which is to be published. In this paper, the emphasis is on the intelligent part in which we will discuss the online path planning of the USV in dynamic, sophisticated environments of oceans. When there is many vehicles ahead the USV, negotiate obstacle avoidance part is applied to choose the best behavior to avoid them. The Last part is actuators which is the force the USV to the expected motion status.

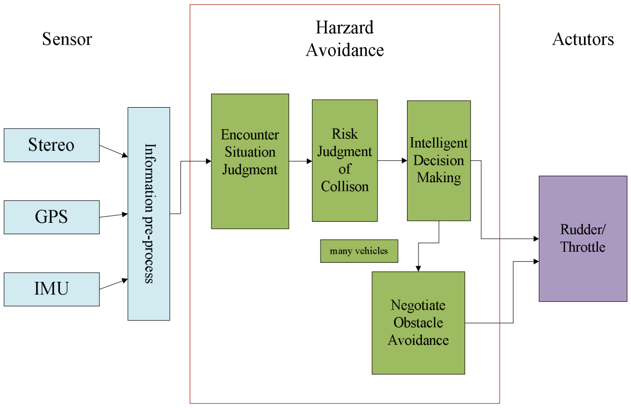


Fig. 1. The structure chart of ACAS

III. INTELLIGENT DECISION MAKING PLANNER

The Intelligent decision making planner contains the local path planner and the global path planner. Velocity Obstacle is utilized to avoid the moving obstacles in the local path planning. The VO chooses the right direction when the USV is avoiding moving obstacles by being one constraint in the constrain set of MILP. The MILP has a constrain set in which the velocity change is the decision variable which determines the objective function. VO is one of the constraints, so that the MILP can avoid going into the local minimum value.

A. Problen Statement

The problem considered in this paper is stated as follows. Given

- The velocity of the obstacle is lower than that of the USV
- All moving obstacles maintain the constant velocity within the tiny time step
- The relative position between the USV and obstacles remain unchanged within the tiny time step

In every tiny time step, the goal is to find the best velocity command that avoids the moving hazards and reaches the target point.

B. The standard model of Mixed Integer Linear Programming

In order to establish the linear programming model of the path planning problem, the key issue is to establish the model of constrains and objective functions. Mixed Integer Linear Programming is a kind of mathematical optimization model in which some local variables have to be integer. The standard form is defined as

$$\min(\max)\mathbf{c}^{\mathrm{T}}\mathbf{x}, \quad s.t.\mathbf{A}\mathbf{x} \le (\ge, =)\mathbf{b}, \quad \mathbf{x} \ge 0 \qquad (1)$$

In equation (1), $\mathbf{x} \in \Re^n$ is the variable to be solved, $c, A, b \in \Re^n$ are the known vector or matrix. Integer is integrated into MILP to solve the logical relation of and in constrains which can't be expressed in LP. In solving the path planning problem, the dynamics and kinematics of the USV, sensors and uncertainty constrains of the environment all can be taken into consideration. MILP is the optimization for the all free space. As a result, when the free space is not null, the algorithm is sure to find an optimal value. In the problem of path planning, if there is a feasible trajectory, an optimal trajectory can be positively found. The convergence of the algorithm can be guaranteed, and the found trajectory is the optimal trajectory about the entire free space.

C. Constrains and objective function

1) The linearization of Velocity Obstacle constrains : The VO can expressed as one of the constrain set.

The obstacle is expanded into a circle to simply the calulation [5]. Let $\mathbf{v}_u$ an $\mathbf{v}_o$ denote the velocity of the USV and the obstacle. Let $\mathbf{v}_{uo}$ and $\mathbf{L}_{uo}$ denote the relative velocity and position between the USV and the obstacle. Let γ_{uo} denote the angle between $\mathbf{L}_{uo}$ and $\mathbf{v}_{uo}$. Let UM and UN denote the tangent of the obstacle circle.Let γ_{uoc} denote the angle between UM and UO. The relative velocity global coordinate system is as shown in figure 2.

