Accurate FDTD Analysis of Underwater Acoustic Modal Equation Using Transparent Source

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***Abstract*—A transparent source is implemented for investigation of its accuracy as an optimal type of source in the analysis of wideband path loss in underwater acoustic channels based on finite-difference time-domain (FDTD) scheme. By increasing the cut-off time of transparent source scheme, the exact transparent source for the underwater acoustic modal equation is implemented. The accuracy of the transparent source is verified by comparing the wideband path-loss results with famous normal mode solution, KRAKEN.**

I. Introduction

The normal mode theory is a well-known algorithm that can predict acoustic wave propagation accurately in an underwater channel [1], [2]. However, the method calculates the response at a single frequency only, which may be time-consuming for a wideband analysis since it must be repeated at every frequency point. Therefore, a new normal mode algorithm based on the finite-difference time-domain (FDTD) scheme has been proposed which is capable of calculating the response over a wideband through just one solution in the time domain [3].

However, it has been reported that the accuracy of FDTD analysis depends largely on the source implemetation method [4], [5]. Therefore, it is very important not only to analyze effects of FDTD source schemes on the results of normal mode analysis but also to implement exact source to predict the wideband path-loss exactly. In this paper, the transparent source is implemented as an optimal type of source, which can preserve the original magnitude while separating the excited and reflected waves. Also, the simulation condition, cut-off time, is proposed to implement the transparent source accurately even if the horizontal wavenumber is variously changed, which generates dispersive characteristic of the propagating wave. Moreover, the accuracy of wideband path-loss result using transparent source is verified comparing with famous normal mode solution, KRAKEN.

II. Analysis of FDTD Sources

A. Soft Source

The following is the underwater acoustic modal equation derived by applying the separation of variables technique to the acoustic wave equation, which is then solved by FDTD scheme in the proposed algorithm [3], [7]. Equation (1) is the modal equation in the time domain, whereas Equation (2) is the update equation of the FDTD scheme to solve Equation (1).

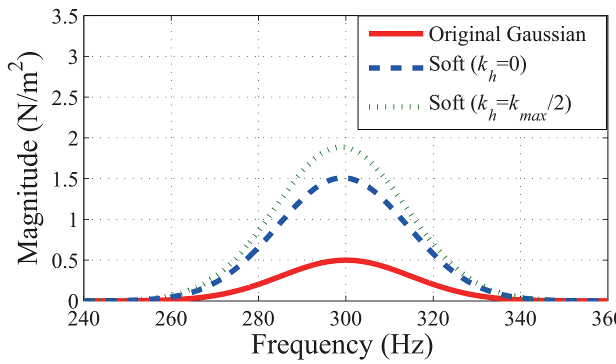

Fig. 1. Frequency domain responses of original Gaussian pulse and soft sources with different horizontal wavenumbers.

$$\frac{\partial^2 Z(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 Z(z,t)}{\partial t^2} - k^2 Z(z) = \delta\left(z - z_s\right) s\left(t\right). \quad (1)$$

$$\begin{aligned} Z_i^{n+1} = \;& (c\Delta t)^2 \left(\frac{Z_{i+1}^n - 2Z_i^n + Z_{i-1}^n}{\Delta z^2}\right) \\ & + 2Z_i^n - Z_i^{n-1} - (c\Delta t k_h^m)^2 Z_i^n. \end{aligned} \quad (2)$$

Where $Z(\cdot)$ is the pressure, and k_h is a separation constant known as the horizontal wavenumber. The superscript n and the subscript i in $Z(\cdot)$ are the indexes of the discretized time and space, respectively. The superscript m of k_h is the discretized number of the horizontal wavenumber. The size of time step and spatial grid are determined as $\Delta z = \lambda_{min}/20$ and $\Delta t = S\Delta z/c$, respectively. where S is Courant stability constant which is assumed as 0.8 in this work.

Equation (2) applies to everywhere in an underwater channel, including the source node. However, a problem occurs at the source node when excites a soft source. The source term, $s(t)$, of Equation (1) is generated as discretized form of s_i^n, and this source term has to substitute the left-hand side of Equation (2) at the source node. However, the right-hand side of Equation (2) is added by algorithm of the soft source, even though the magnitude at source node should have just original level. Therefore the soft source suffers from relatively

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large error of magnitude modification, even though the scheme can consider the interference between the excited and reflected wave from other boundaries. In addition, the errors are changed as discretized horizontal wavenumber, k_h^m, of Equation (2) as shown in Figure 1. Therefore, it is very difficult to normalize these magnitude modified results to original Gaussian pulse.

