Second Order Divided Difference Filter applied in Underwater Bearing Only Target Tracking

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Abstract: A new method based on the second order divided difference filter (DDF2) is proposed and applied for underwater target tacking using passive bearings only measurements of Unmanned Underwater Vehicle (UUV) in this paper. The target kinematics model and passive sonar measurement model of UUV are built in Cartesian-Coordinates. In contrast to Taylor's expansion of extended Kalman filter (EKF), the second order divided difference filter (DDF2) algorithm is a nonlinear estimator , it makes use of second order stirling's interpolation to achieve the polynomial approximation and requires no derivatives. By using the presented nonlinear target tracking model, the performance of three filters EKF, the first order divided difference filter (DDF1) and DDF2 is compared. Simulation results show that, under the same initial conditions, DDF2 and DDF1 are more robust than EKF, and the performance of DDF2 is slightly better than DDF1.
Keywords: *Second Order Ddivided Difference Filter; Unmanned Underwater Vehicle; Bearing-Only Target Tracking; First Order Divided Difference Filter; Extended Kalman Filter.*

1. INTRDUCTION

In order to be able to track an enemy underwater target, the UUV must be able to perform target motion analysis (TMA). TMA is the mission of the UUV which needs the UUV to have the ability to determine the position, course, speed and range of the target. For maintaining secrecies, the only information can be used would be the bearings between the UUV and the target, resulting in bearing-only target tracking (BOTT) problems.

Under the condition of noise measurement, a high quality estimation algorithm should have unbiasedness, convergence, stability, etc. Due to the nonlinear characteristics of the BOTT problems, the solving methods have diversities and complexities. In the early days, the deterministic solution algorithm is the mainly solving method[1]. The target is under the assumption of uniform linear motions, so the system needs to four input variables, which is mainly composed of bearings and/ or the order rate measurements. The deterministic solution algorithm has advantage in solving target motion parameters fast, but the algorithm is sensitive to measurement errors, and the parameter estimation accuracy is difficult to guarantee. In the case of multiple measuring bearings, the simplest method is the least square method[2]. It uses the linearized mathematical measurement equation and the time-varying linear least squares method to estimate parameters such as the target position and velocity vectors. The least square method mainly drawback is that it needs large amount of calculation. In the 1960s, researchers introduce the kalman filter into BOTT problem[3, 4]. There are several successful improved Kalman filter algorithm to solve the BOTT problem, such as the extended Kalman filter[5-7] (EKF) and unscented Kalman filte[r][8, 9] algorithm. The EKF uses the first-order Taylor approximations to linearize the nonlinear dynamic and measurement functions. It needs to calculate the Jacobian matrix or Hessian matrix. Application of the EKF is therefore contingent on the assumption that it required derivatives exist. The UKF realizes the state estimated by constructing a set of weighted sample point to avoid the nonlinear terms linearization and calculation of the Jacobian Matrix. UKF can be directly applied to nonlinear system as a state estimator. The EKF and UKF have the filter divergence problem that caused by the noise signal, the calculation error or other factors.

In this paper we propose a new algorithm, which is referred to as the second order divided difference filter (DDF2) and based on polynomial approximations of the nonlinear transformations obtained with second order Stirling's interpolation formula. The new algorithm's implementation principle resembles that of the EKF, however, is significantly simpler for it does not need to formulate the Jacobian and Hessian matrices of partial derivatives of the system.

In 2005, Lee[10] presentes the DDF filtering method in his doctoral thesis, one of which is the first order divided difference filter (DDF1) that based on the first order Stirling's interpolation, the other is the DDF2 that based on the second order Stirling's interpolation. Ref.[11] proposes the ballistic target tracking based on DDF2. The paper compares DDF2 with EKF and UKF by Monte Carlo simulation approach and simulation results show that the DDF2 method has relatively consistent stability. Ref.[12, 13] propose an algorithm based on data fusion DDF2 and apply to a rigid body attitude angle estimation. Ref.[14] uses the DDF2 algorithm in global position navigation system, and proves that the estimation accuracy of DDF2 is significantly higher than the DDF1 and EKF.