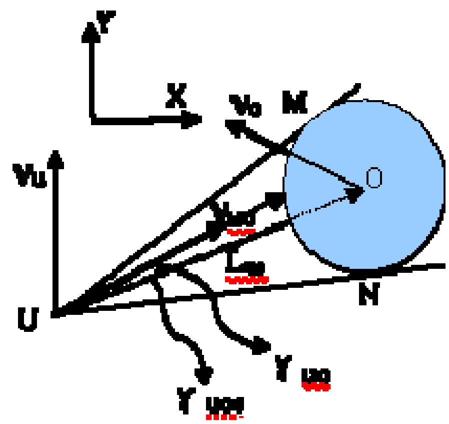


Fig. 2. The relative velocity global coordinate system

Let $\Delta\gamma_{uo}$ denote the changes in γ_{uo} . The Avoidance Angle (AA) is the angle that $\mathbf{v}_{uo}$ should steer to in order to avoid the obstacle and it is given as

$$\Delta\gamma_{UO} = -\frac{1}{\sqrt{V_{UO}^{2} L_{UO}^{2} - P^{2}}}\left[\mathbf{L}_{\mathbf{UO}} - \frac{P}{V_{UO}^{2}}\mathbf{v}_{\mathbf{uo}}\right].\Delta\mathbf{v}_{\mathbf{uo}} \qquad (2)$$

where $P = \mathbf{L}_{\mathbf{uo}} \bullet \mathbf{v}_{\mathbf{uo}}$, $\Delta\mathbf{v}_{uo}$ is the variable that need to be planned.

Rotating to the direction UM that is altering its course to portside and rotating to the direction UM that is alter its course to starboard. The linear constraints model of obstacle avoidance of the USV is given as

Steer to starboard:

$$\gamma_{UOC} \leq \gamma_{UO} + \Delta\gamma_{UO} \leq 2\pi - \gamma_{UOC} \tag{3}$$

Steer to portside

$$\gamma_{UO} + \Delta\gamma_{UO} \leq -\gamma_{UOC} \tag{4}$$

2) The linearization of the dynamics and kinematics constrains of the USV: the dynamics and kinematics can be expressed as another one of the constrain set.

The velocity and accelerated velocity must meet constrains as follows:

$$\begin{cases} -V_{u\max} \leq \mathbf{v}_{uj} \leq V_{u\max} \\ -\Delta_{u\max} \leq \Delta\mathbf{v}_{uj} \leq \Delta_{u\max} \end{cases} \tag{5}$$

Let $V_{u\max}$ denote the maximum velocity. Let $\mathbf{v}_{uj}$ denote one component of velocity vector. Let $\Delta_{u\max}$ denote the largest changes in velocity of the USV in one planning period. Let $\Delta\mathbf{v}_{uj}$ denote the changes of the $\mathbf{v}_{uj}$.

Furthermore, according to the velocity constraint, the following constrain can be reasoned.

$$-V_{u\max} \leq \mathbf{v}_{uj} + \Delta\mathbf{v}_{uj} \leq V_{u\max} \tag{6}$$

Combining equation (5) and equation (6), the acceleration component constraint can be expressed as follows:

$$\max\{-\Delta_{u\max}, -V_{u\max} - \mathbf{v}_{uj}\} \leq \Delta\mathbf{v}_{uj} \leq \min\{\Delta_{u\max}, V_{u\max} - \mathbf{v}_{uj}\} \tag{7}$$

Also the maximum rotation $\Delta\gamma_{uo\max}$ of the relative velocity between the USV and the obstacle is under constrains that the AUV can alter.

$\Delta\gamma_{uo\max}$ relate to the maximum instantaneous acceleration. Let $\Delta\gamma_{u\max}$ denote the maximum rotation of the USV. The definition of $\Delta\gamma_{uo\max}$ is as follows:

$$\Delta\gamma_{uo\max} = \begin{cases} \arcsin\dfrac{\Delta V_{u\max}}{V_{uo}} (\mathbf{v}_{uo} > \Delta\mathbf{v}_{uo\max}) \\ \Delta\gamma_{u\max} (\mathbf{v}_{uo} < \Delta\mathbf{v}_{uo\max}) \end{cases} \tag{8}$$

The rotation constraint of the USV is as follows:

$$-\Delta\gamma_{uo\max} \leq \Delta\gamma_{uo} \leq \Delta\gamma_{uo\max} \tag{9}$$

3) The linearization of the objective function:

Let L_{UG} and v_{UG} denote the relative distance and velocity between the USV and the target point. Let Δv_{UG} denote the changes in v_{UG}.

$$\min : \left| L_{UG} - (v_{UG} + \Delta v_{UG})\Delta t \right| \tag{10}$$

D. Algorithm flow.

The first step in the algorithm flow is to judge if the USV is in danger with the coming obstacles when stay course. If the USV is in danger, judging a moving vessel is in a specific situation that encounter situations divided. The division of encounter situation is the quantization based on COLREGS.

