

Nonlinear Model Predictive Control Based Ship Control System for Path Following

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Abstract—In many applications, it is of primary importance to steer an object along a desired path. For different controlled objectives and the dimension of the control forces, the path following control methods are usually classified into two kinds: the full-actuated and under-actuated control. Many conventional and adaptive control methods or schemes are presented for the path following control system of surface vessels. The path following control systems in some situation are required to operate at the limits of their capabilities so as to maximize the performance. In this paper, a nonlinear model predictive control based ship control system is presented for path following of surface vessels. For a nonlinear model predictive control system, it can directly take the saturation constraints into account. And the simulation results show that the nonlinear model predictive control strategy improves the system performance when the surface vessel is conduct the path following functionality.

Keywords—nonlinear model predictive control; path following; extend Kalman filter; ship model identification

I. INTRODUCTION

For the ship motion control problems, the ability to accurately maneuver a surface vessel along a given path is primarily important for most applications. In many practical applications, steering a ship to follow a predefined path is far more frequently adopted than stabilizing the vehicle in a fixed posture [1]. Hence, the path following problem attracts many concerns in practice. The ship path following control systems in some situations are required to operate at the limits of their capabilities so as to maximize the system performance. These constraints, in general, produce a degradation of the closed-loop performance, a reduction of the lifespan of the actuator and, and in some cases stability problems. Thus the constrained control strategy is welcome to the ship motion control systems. The nonlinear model predictive control (NMPC) strategy is one of the constraints control strategies which can handles constraints directly. The NMPC strategy is hopeful but challenging. It requires the formulation of an optimization problem based on the predictive model and the fidelity of the predictive model is of primary importance to the control performance. In this paper, an off-line identified predictive model is adopted to improve the fidelity of the predictive model..

Five parts is included in this paper. Part 2 describes the reference frame in the ship motion control systems and the 3 degree of freedom (DOF) horizontal ship motion model. Part 3

presents an EKF identification algorithm by which a predictive model is identified off-line from the sea trial data. Part 4 describe the path following problems, and part 5 proposes an NMPC based control strategy for path following problems. The simulation results are shown in Part 6, and the conclusions are made in part 7.

II. HORIZONTAL SHIP MOTION MODEL

For ship motion control systems, a simple horizontal ship motion control model will be employed for the controller design. In the NMPC scheme, the ship motion control model usually is taken as the predictive model. To highlights the robustness of the NMPC scheme, an off-line identified predictive model is taken with model mismatch.

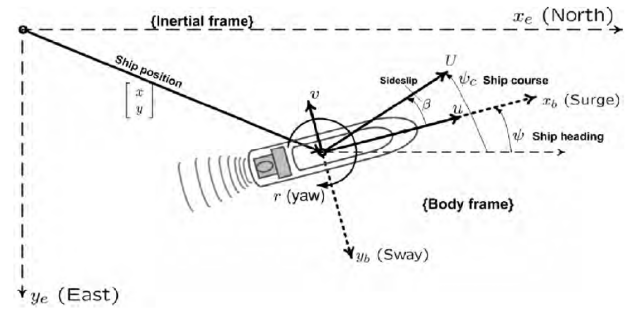


Figure 1. Notations and frame definitions for ship motion descriptions

A. Reference Frame

For marine vessels moving in the horizontal plain, the North-East-Down coordinate system $x_e y_e z_e$ (NED frame) and the body-fixed reference frame $x_b y_b z_b$ (B frame) is necessary to determine the position and the orientation. In the NED frame, the x -axis points towards the true North, and the y -axis points towards the East, and the z -axis points downwards normal to the Earth surface. The B frame is a moving coordinate frame which is fixed to the vessel. The position and orientation of the vessel are described relative to the NED frame while the linear and angular velocities of the vessel are expressed in the B frame, see Fig. 1.