B. Hard Source

The hard source is one of the famous FDTD source, which can prevent the error of the soft source. It is very simple to implement by imposing the Gaussian pulse on the source node such as Equation (3).

$$Z_i^{n+1} = s_i^n. \tag{3}$$

The source node does not suffer from the problem of magnitude modification, however, the hard source can not transmit the reflected wave by other boundaries. For example, Figure 2 shows that a hard source is located immediately above a boundary of two layers. In this case, the hard source reflects back the reflected wave from the boundary, and maintains the magnitude of original Gaussian pulse at the source node as shown in Figure 3. When speeds of two mediums are different, the reflection coefficient is given by $\mathit{\Gamma} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{1}{3}$ where $Z_1 = c\rho = 1500$ and $Z_2 = c\rho = 3000$. Therefore, there exists reflected wave which have to be superposed with original Gaussian pulse, however, the magnitude at the source node is fixed with the original Gaussian pulse. Figure 3 shows the reflected wave which can be analytically obtained by multiplying the original Gaussian pulse with reflection coefficient. However, the hard source scheme can not consider this reflected wave as shown in Figure 3.

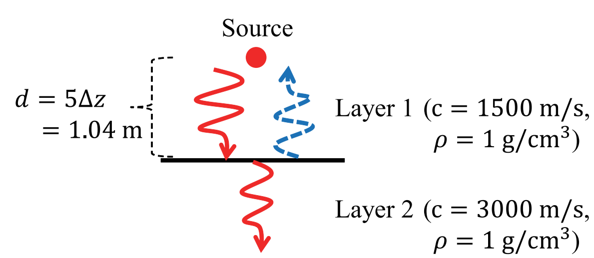


Fig. 2. Schematic when source is immediately above boundary of two layers.

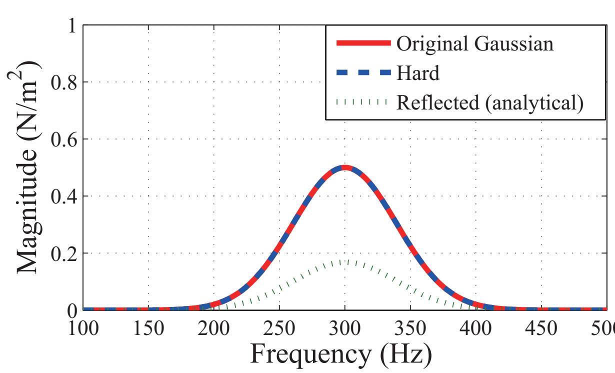
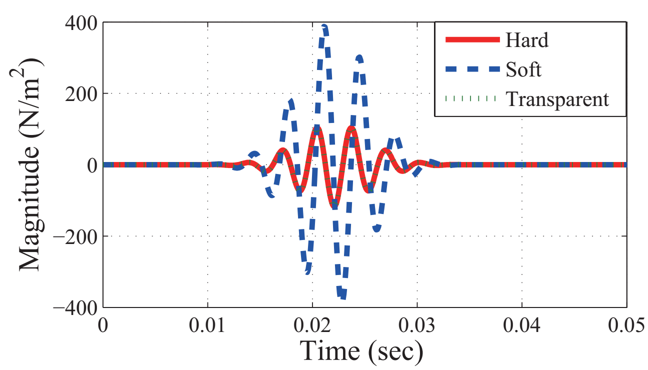


Fig. 3. Frequency domain responses of original Gaussian pulse, hard source, and reflected wave obtained analytically.

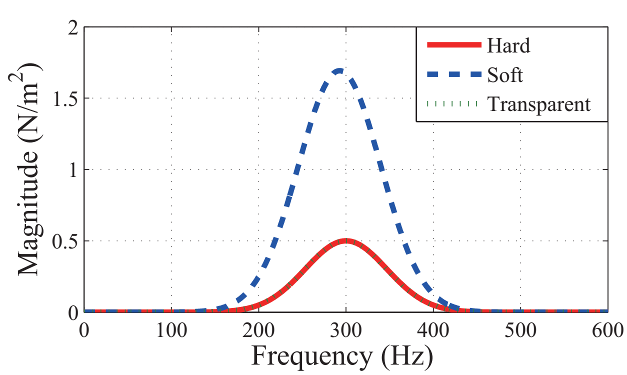
C. Transparent Source

The transparent source maintains the power level of the original source, while it separates the wave generated by the source and those reflected by other boundaries, as in the case with the soft source. Hence, the transparent source scheme preserves the spectrum of the source while precisely considers the interference between the excited and the reflected waves. To implement the transparent source, the cause of magnitude modulation have to be removed, which come from right-hand side of Equation (2). Therefore, the differences between the soft and hard source are precalculated as increasing the time step, and compensated by subtracting it from the soft source scheme [6].