1) Checking Collision: To check the collision of the USV with vessels, CPA (Closest Point of Approach) needs to be calculated. $\mathbf{P}_{\mathrm{U}}$ and $\mathbf{P}_{\mathrm{U}}$ denote the position of the USV and a vessel, As defined in [3], the time to CPA is given by

$$t_{CPA} = \frac{(\mathbf{P}_{\mathrm{U}} - \mathbf{P}_{\mathrm{O}}) \bullet (\mathbf{v}_{\mathrm{U}} - \mathbf{v}_{\mathrm{O}})}{\left\| \mathbf{v}_{\mathrm{U}} - \mathbf{v}_{\mathrm{O}} \right\|^2} \tag{11}$$

The distance at CPA d_{CPA} is then calculated by

$$d_{CPA} = \left\| (\mathbf{P}_{\mathrm{U}} + \mathbf{v}_{\mathrm{u}} t_{CPA}) - (\mathbf{P}_{\mathrm{O}} + \mathbf{v}_{\mathrm{O}} t_{CPA}) \right\| \tag{12}$$

The checking collision rule of the USV can stay course by checking

$$0 \leq t_{CPA} \leq t_{\max} \quad \text{and} \quad d_{CPA} \leq d_{\min} \tag{13}$$

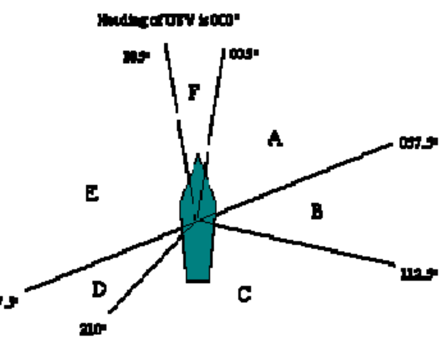


Fig. 3. Division of the encounter situation

2) Division and decision of encounter situation: There are six types of encounter situations. As shown in figure 3, the heading of the USV is 0°. If relative heading between the encouter traffic vessel and the USV is between 5° and 67.5°, the encounter situation is in A section. If relative heading between the encouter traffic vessel and the USV is between 67.5° and112.5°, the encounter situation is in B section. If relative heading between the encouter traffic vessel and the USV is between 112.5° and 210°, the encounter situation is in

C section. If relative heading between the encouter traffic vessel and the USV is between 210° and 247.5°,the encounter situation is in D section. If relative heading between the encouter traffic vessel and the USV is between 247.5° and 335°, the encounter situation is in E section. If relative heading between the encouter traffic vessel and the USV is between 335° and5°, the encounter situation is in F section.The division and decision of encounter situation are described in another paper.

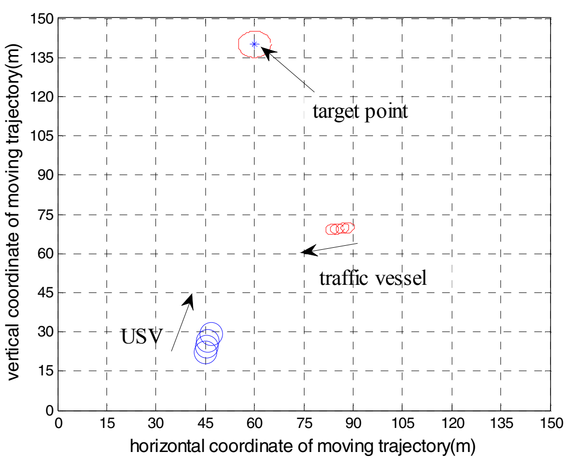
IV. SIMULATION

The simulations were made in Matlab2012b. There are some encounter situations in the simulation, the result shows the algorithm is very efficient and situated in dynamic oceans.