B. The 3 DOF Ship Model

The horizontal motion of a ship is described by the motion components in surge, sway and yaw. Therefore, we choose $\mathbf{v} = [u \ v \ r]^T$ and $\boldsymbol{\eta} = [n \ e \ \psi]^T$ for the horizontal motion of surface vessels. For the horizontal motion of a vessel, the kinematic equations take the form as follows:

$$R(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The vessel kinematics and the dynamics are conventionally described as in [2]:

$$\dot{\boldsymbol{\eta}} = R(\psi) \mathbf{v} \quad (2)$$

$$M\dot{\mathbf{v}} = -C(\mathbf{v})\mathbf{v} - D\mathbf{v} - D_{NL}(\mathbf{v})\mathbf{v} + R^T(\psi)\mathbf{b} + \boldsymbol{\tau} + \mathbf{w}(t) \quad (3)$$

where $\boldsymbol{\eta} = [n \ e \ \psi]^T$ denotes the position and orientation of the vessel in the NED frame $x_e y_e z_e$ (see Fig. 1), and $\mathbf{v} = [u \ v \ r]^T$ denotes the translation velocities and angular rate in the B frame $x_b y_b z_b$, and $\boldsymbol{\tau}$ denotes the control forces and moments as system inputs. $R(\psi)$ is a state-dependent transformation matrix, as in (1). $M = M^T > 0$ is the mass and inertial matrix. $C(\mathbf{v})$ is the Coriolis and centripetal matrix. D is the damping matrix. $\mathbf{b}$ is a bias term representing slowly varying environmental forces and moment.

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix} \quad (4)$$

$$C(\mathbf{v}) = \begin{bmatrix} 0 & 0 & c_{13} \\ 0 & 0 & c_{23} \\ -c_{13} & -c_{23} & 0 \end{bmatrix} \quad (5)$$

$$D = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix} \quad (6)$$

$$D_{NL}(\mathbf{v}) = \begin{bmatrix} -X_{|u|u}|u| - X_{uuu}u^2 & 0 & 0 \\ 0 & -Y_{|v|v}|v| - Y_{|r|v}|r| & -Y_{|v|r}|v| - Y_{|r|r}|r| \\ 0 & -N_{|v|v}|v| - N_{|r|v}|r| & -N_{|v|r}|v| - N_{|r|r}|r| \end{bmatrix} \quad (7)$$

where $c_{13} = -(m - Y_{\dot{v}})v - (mx_g - Y_{\dot{r}})r$ and $c_{23} = (m - X_{\dot{u}})u$.

III. THE PREDICTIVE MODEL IDENTIFIED BY EKF

For many ships, safe and successful operation in the performance of its mission depends significantly on the ability of motion control of the ship in the environment in which it operates (current, wind, wave, shallow water etc.). For NMPC, the fidelity of the predictive model is of primary importance in the motion control abilities. Thus a proper ship motion model is demanded for the NMPC design. The hydrodynamic coefficients are the principal terms in the mathematical model used in the simulation, controller design and prediction of ship maneuvers. The application of system identification analysis techniques to specified ship trial maneuvers provides a way of “measuring” the hydrodynamic coefficients of the ship and helps to verify the proper form of the equations of motion used in the simulation, prediction and controller design.

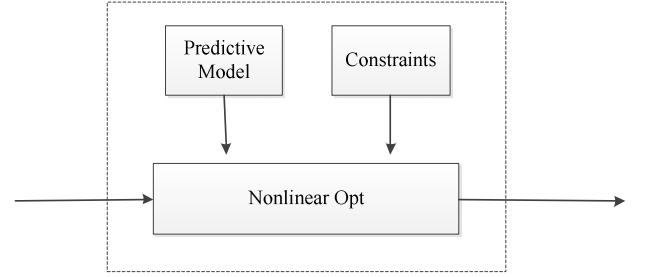


Figure 2. The predictive model is of primary importance in the NMPC frame.

Most system identification techniques essentially go through a process whereby the predicted values of state variables (the motion variables in our case of u , v , r and ψ as functions of time) are compared with the measured variables and the difference is the error in the estimation. The values of the coefficients are then continuously updated, as the data are passed through, by an algorithm which tends to reduce the error function, eventually seeking a set of coefficients which minimizes the error function. In the EKF approach, the hydrodynamic coefficients are treated as additional state variables but must be constant in time. A very simple qualitative description of system identification has been given in Fig.3.