This paper introduces the DDF2 algorithm into BOTT. The simulation results showed that the filtering accuracy of DDF2 and DDF1 algorithms is much better than the EKF. For the DDF2 algorithm uses the second order

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Stirling's interpolation to linearize the nonlinear terms, the accuracy of DDF2 algorithm is higher than DDF1, and DDF2 filtering stability is superior to the other two methods.

The paper is organized as follows. First we introduce the mathematical model of the target tracking system. Then the DDF2 filter based on the model is proposed. The performance of the algorithm is given by Monte Carlo simulation approach.

2. PROBLEM FORMULATION

The UUV and target can be considered as two points in the space, the acceleration of the target can be assumed to be Gaussian random process.

The measurement information of UUV's passive sonar is generally obtained from Polar/Cartesian coordinate system, which is transforming the BOTT problem into nonlinear state estimation problem.

2.1 Target Tracking System Model

This paper considers the two-dimensional plane of uniform linear motion target in Cartesian coordinate system only. The target position and speed can be denoted as (x^t, y^t) and $(\dot{x}^t, \dot{y}^t)$, then target state vector is defined as:

$$\mathbf{x}^t = [x^t \ \dot{x}^t \ y^t \ \dot{y}^t]^T \tag{1}$$

The corresponding UUV state vector is defined as:

$$\mathbf{x}^o = [x^o \ \dot{x}^o \ y^o \ \dot{y}^o]^T \tag{2}$$

Where (x^o, y^o) and $(\dot{x}^o, \dot{y}^o)$ is the position and speed of the UUV. The state vector of the relative movement of the target and platform can be expressed as:

$$\mathbf{x} = \mathbf{x}^t - \mathbf{x}^o = [x \ \dot{x} \ y \ \dot{y}]^T \tag{3}$$

According to the ref.[15], the equation of the state of the system model is expressed as:

$$\mathbf{x}_{k+1} = \mathrm{f}(\mathbf{x}_k, \mathbf{v}_k, k) = \mathbf{F}\mathbf{x}_k + \mathbf{\Gamma}\mathbf{v}_k \tag{4}$$

Where $\mathbf{F}$ and $\mathbf{\Gamma}$ are transition matrix as equation (5) and (6) shows, respectively. T is the sample period, $\mathbf{v}_k$ is the process noise and satisfies Gaussian distribution vector $\mathbf{v}_k \sim N(\bar{\mathbf{v}}, \mathbf{Q}_k)$, $\mathbf{Q}_k = \sigma_a I_2$, σ_a is a scalar, $\mathbf{I}_2$ is a 2×2 unit matrix.

$$\mathbf{F} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & T & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$\mathbf{\Gamma} = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix} \tag{6}$$

The relative motion of the system is shown in Fig.1, ψ^t is the heading of target, V^t is the target velocity, V^o is the UUV's velocity, $(r_0 \cdots r_k)$ is the relative distance sequence.

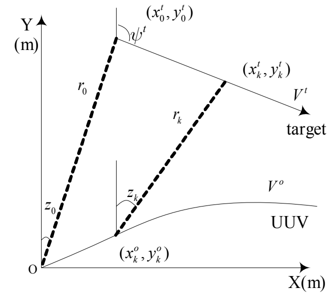

Fig.1 Geometry relationship between UUV and target

2.2 UUV's Passive Sonar Measurement Model

For the UUV's passive sonar accomplish target tracking only by the relative bearings of the target, the measurement model of the measurement model can be expressed as:

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, \boldsymbol{\omega}_k, k) = \mathbf{H}(\mathbf{x}_k) + \boldsymbol{\omega}_k \tag{7}$$

Where $\mathbf{H}(\mathbf{x}_k)$ is the truly bearings, as (8) shows. $\boldsymbol{\omega}_k$ is the measurement noise and satisfy Gaussian distribution vector $\boldsymbol{\omega}_k \sim N(\bar{\boldsymbol{\omega}}_k, \mathbf{R}_k)$.