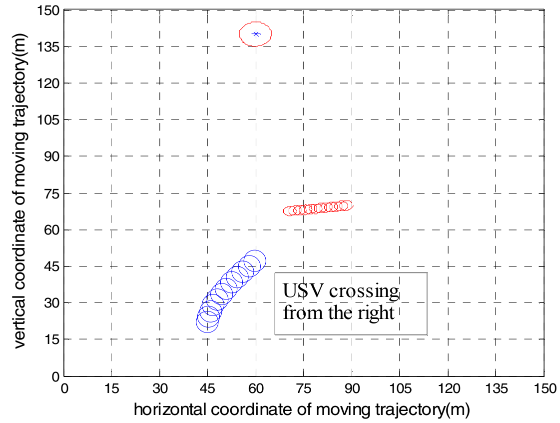
Figure 4(a) shows traffic vessel is crossing right of the USV. The blue circle represents the USV and the red smaller circle represents the traffic vessel.

The data in this simulation test during the right crossing process as shown in Figure 4(d) .The scalar velocity of the USV and the vessel are in the first column, blue represents the USV's speed, and green represents the traffic vessel, from which the initial velocity of the USV and the traffic vessel are 1.1m/s and 0.8m/s. The heading of the USV is in the second column in which the initial heading of the USV is 90 degree. In the last column the relative distance between the USV and the traffic vessel is shown from which the initial velocity of distance between them is about 65 meters. The distance can be justified to be dangerous distance, so the USV alter its velocity and heading constantly to avoid the traffic vessel. Figure 4(b) shows how the USV overtakes the traffic vessel. The USV speeded up and steered to the right during the first 1s and then slowed down to give way. Figure 4(c) shows the whole paths of the USV and the traffic vessel. When the USV reached the target point, the optimization process terminated.

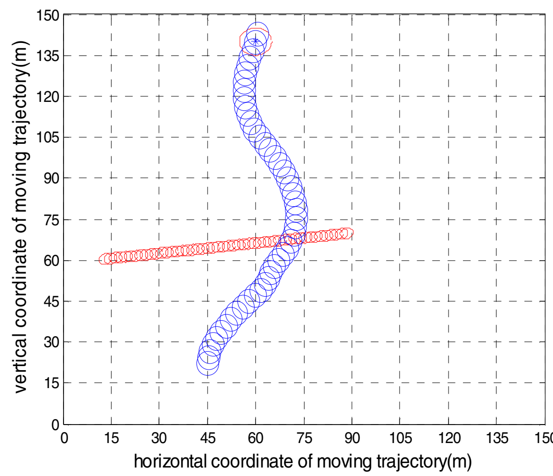
The iteration time of one optimization is 0.1s.The whole optimization time last 5s.



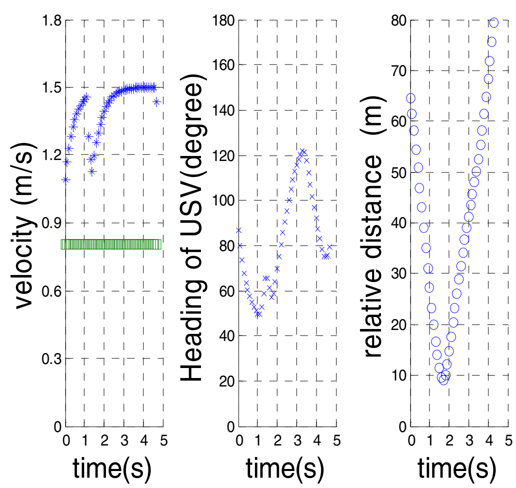
(a) The traffic vessel is crossing right of USV