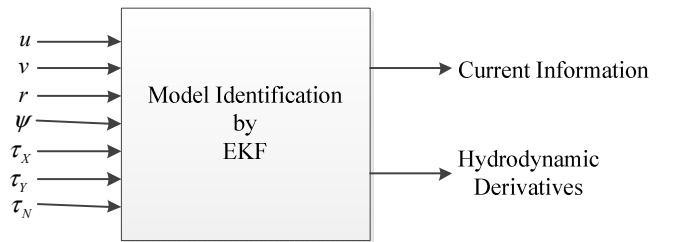


Figure 3. The block diagram of the model identification

In the EKF identification algorithm, the above 3 DOF horizontal model is adapted as the model structure. To take the surge freedom for example, an equivalent model to be identified is given as follows:

$$(m - X_{\dot{u}})\dot{u} = X_u u_r + X_{|u|u}|u_r|u_r + \tau_x \quad (7)$$

which equals

$$\dot{\mathbf{u}} = \frac{X_u}{m-X_u} \mathbf{u}_r + \frac{X_{|u|u}}{m-X_u} |u_r| u_r + \frac{1}{m-X_u} \boldsymbol{\tau}_X \quad (8)$$

we denote as follows:

$$a_1 = \frac{X_u}{m-X_u}$$

$$a_2 = \frac{X_{|u|u}}{m-X_u}$$

Then we get

$$\dot{\mathbf{u}} = a_1 \mathbf{u}_r + a_2 |u_r| u_r + \frac{1}{m-X_u} \boldsymbol{\tau}_X \quad (9)$$

Step 1 Import corresponding sea trial data

Step 2 Initialize the EKF algorithm

$$\hat{\mathbf{x}}_0 = \mathbf{x}_0$$

$$P_0 = \text{diag}\{10^6 \ 10^6 \ 10^6 \ 10^6 \ 10^6\}$$

Step 3 execute the EKF algorithm

$$\hat{\mathbf{x}}(k+1|k) = \mathbf{f}(\hat{\mathbf{x}}(k), \mathbf{u}(k))$$

$$P(k+1|k) = \mathbf{F}(k) \cdot P(k) \cdot \mathbf{F}^T(k)$$

$$K(k+1) = P(k+1|k) \cdot \mathbf{C}^T(k+1) \cdot [\mathbf{C}(k+1) \cdot P(k+1|k) \cdot \mathbf{C}^T(k+1)]^{-1}$$

$$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{x}}(k+1|k) + K(k+1)[\mathbf{y}(k+1) - \mathbf{C}(k+1)\hat{\mathbf{x}}(k+1|k)]$$

$$P(k+1) = [I - K(k+1) \cdot \mathbf{C}(k+1)] \cdot P(k+1|k)$$

Step 4 Validate the hydrodynamic derivatives by identification

Figure 4. The extended Kalman filtering identification steps

On account of the ability of the NMPC strategy and the EKF technique to naturally handle the discrete-time system, the mathematical model of the ship motion is discretized into the followings form:

$$u(k+1) = u(k) + \Delta T(a_1 u_r(k) + a_2 |u_r(k)| u_r(k) + \frac{1}{m-X_u} \tau_X) \quad (10)$$

where ΔT is the sample interval of the system. Then we get the system extended states $\mathbf{x} = [u \ U_C \ \psi_C \ a_1 \ a_2]^T$ and the system inputs $\mathbf{u} = [\tau_X \ \tau_Y \ \tau_N]^T$. The extended system state equation is:

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k))$$

$$\mathbf{y}(k+1) = \mathbf{h}\mathbf{x}(k+1) \quad (11)$$

where

$$\mathbf{h} = [1 \ 0 \ 0 \ 0 \ 0]$$

$$\mathbf{f} = [f_1 \ f_2 \ f_3 \ f_4 \ f_5]^T$$

$$f_1 = u(k) + \Delta T(a_1 u_r(k) + a_2 |u_r(k)| u_r(k) + \frac{1}{m-X_u} \tau_X)$$

$$f_2 = U_C(k)$$

$$f_3 = \psi_C(k)$$

$$f_4 = a_1(k)$$

$$f_5 = a_2(k)$$

The Fig. 4 shows the steps of the EKF based identification. The 3 DOF ship model identification results are shown in Fig. 5 and Fig. 6.