$$\mathbf{H}(\mathbf{x}_k) = \theta_k = \arctan(x_k / y_k) \tag{8}$$

As above, the essence of the BOTT problem is the process of giving bearings measurement sequence $\mathbf{Z}_k = \{\mathbf{z}_i\}_{i=1}^k$ to estimate the state vector $\mathbf{x}_k$. As the passive sonar measurement model is a nonlinear model about state vector $\mathbf{x}_k$, BOTT problem is a nonlinear filtering problem. Ref.[16, 17] pointed out that the UUV must make an appropriate maneuver to complete target tracking.

3. SECOND ORDER DIVIDED DIFFERENCE FILETR

The DDF2 is obtained by using the calculation of the mean and covariance in the second order polynomial approximation section, and taken the same predictor-corrector structure as the EKF [18].

3.1 Second Order Stirling's Interpolation

Consider the nonlinear function as follows:

$$\mathbf{y} = f(\mathbf{x}) \tag{9}$$

The random variable $\mathbf{x} \in \mathbf{R}^n$ has gaussian density with mean $\bar{\mathbf{x}}$ and covariance $\mathbf{P}_\mathbf{x}$. A linear transformation of $\mathbf{x}$ is introduced as follows:

$$\mathbf{z} = \mathbf{S}_\mathbf{x}^{-1}\mathbf{x} \tag{10}$$

The matrix $\mathbf{S}_\mathbf{x}$ is the square Cholesky factor of the covariance $\mathbf{P}_\mathbf{x}$, such as $\mathbf{P}_\mathbf{x} = \mathbf{S}_\mathbf{x}\mathbf{S}_\mathbf{x}^T$. The elements of $\mathbf{z}$ become mutually uncorrelated. A new function $\tilde{f}$ is defined as:

$$\mathbf{y} = f(\mathbf{S}_\mathbf{x}\mathbf{x}) = \tilde{f}(\mathbf{z}) \tag{11}$$

The second order stirling's interpolation about $\bar{\mathbf{z}}$ is show as:

$$\mathbf{y} \cong \tilde{f}(\bar{\mathbf{z}}) + \tilde{\mathrm{D}}_{\Delta\mathbf{z}}\tilde{f} + \frac{1}{2!}\tilde{\mathrm{D}}^2_{\Delta\mathbf{z}}\tilde{f} \tag{12}$$

Where the operators $\tilde{\mathrm{D}}_{\Delta\mathbf{z}}\tilde{f}$ and $\tilde{\mathrm{D}}^2_{\Delta\mathbf{z}}\tilde{f}$ are defined as follows:

$$\tilde{\mathrm{D}}_{\Delta\mathbf{z}}\tilde{f} = \frac{1}{h}\left(\sum_{j=1}^{n}\Delta\mathbf{z}_j\mu_j\delta_j\right)\tilde{f}(\bar{\mathbf{z}}) \tag{13}$$

$$\tilde{\mathrm{D}}^2_{\Delta\mathbf{z}}\tilde{f} = \frac{1}{h^2}\left(\sum_{j=1}^{n}\Delta\mathbf{z}_j^2\delta_j^2 + \sum_{j=1}^{n}\sum_{\substack{i=1\\ i\neq j}}^{n}\Delta\mathbf{z}_j\Delta\mathbf{z}_i(\mu_j\delta_j)(\mu_i\delta_i)\right)\tilde{f}(\bar{\mathbf{z}}) \tag{14}$$

Where $\Delta\mathbf{z} = \mathbf{z} - \bar{\mathbf{z}}$, $\Delta\mathbf{z}_j$ is the *j*-th element of $\Delta\mathbf{z}$. h is the selected interval length.