(b) USV overtaking the traffic vessel from right

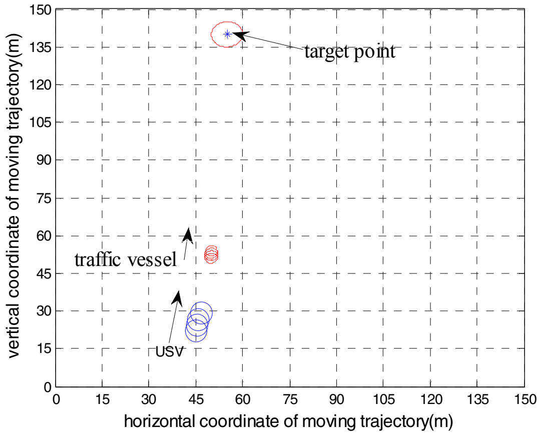


(c) The whole trajectory the USV overtaking the vessel and reach the target

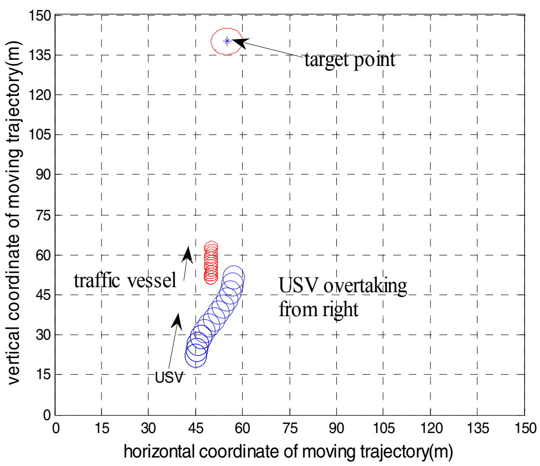


(d)Velocity, heading and relative distance between the USV and the traffic vessel

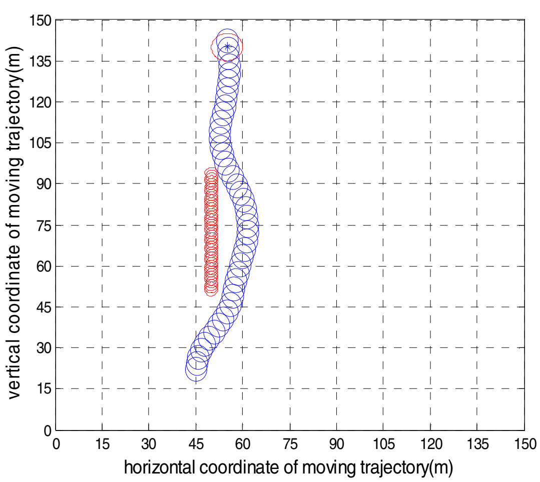
Fig. 4. Crossing from right



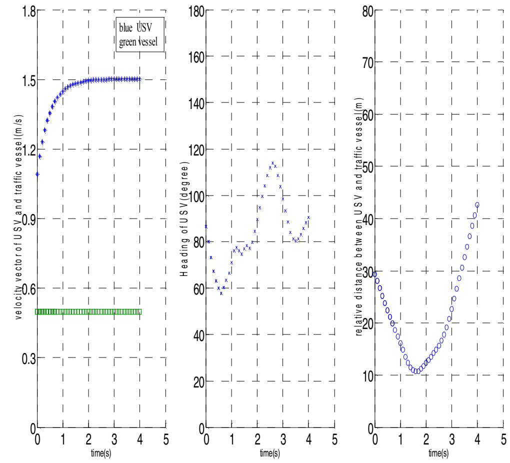
(a) The traffic vessel is ahead USV



(b)USV overtaking the traffic vessel from right



(c) The whole trajectory the USV overtaking the vessel and reach the target

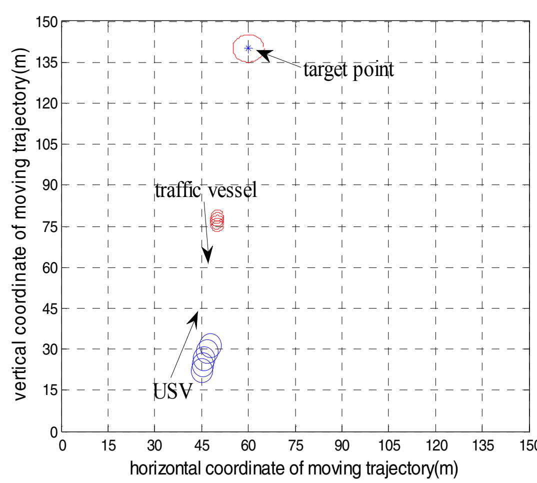


(d)Velocity, heading and relative distance between the USV and the traffic vessel