To implement transparent source efficiently and exactly, there is one important simulation condition that determines the end of time step of compensating process, which is called as cut-off time. The cut-off time is needed to implement FDTD source efficiently and exactly as introduced at ref. [5] which determines the cut-off time of $2T_0$. T_0 is time duration between $t = 0$ and time step that Gaussian pulse reaches the maximum magnitude. Equation (4) shows the original Gaussian pulse in time domain which is frequency modulated as f_0 and time shifted as T_0.

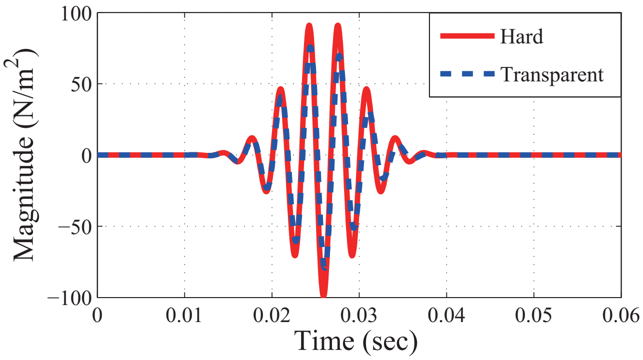


(a) Time domain responses.

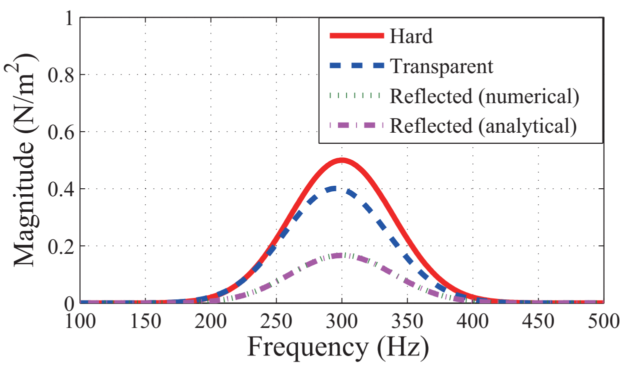


(b) Frequency domain responses.

Fig. 4. Responses of hard, soft, and transparent sources with cut-off time of $2T_0$. For all cases, $k_h = 0$.

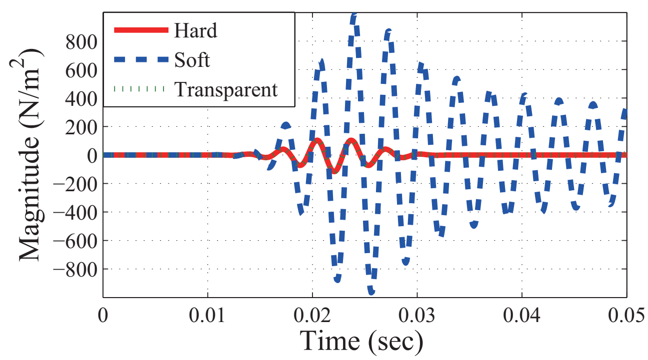


(a) Time domain responses.

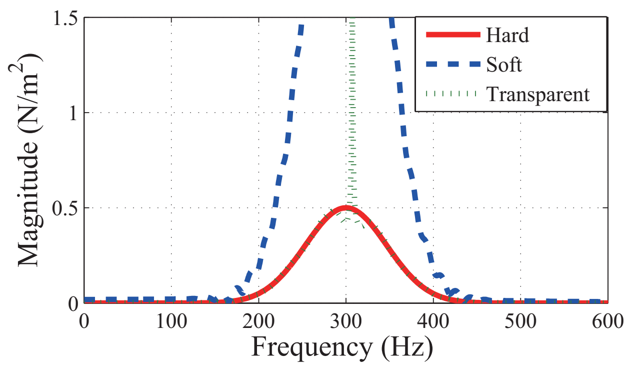


(b) Frequency domain responses.

Fig. 5. Responses of hard, transparent sources and reflected waves obtained numerically, and analytically. For all cases, $k_h = 0$.



(a) Time domain responses.