The partial operators μ and δ are defined as follows:

$$\mu_j f(\bar{\mathbf{z}}) = \frac{1}{2}\left[\tilde{f}(\bar{\mathbf{z}} + \frac{h}{2}\mathbf{e}_j) + \tilde{f}(\bar{\mathbf{z}} - \frac{h}{2}\mathbf{e}_j)\right] \tag{15}$$

$$\delta_j f(\bar{\mathbf{z}}) = \tilde{f}(\bar{\mathbf{z}} + \frac{h}{2}\mathbf{e}_j) - \tilde{f}(\bar{\mathbf{z}} - \frac{h}{2}\mathbf{e}_j) \tag{16}$$

Where $\mathbf{e}_j$ is the unit column vector.

Now ,the approximate mean $\bar{\mathbf{y}}$, covariance $\mathbf{P}_{\mathbf{yy}}$, and cross-covariance $\mathbf{P}_{\mathbf{xy}}$ of $\mathbf{y}$ is calculated as follows:

$$\bar{\mathbf{y}} = \mathbf{E}[\mathbf{y}] = \frac{h^2-n}{h^2}f(\bar{\mathbf{x}}) + \frac{1}{2h^2}\sum_{j=1}^{n}[f(\bar{\mathbf{x}} + h\mathbf{s}_{\mathbf{x},j}) + f(\bar{\mathbf{x}} - h\mathbf{s}_{\mathbf{x},j})] \tag{17}$$

$$\begin{aligned}\mathbf{P}_{\mathbf{yy}} &= \mathbf{E}\left[(\mathbf{y}-\bar{\mathbf{y}})(\mathbf{y}-\bar{\mathbf{y}})^T\right] \\ &= \frac{1}{4h^2}\sum_{j=1}^{n}\left[f(\bar{\mathbf{x}}+h\mathbf{s}_{\mathbf{x},j}) - f(\bar{\mathbf{x}}-h\mathbf{s}_{\mathbf{x},j})\right]\times\left[f(\bar{\mathbf{x}}+h\mathbf{s}_{\mathbf{x},j}) - f(\bar{\mathbf{x}}-h\mathbf{s}_{\mathbf{x},j})\right]^T \\ &= +\frac{h^2-1}{4h^2}\sum_{j=1}^{n}\left[f(\bar{\mathbf{x}}+h\mathbf{s}_{\mathbf{x},j}) + f(\bar{\mathbf{x}}-h\mathbf{s}_{\mathbf{x},j}) - 2f(\bar{\mathbf{x}})\right]\times\left[f(\bar{\mathbf{x}}+h\mathbf{s}_{\mathbf{x},j}) + f(\bar{\mathbf{x}}-h\mathbf{s}_{\mathbf{x},j}) - 2f(\bar{\mathbf{x}})\right]^T\end{aligned} \tag{18}$$

$$\mathbf{P}_{\mathbf{xy}} = \mathbf{E}\left[(\mathbf{x}-\bar{\mathbf{x}})(\mathbf{y}-\bar{\mathbf{y}})^T\right] = \frac{1}{2h}\sum_{j=1}^{n}\mathbf{s}_{\mathbf{x},j}[f(\bar{\mathbf{x}}+h\mathbf{s}_{\mathbf{x},j}) - f(\bar{\mathbf{x}}-h\mathbf{s}_{\mathbf{x},j})]^T \tag{19}$$

Where $\mathbf{s}_{\mathbf{x},j}$ is the *j*-th column of $\mathbf{S}_\mathbf{x}$.

3.2 Second Order Divided Difference Filter based BOTT

The BOTT system is composed by equations (4) and (7). First, define the following square Cholesky factorizations [19]:

$$\mathbf{P}_k = \mathbf{S}_\mathbf{x}\mathbf{S}_\mathbf{x}^T \tag{20}$$

$$\mathbf{Q}_k = \mathbf{S}_\mathbf{v}\mathbf{S}_\mathbf{v}^T \tag{21}$$

$$\mathbf{R}_k = \mathbf{S}_\boldsymbol{\omega}\mathbf{S}_\boldsymbol{\omega}^T \tag{22}$$