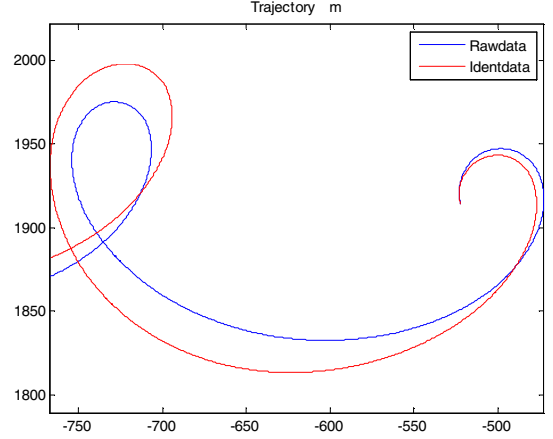


Figure 5. Comparing the trajectories between the sea trial data and the data generated by the identified ship model with the same inputs.

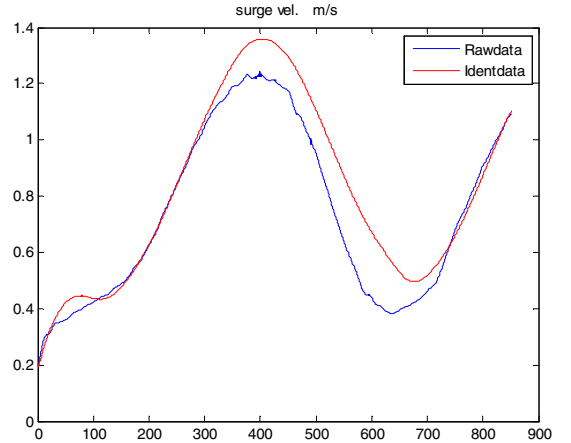


Figure 6. Comparing the surge velocities between the sea trial data and the data generated by the identified ship model with the same inputs.

IV. THE SHIP PATH FOLLOWING PROBLEM

The route of a surface vessel is usually specified in terms of way-point. Each way-point is defined using Cartesian coordinates (x_k, y_k) for $k=1, \dots, n$. An attractive method for the path following control is the line-of-sight guidance [4]. A line-of-sight (LOS) vector from the vessel to the next way-

point or a point on the path between the two way-points can be computed for a heading control. In many situation, the line-of-sight vector is taken as a vector from the body-fixed origin (x, y) to the next way-point (x_k, y_k) [5], as in Fig. 7. This suggests the guidance heading of the surface vessel should be chosen as:

$$\psi_d(t) = \text{atan2}(y_k - y(t), x_k - x(t)) \quad (12)$$

At each line segment, we attach a local path reference frame R_i with origin at past way-point $P_{i-1} = (x_{i-1}, y_{i-1})$ and the x-axis points towards the next way-point $P_i = (x_i, y_i)$.

Letting $\varepsilon_1 = [10]^T$, $\varepsilon_2 = [01]^T$ and ψ_i be the angle of rotation of frame R_i in the NED frame [6]. From above we can obtain

$$\psi_i = \text{atan2}\left(\frac{\varepsilon_2^T(P_i - P_{i-1})}{\varepsilon_1^T(P_i - P_{i-1})}\right) \quad (13)$$

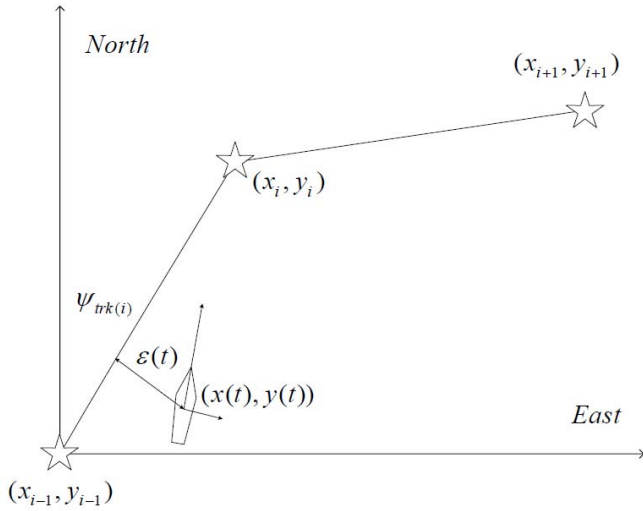


Figure 7. A surface vessel is performing the way-point path following functionality.