(b) Frequency domain responses.

Fig. 6. Results of hard, soft, and transparent sources with cut-off time of $2T_0$. For all cases, $k_h = 0.767k_{max}$.

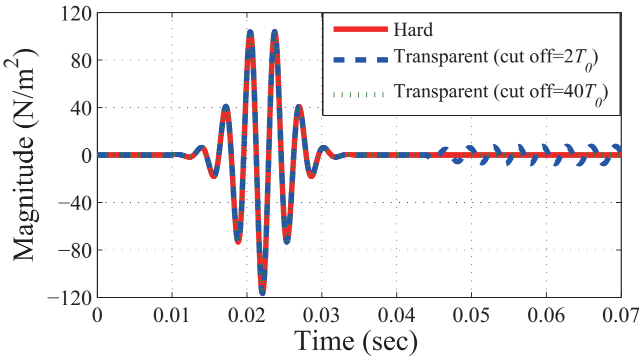
$$s_i^{n+1} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(n\Delta t - T_0)^2}{2\sigma^2}} \cos(2\pi f_0(n\Delta t - T_0)) \quad (4)$$

To implement Equation (4) as the transparent source exactly, the compensating process of subtracting the precalculated errors from the soft source scheme is applied during the cut-off time.

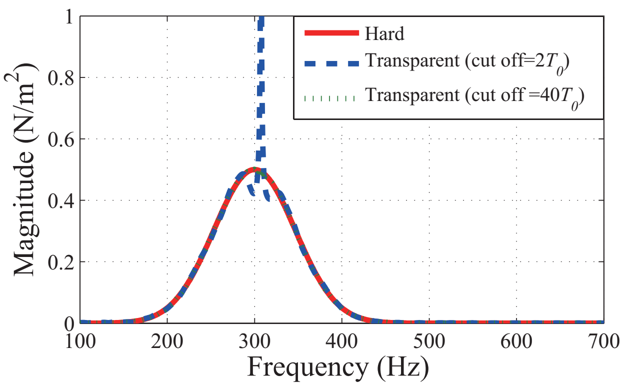
Figure 4 shows the comparison of the Gaussian pulses with the horizontal wavenumber, $k_h = 0$, in the time and frequency domain at the location of the sources recorded. As seen, the magnitude of the soft Gaussian pulse is larger than the hard source. However, the transparent source maintains its level same with the hard source using the cut-off time of $2T_0$. In addition, to show the transparency of the source node comparing with the hard source scheme, an example shown in Figure 2 is simulated using the transparent source. Figure 5(a) shows the time domain response superposed with excited and reflected wave at the source node. In addition, Figure 5(b) shows the frequency domain response of superposed result at the source node comparing with the hard source sheme. Moreover, Figure 5(b) shows that the fourier transform of difference between transparent and hard source is well coincide with reflected wave by boundary, which can be obtained exactly by multiplying the original Gaussian pulse with the reflection coefficient.

However, significant problem happens when the horizontal wavenumber, $k_h \neq 0$. When $k_h = 0$, the Equation (1) is Helmholtz's equation in time domain, however, the Equation (1) is changed to modal equation which has a dispersive characteristic when $k_h \neq 0$. The group velocity is different for all of frequency components, therefore, the cut-off time is needed to extend to compensate the wave which has the slowest group velocity in time domain even though the optimal cut-off time is proposed for Helmholtz's equation in Ref. [5]. Figure 6(a) shows the time domain response with $k_h = 0.767k_{max}$ at the source node. The response of soft source scheme has large magnitude modulation with significant dispersive characteristic which is shown as protractive response in time domain. Moreover, Figure 6(b) shows the frequency domain responses that the transparent source dose not correspond with the hard source when cut-off time is used as $2T_0$.

Therefore, the sufficient cut-off time have to be applied to implement the transparent source for the modal equation. In this paper, the sufficient cut-off time is used as $40T_0$. Figure 7(a) shows the time domain responses of transparent sources with different lengths of cut-off time. The response of transparent source scheme using cut-off time of $2T_0$ has small ripple after 0.04 sec, which is not perfectly compen-



(a) Time domain responses.



(b) Frequency domain responses.

Fig. 7. Responses of hard, soft, and transparent sources with cut-off time of $40T_0$. For all cases, $k_h = 0.767k_{max}$.

sated by transparent source scheme. However, the transparent source using cut-off time of $40T_0$ is exactly coincide with the response of hard source. Moreover, Figure 7(b) shows the frequency domain responses that the transparent source is well correspond with the hard source when cut-off time is used as $40T_0$.