DDF2 computes the predicted state vector $\hat{\mathbf{x}}_{k+1}^-$ as below:

$$\begin{aligned}\hat{\mathbf{x}}_{k+1}^- &= \frac{h^2-(n_\mathbf{x}+n_\mathbf{v})}{2h^2}\mathbf{f}(\hat{\mathbf{x}}_k^+, \bar{\mathbf{v}}_k) \\ &+ \frac{1}{2h^2}\sum_{p=1}^{n_\mathbf{x}}\left\{\mathbf{f}(\hat{\mathbf{x}}_k^+ + h\mathbf{S}_{\mathbf{x},p}, \bar{\mathbf{v}}_k) + \mathbf{f}(\hat{\mathbf{x}}_k^+ - h\mathbf{S}_{\mathbf{x},p}, \bar{\mathbf{v}}_k)\right\} \\ &+ \frac{1}{2h^2}\sum_{p=1}^{n_\mathbf{v}}\left\{\mathbf{f}(\hat{\mathbf{x}}_k, \bar{\mathbf{v}}_k + h\mathbf{S}_{\mathbf{v},p}) + \mathbf{f}(\hat{\mathbf{x}}_k, \bar{\mathbf{v}}_k - h\mathbf{S}_{\mathbf{v},p})\right\}\end{aligned} \tag{23}$$

Where $h=\sqrt{3}$ is usually set for a Gaussian distribution, $n_\mathbf{x}=4$ and $n_\mathbf{v}=2$ are the dimension of the state vector and noise vector, respectively, $\mathbf{S}_{\mathbf{x},p}$ is the p th column of $\mathbf{S}_\mathbf{x}$ matrix, $\mathbf{S}_{\mathbf{v},p}$ is the p th column of $\mathbf{S}_\mathbf{v}$ matrix.

The state prediction covariance matrix $\mathbf{P}_{k+1}^-$ is calculated by the following formula:

$$\mathbf{P}_{k+1}^- = \mathbf{S}_\mathbf{x}^-(k+1)(\mathbf{S}_\mathbf{x}^-(k+1))^T \tag{24}$$

Where $\mathbf{S}_\mathbf{x}^-(k+1)$ is a symmetric matrix, as shown in equation (12). Equation (11) shows that the prediction covariance matrix calculated by the method meeting the positive definiteness.

$$\mathbf{S}_\mathbf{x}^-(k+1) = [\mathbf{S}_{\mathbf{x}\hat{\mathbf{x}}}^{(1)}\,\mathbf{S}_{\mathbf{x}\hat{\mathbf{v}}}^{(1)}\,\mathbf{S}_{\mathbf{x}\hat{\mathbf{x}}}^{(2)}\,\mathbf{S}_{\mathbf{x}\hat{\mathbf{v}}}^{(2)}] \tag{25}$$

$$\mathbf{S}_{\mathbf{x}\hat{\mathbf{x}}}^{(1)}(k+1) = \left\{\left[\mathbf{f}_i\left(\hat{\mathbf{x}}_k^+ + h\mathbf{S}_{\mathbf{x},j}, \bar{\mathbf{v}}_k\right) - \mathbf{f}_i\left(\hat{\mathbf{x}}_k^+ - h\mathbf{S}_{\mathbf{x},j}, \bar{\mathbf{v}}_k\right)\right]\Big/2h\right\} \tag{26}$$

$$\mathbf{S}_{\mathbf{x}\mathbf{v}}^{(1)}(k+1) = \left\{\left[\mathbf{f}_i\left(\hat{\mathbf{x}}_k^+, \bar{\mathbf{v}}_k + h\mathbf{S}_{\mathbf{v},j}\right) - \mathbf{f}_i\left(\hat{\mathbf{x}}_k^+, \bar{\mathbf{v}}_k - h\mathbf{S}_{\mathbf{v},j}\right)\right]\Big/2h\right\} \tag{27}$$