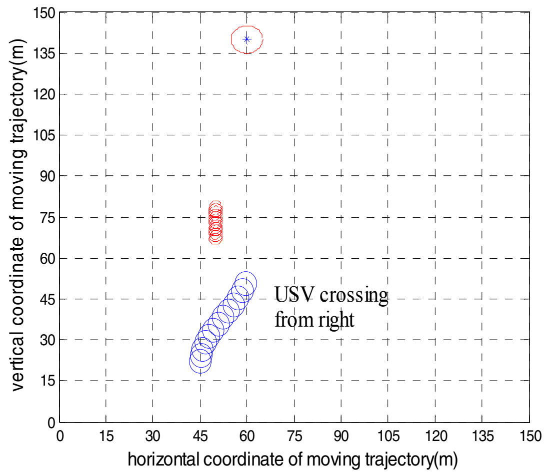
Fig. 5. Overtaking

Figure 5(a) shows the USV is going to overtake the traffic vessel which is ahead of it. The initial velocity of the USV and the traffic vessel are 1.1m/s and 0.5m/s. The initial heading of the USV is 90 degree. The initial the relative distance between the USV and the traffic vessel is 30 meters. The minimum distance between the USV and the vessel is 10 meters which is a very close distance. The velocity of USV must be larger than that of the vessel, and can speed up faster and faster which can be shown in second column of figure 5(d).

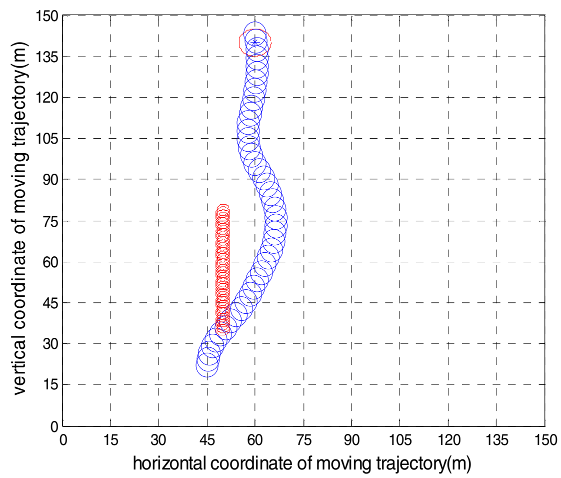
Figure 6(a) shows the USV is going to head to head to the traffic vesselThe initial velocity of the USV is 1.0m/s. The initial heading of the USV is 90 degree. The initial the relative distance between the USV and the traffic vessel is 60 meters. The minimum distance between the USV and the vessel is 13 meters which is a very close distance. The velocity of USV speed up faster which can be shown in second column of figure 6(d).



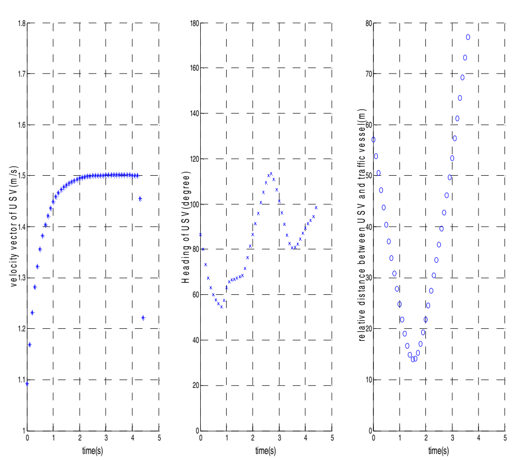
(a) The traffic vessel is head to head to the USV