A vector in the R_i frame is mapped to the NED frame by the rotation matrix $R_2(\psi_i)$:

$$R_2(\psi_i) = \begin{bmatrix} \cos \psi_i & -\sin \psi_i \\ \sin \psi_i & \cos \psi_i \end{bmatrix} \quad (14)$$

For a ship current position point p in form of a vector in the NED frame, letting $\eta(p, i) = R_2^T(p - p_i)$ be the local representation of the point p expressed as a vector in the R_i reference frame. Thus $\varepsilon_1^T \eta(p, i)$ is the projection of the

vector p onto the line segment, and the term $|\varepsilon_2^T \eta(p, i)|$ is the norm distance from the point p to the line segment (often referred to as the cross-track error). In the low-speed path following problems, the ship follows the carrot points generated on the path segments, who's initial point is given based on the projection point of p to the line segment. The proportional coefficient α to generate the objective carrot point is defined as follows:

$$\alpha = \frac{|\varepsilon_1^T \eta(p, i)|}{|p_{i-1} - p_i|} \quad (15)$$

The norm line through the point p to the line segment is the joint with the line segment, denoted by the point Q . Then we can obtain

$$Q = \alpha p_{i-1} + (1 - \alpha) p_i \quad (16)$$

V. THE NMPC FOR THE PATH FOLLOWING PROBLEM

The path following control system is depicted in Fig. 3. Here the controller in the block diagram is referred to the NMPC scheme. For the convenience of analysis, we assume that:

- a) The whole states $\mathbf{x}$ and the bias term $\mathbf{b}$ are known (measurable).
- b) The noise and the unmodeled dynamics are ignored, that is $\mathbf{w}(t) = 0$.

The NMPC strategy is a feedback control strategy based on the online solution of a moving horizon optimal control problem (OCP). The OCP to be solved is to compute a sequence of inputs $\{\mathbf{u}(k)\}$ that will take the process from its current states $\mathbf{x}(k)$ to desired states $\mathbf{x}^{ref}(k)$. Feedback disturbances are typically handled by assuming that step disturbances have entered at the outputs and that it will remain constant in the whole future prediction horizon. Assuming that X is an admissible state set, and U is an admissible input set, and U^{N_c} is an admissible input sequence set. At each sampling time instant k , $k = 0, 1, 2, \dots$. The NMPC algorithm is stated as follows:

- a) To measure the states $\mathbf{x}(k) \in X$
- c) Based on the current states $\mathbf{x}(k)$, to solve the optimal control problem:

minimize

$$\begin{aligned} V^*(\mathbf{x}(k)) &= \min_{\mathbf{u}(\cdot)} J(\mathbf{x}(k), \mathbf{u}(\cdot), \mathbf{x}^{ref}(k)) \\ &= \min_{\mathbf{u}(\cdot)} \left(\sum_{i=0}^{N_p-1} \ell(\mathbf{x}_{k+i|k}, \mathbf{u}_{k+i}, \mathbf{x}^{ref}_k) + V_f(\mathbf{x}_{k+N_p|k}) \right) \end{aligned} \quad (17)$$

with respect to

$$u(\cdot) \in U^{N_c}(\mathbf{x}(k)) \quad (18)$$

$$\mathbf{x}_{k|k} = \mathbf{x}(k) \quad (19)$$

subject to

$$\mathbf{x}_{k+i|k} = \mathbf{f}(\mathbf{x}_{k+i|k}, \mathbf{u}_{k+i}) \quad (20)$$

$$\mathbf{y}_{k+i|k} = \mathbf{C}\mathbf{x}_{k+i|k} + \mathbf{b}_k \quad (21)$$

$$\mathbf{y}_{\min} \leq \mathbf{y}_{k+i|k} \leq \mathbf{y}_{\max}, i=1, \dots, N \quad (22)$$