$$\begin{aligned}\mathbf{S}_{\mathbf{x}\hat{\mathbf{x}}}^{(2)}(k+1) = \frac{\sqrt{h^2-1}}{2h^2}&\left\{\mathbf{f}_i\left(\hat{\mathbf{x}}_k^+ + h\mathbf{S}_{\mathbf{x},j}, \bar{\mathbf{v}}_k\right)\right. \\ &\left. + \mathbf{f}_i\left(\hat{\mathbf{x}}_k^+ - h\mathbf{S}_{\mathbf{x},j}, \bar{\mathbf{v}}_k\right) - 2\mathbf{f}_i\left(\hat{\mathbf{x}}_k^+, \bar{\mathbf{v}}_k\right)\right\}\end{aligned} \tag{28}$$

$$\begin{aligned}\mathbf{S}_{\mathbf{x}\mathbf{v}}^{(2)}(k+1) = \frac{\sqrt{h^2-1}}{2h^2}&\left\{\mathbf{f}_i\left(\hat{\mathbf{x}}_k^+, \bar{\mathbf{v}}_k + h\mathbf{S}_{\mathbf{v},j}\right)\right. \\ &\left. + \mathbf{f}_i\left(\hat{\mathbf{x}}_k^+, \bar{\mathbf{v}}_k - h\mathbf{S}_{\mathbf{v},j}\right) - 2\mathbf{f}_i\left(\hat{\mathbf{x}}_k^+, \bar{\mathbf{v}}_k\right)\right\}\end{aligned} \tag{29}$$

Where $i, j = 1, 2 \cdots 4$, $\mathbf{S}_{\mathbf{x},j}$ is the *j*-th column vector of the matrix $\mathbf{S}_\mathbf{x}$, $\mathbf{S}_{\mathbf{v},j}$ is the *j*-th column vector of the matrix $\mathbf{S}_\mathbf{v}$.

The predicted observation vector $\hat{\mathbf{z}}_{k+1}^-$ and its predicted covariance $\mathbf{P}_{k+1}^\mathbf{z}$ $\hat{\mathbf{z}}_{k+1}^-$ are calculated as follows:

$$\begin{aligned}\hat{\mathbf{z}}_{k+1}^- &= \frac{h^2-(n_\mathbf{x}+n_\boldsymbol{\omega})}{2h^2}\mathbf{h}(\hat{\mathbf{x}}_{k+1}^-, \bar{\boldsymbol{\omega}}) \\ &+ \frac{1}{2h^2}\sum_{p=1}^{n_\mathbf{x}}\left\{\mathbf{h}(\hat{\mathbf{x}}_{k+1}^- + h\mathbf{S}_{\mathbf{x},p}^-, \bar{\boldsymbol{\omega}}) + \mathbf{h}(\hat{\mathbf{x}}_{k+1}^- - h\mathbf{S}_{\mathbf{x},p}^-, \bar{\boldsymbol{\omega}})\right\} \\ &+ \frac{1}{2h^2}\sum_{p=1}^{n_\boldsymbol{\omega}}\left\{\mathbf{h}(\hat{\mathbf{x}}_{k+1}^-, \bar{\boldsymbol{\omega}} + h\mathbf{S}_{\boldsymbol{\omega},p}) + \mathbf{h}(\hat{\mathbf{x}}_{k+1}^-, \bar{\boldsymbol{\omega}} - h\mathbf{S}_{\boldsymbol{\omega},p})\right\}\end{aligned} \tag{30}$$

$$\mathbf{P}_{k+1}^\mathbf{z} = \mathbf{S}_\mathbf{z}^-(k+1)(\mathbf{S}_\mathbf{z}^-(k+1))^T \tag{31}$$