(b) The USV makes a detour in order to avoid the head-on vessel

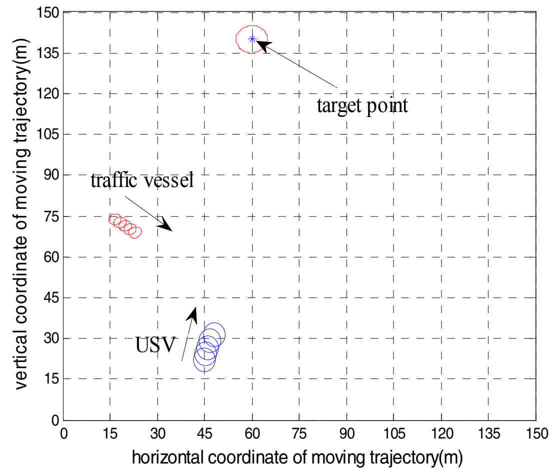


(c) The whole trajectory the USV overtaking the vessel and reach the target

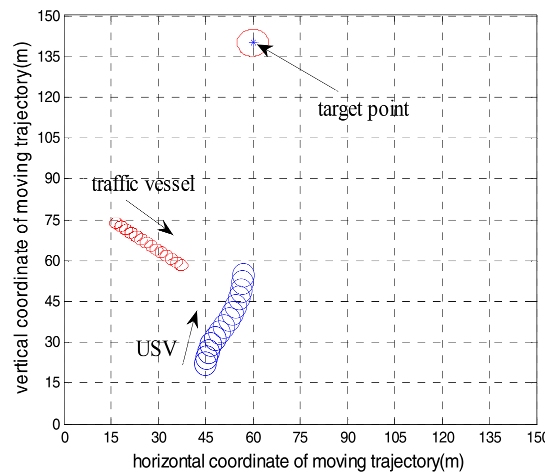


(d)Velocity, heading and relative distance between the USV and the traffic vessel

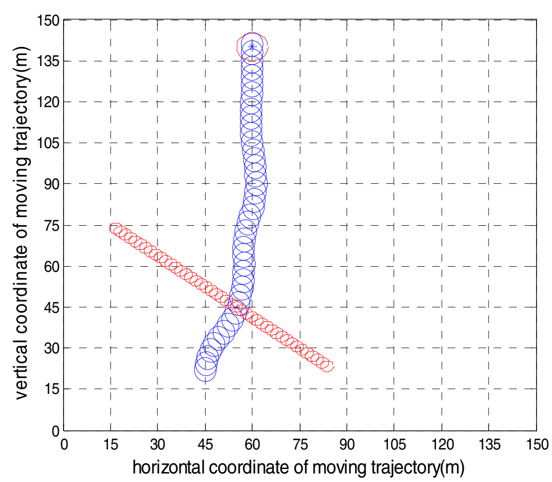
Fig. 6. Head-on



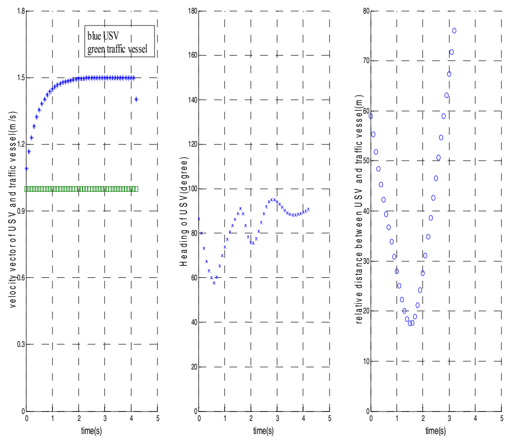
(a) The traffic vessel is crossing right of USV



(b) The USV steer to its right



(c) The traffic vessel is ahead USV



(d)Velocity, heading and relative distance between the USV and the traffic vessel

Fig. 7. Crossing from left

Figure 7(a) shows the traffic vessel is crossing from the left of the USV.The initial velocity of the USV and vessel are 1.1m/s and 1.0m/s. The initial heading of the USV is 90 degree. The initial the relative distance between the USV and the traffic vessel is 80 meters. The minimum distance between the USV and the vessel is 18 meters. The USV alters its heading so that the distance between them was increased.

V. CONCLUSION

This paper presented an online path planning algorithm using Mixed Integer Linear Programming and Velocity Obstacle. By regarding VO as one of the constraints of MILP, the global and optimal solution can be solved which avoids the moving obstacles and reaches the target point. The linearization of the dynamics and kinematics constrains of the USV and Velocity Obstacle constrains are presented. The algorithm flow also presented including checking collision using CPA, making a division and decision of encounter situation in which six types of situations are included. The algorithm has been demonstrated in simulations.

ACKNOWLEDGMENT

This research is supported by the Key Research Program of the Chinese Academy of Sciences, Grant NO.KGFZD-125-014.

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