III. Path-Loss Results

Based on the transparent source, the path-loss simulation results are obtained and compared with the famous normal mode algorithm, the KRAKEN code [1]. To calculate the path-loss, the Fourier-Bessel integral shown as Equation (5) is analyzed, which is well known as the mathematical expression of the path-loss [1], [2].

$$P(r, z_r) = \frac{1}{2\pi}\int_0^\infty G(z_s, z_r; k_h) J_0(k_h r) k_h dk_h. \qquad (5)$$

As proposed in Ref. [3], The Green's function of Equation (5) is analyzed based on FDTD scheme using the exactly implemeted transparent source. In addition, the path-loss integral is analytically calculated based on pole-residue estimation.

In this paper, the Gaussian pulse to analyze wideband

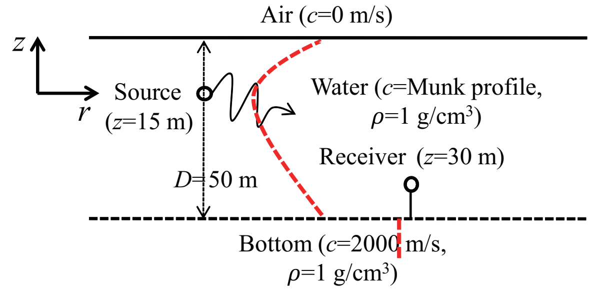
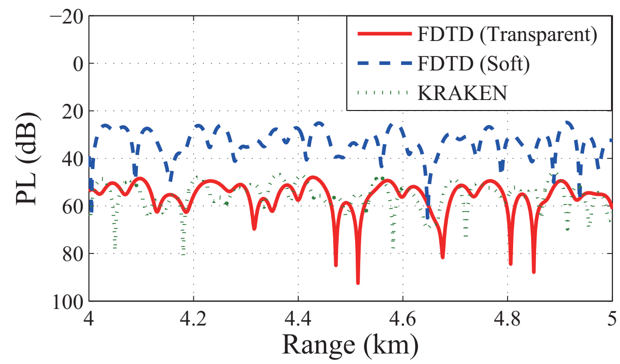
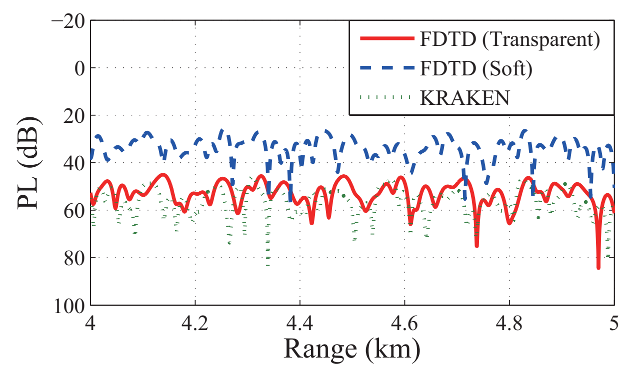


Fig. 8. Schematic of an underwater acoustic channel with the Munk speed profile.



(a) f=240 Hz



(b) f=360 Hz

Fig. 9. Calculated path losses in Munk speed profile using transparent and soft sources with f_c = 300 Hz and BW=120 Hz. For comparison, KRAKEN results are shown also.

response is centered at 300 Hz with a bandwidth of 120 Hz. The transmission geometry is shown in Figure 8 for the Munk speed profile. For the FDTD simulation, 60000 time steps and discretizing number of the horizontal wavenumber domain, $N_k = 3000$, are used. Figure 9 shows the path-loss results for the Munk speed profile at the lowest and highest frequency, 240 Hz and 360 Hz, respectively. As shown, the proposed normal mode analysis based on FDTD scheme adapting a transparent source shows much accurate results, with the path-loss results nearly matching those from the KRAKEN code.

IV. CONCLUSION

This paper demonstrates the accuracy of the transparent source for analyzing the underwater acoustic modal equation. The simulation condition, cut-off time, is proposed to implement transparent source exactly. By compensating the slowly propagating wave using sufficient cut-off time, the transparent source preserves the original magnitude for different horizontal wavenumber, and maintains characteristic of transparency which can separate the excited and reflected wave. Moreover, the path-loss simulation results are obtained based on the exactly implemented transparent source. The accuracy of the path-loss simulation results are verified by comparing with the famous normal mode algorithm, the KRAKEN code.

ACKNOWLEDGMENT

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