$$\mathbf{u}_{\min} \leq \mathbf{u}_{k+i} \leq \mathbf{u}_{\max}, i=1, \dots, N \quad (23)$$

$$\Delta \mathbf{u}_{\min} \leq \Delta \mathbf{u}_{k+i} \leq \Delta \mathbf{u}_{\max}, i=1, \dots, N \quad (24)$$

d) Define the NMPC control strategy as $\mathbf{u}^*(0) \in U$ and use this control values in the next sampling period.

The cost function of the OCP includes two parts. The former part is the running cost and the latter part is the terminal cost. Usually, the cost function takes the forms as follows

$$\begin{aligned} J(\mathbf{x}(k), \mathbf{u}(\cdot)) = & \sum_{i=0}^{N_p} (\mathbf{x}_{k+i|k} - \mathbf{x}^{ref}(k))^T \mathbf{Q} (\mathbf{x}_{k+i|k} - \mathbf{x}^{ref}(k)) \\ & + \sum_{i=0}^{N_c} \Delta \mathbf{u}_{k+i}^T \mathbf{R} \Delta \mathbf{u}_{k+i} \\ & + ((\mathbf{x}_{k+N_p|k} - \mathbf{x}^{ref}(k))^T \tilde{\mathbf{Q}} (\mathbf{x}_{k+N_p|k} - \mathbf{x}^{ref}(k))) \end{aligned} \quad (25)$$

In theory, the most straightforward way to modify the NMPC algorithm to achieve the nominal stability involves setting the prediction and control horizons to infinity. From a practical point of view, however, it is simply not possible to solve the OCP of the NMPC scheme with infinite horizons for a realistic problem. The focus of recent research efforts has been to obtain a computationally tractable approximation of the infinite horizon problem that still retains desirable closed-loop properties. Thus we add to the NMPC algorithm a terminal constraint

$$(\mathbf{x}_{k+N+1} - \mathbf{x}^{ref}) \in W \quad (26)$$

With such an enforced constraint, the objective function of the NMPC scheme becomes a Lyapunov function for the closed-loop system, leading the system to nominal stability.

VI. THE SIMULATION

The controlled ship is a multifunctional support vessel. The mass is $m = 6.12 \times 10^6 \text{ kg}$. It's fully actuated, and the path following control design is based on the full state measured assumption. The hydrodynamic parameters are identified by the EKF technique.

For the low-speed path following problem, we assume that only the position information is of interest. So the orientation information is just settled without changing. The controller settings are as follows: the control horizon $N_c = 3$, the prediction horizon $N_p = 15$, and the sampling period $\Delta t = 0.3 \text{ sec}$. Considering the inputs limits, the thruster

forces and moment limits are $|\tau_x| < 900 \text{ kN}$, $|\tau_y| < 320 \text{ kN}$ and $|\tau_N| < 1280 \text{ kNm}$ respectively. The state, input and the terminal constraint weight matrices are $\mathbf{Q} = \text{diag}\{10^6, 10^6, 0, 0, 0, 0\}$, $\mathbf{R} = \text{diag}\{10^7, 10^7, 10^8\}$ and $\tilde{\mathbf{Q}} = \text{diag}\{10^5, 10^5, 0, 0, 0, 0\}$ respectively.

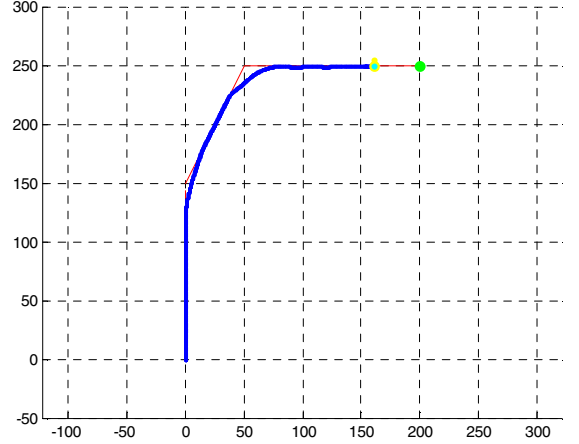


Figure 8. The ship follows a three-segment path.

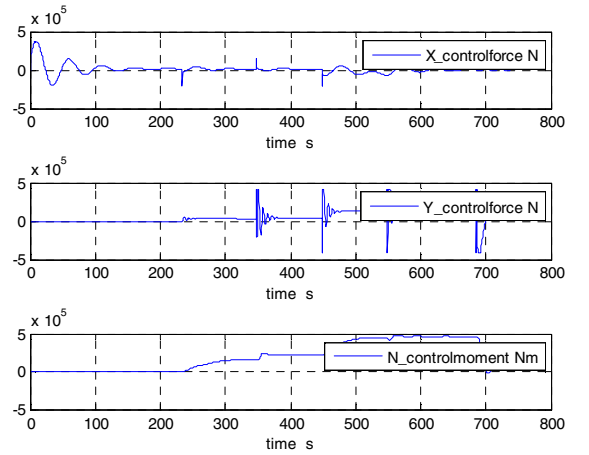


Figure 9. The control forces and moment calculated by the NMPC method during the way-point path following

The ship firstly follows a three-segment path, see Fig. 8. The red path is the reference path. The yellow point runs from the top point. The blue path is the path of the yellow point which represents the ship. The velocity of the ship is 1 knot, and the other states value see Fig. 10. The control forces and moment are shown in Fig. 9.

In conventional path following problems, for example, the PID control strategy, the controller parameters must go with the following speed. The PID controller parameters following a path at 1 knot are different from them following a path at 2 knots. One of the advantages of the NMPC strategy is its robustness.

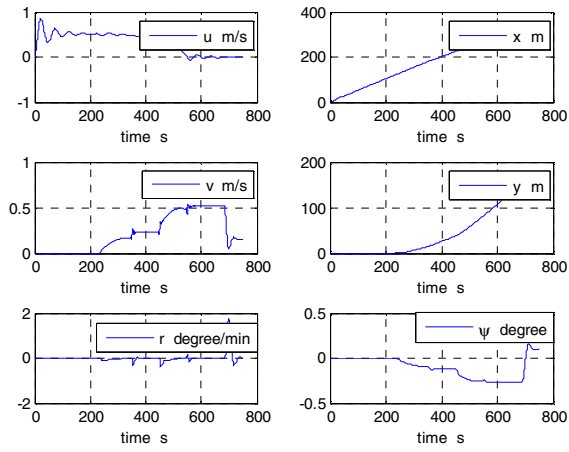


Figure 10. The system states value during the path following

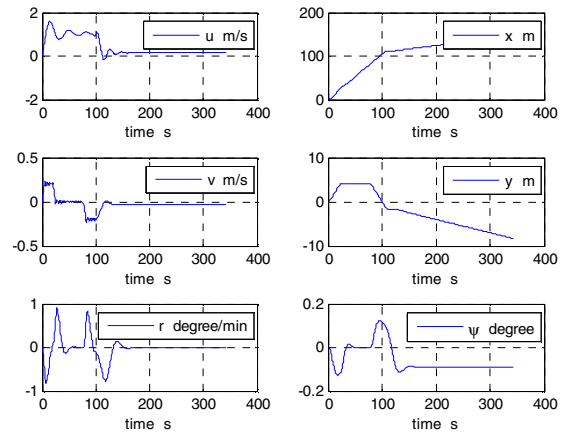


Figure 13. The state values during the following at 2 knots.

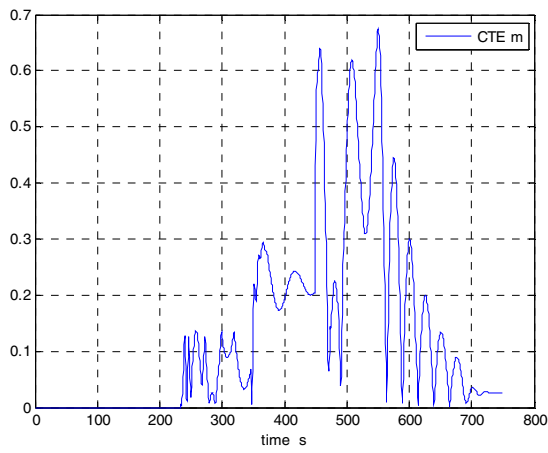


Figure 11. The cross track error curve during the path following

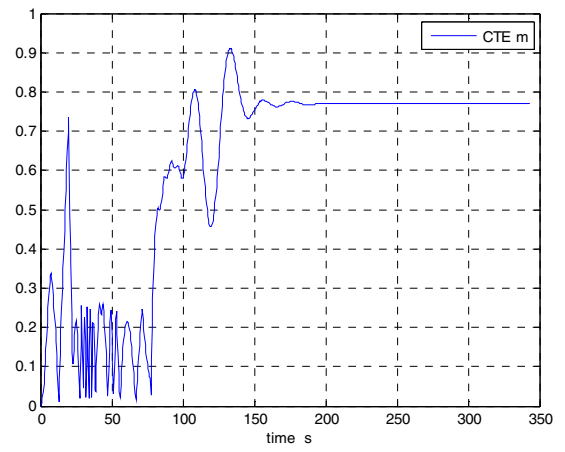


Figure 14. The cross track error during the following at 2 knots.

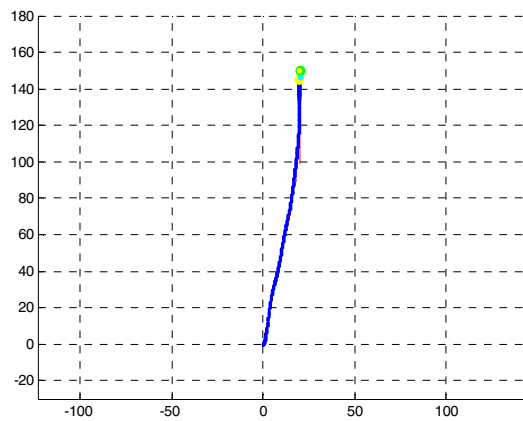


Figure 12. The way-point path with auto-generated curve, the ship is following well.

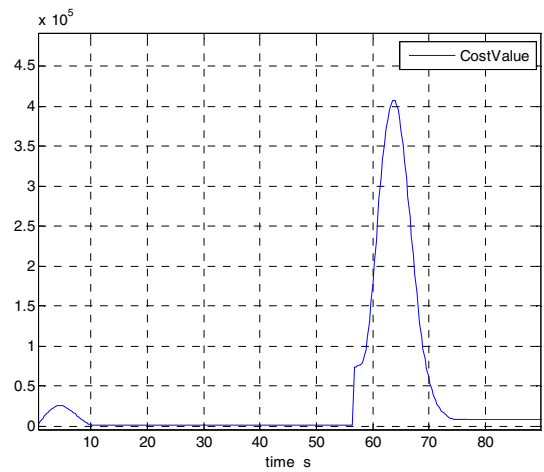


Figure 15. The cost value of the NMPC algorithm during the ship following an ellipsoid path.

The same parameters of the NMPC strategy are taken to follow a path at 2 knot in Fig. 12. The simulation conditions are the same as above, except for the desired speed during the path following. Fig.14 shows that the NMPC can make the ship follow the way-point path effectively without tuning controller parameters.

VII. THE CONCLUSION

This paper concerns the hydrodynamic coefficients identification using the extend Kalman filter technique problem and the NMPC strategy for the surface vessel path following problem. With mild, moderate and wild sea trial data, the corresponding identification is executed. The data generated by the predictive model identified by the EKF are compared with the full-scale sea trial data. The identified data agree the sea trial data well. Meanwhile, an NMPC strategy is proposed for the low-speed way-point path following problems. The NMPC strategy based ship motion controller can handle constraints and multi models effectively with robustness and adaptation. This is of great both theory and practical significances. The ship path following control scheme presented in this paper not only can handle the constraints directly, but also can be adaptive and robust to the different speed, which has a brilliant prospect in the ship motion control field, especially in the low-speed path following problems.

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