Where $n_\boldsymbol{\omega}=1$ is the dimension of the noise vector, $\mathbf{S}_{\mathbf{x},p}^-$ is the p th column vector of the matrix $\mathbf{S}_\mathbf{x}^-$, $\mathbf{S}_{\boldsymbol{\omega},p}$ is the p th column vector of the matrix $\mathbf{S}_\boldsymbol{\omega}$. $\mathbf{S}_\mathbf{z}^-(k+1)$ is derived by the following calculation:

$$\mathbf{S}_\mathbf{z}^-(k+1) = [\mathbf{S}_{\mathbf{z}\hat{\mathbf{x}}}^{(1)}\,\mathbf{S}_{\mathbf{z}\boldsymbol{\omega}}^{(1)}\,\mathbf{S}_{\mathbf{z}\hat{\mathbf{x}}}^{(2)}\,\mathbf{S}_{\mathbf{z}\boldsymbol{\omega}}^{(2)}] \tag{32}$$

$$\mathbf{S}_{\mathbf{z}\hat{\mathbf{x}}}^{(1)}(k+1) = \left\{\left[\mathbf{h}\left(\hat{\mathbf{x}}_{k+1}^- + h\mathbf{S}_{\mathbf{x},j}^-, \bar{\boldsymbol{\omega}}_{k+1}\right) - \mathbf{h}\left(\hat{\mathbf{x}}_{k+1}^- - h\mathbf{S}_{\mathbf{x},j}^-, \bar{\boldsymbol{\omega}}_{k+1}\right)\right]\Big/2h\right\} \tag{33}$$

$$\mathbf{S}^{(1)}_{\mathbf{z}\boldsymbol{\omega}}(k+1)=\left\{\left[\mathbf{h}\left(\hat{\mathbf{x}}^-_{k+1},\bar{\boldsymbol{\omega}}_{k+1}+h\mathbf{S}^-_{\boldsymbol{\omega}}\right)-\mathbf{h}\left(\hat{\mathbf{x}}^-_{k+1},\bar{\boldsymbol{\omega}}_{k+1}-h\mathbf{S}^-_{\boldsymbol{\omega}}\right)\right]\Big/2h\right\} \tag{34}$$

$$\begin{aligned}\mathbf{S}^{(2)}_{\mathbf{z}\hat{\mathbf{x}}}(k+1)=\frac{\sqrt{h^2-1}}{2h^2}&\left\{\mathbf{h}\left(\hat{\mathbf{x}}^-_{k+1}+h\mathbf{S}^-_{\mathbf{x},j},\bar{\boldsymbol{\omega}}_{k+1}\right)\right.\\&\left.+\mathbf{h}\left(\hat{\mathbf{x}}^-_{k+1}-h\mathbf{S}^-_{\mathbf{x},j},\bar{\boldsymbol{\omega}}_{k+1}\right)-2\mathbf{h}\left(\hat{\mathbf{x}}^-_{k+1},\bar{\boldsymbol{\omega}}_{k+1}\right)\right\}\end{aligned} \tag{35}$$

$$\begin{aligned}\mathbf{S}^{(2)}_{\mathbf{z}\boldsymbol{\omega}}(k+1)=\frac{\sqrt{h^2-1}}{2h^2}&\left\{\mathbf{h}\left(\hat{\mathbf{x}}^-_{k+1},\bar{\boldsymbol{\omega}}_{k+1}+h\mathbf{S}^-_{\boldsymbol{\omega}}\right)\right.\\&\left.+\mathbf{h}\left(\hat{\mathbf{x}}^-_{k+1},\bar{\boldsymbol{\omega}}_{k+1}-h\mathbf{S}^-_{\boldsymbol{\omega}}\right)-2\mathbf{h}\left(\hat{\mathbf{x}}^-_{k+1},\bar{\boldsymbol{\omega}}_{k+1}\right)\right\}\end{aligned} \tag{36}$$

The cross correlation matrix $\mathbf{P}^{\mathbf{xz}}_{k+1}$ is shown as below:

$$\mathbf{P}^{\mathbf{xz}}_{k+1}=\mathbf{S}^-_{\mathbf{x}}(k+1)(\mathbf{S}^{(1)}_{\mathbf{z}\hat{\mathbf{x}}}(k+1))^T \tag{37}$$

The filter gain $\boldsymbol{\kappa}_{k+1}$, the state estimate vector $\hat{\mathbf{x}}^+_{k+1}$ and the state covariance matrix $\mathbf{P}^+_{k+1}$ are as follows:

$$\boldsymbol{\kappa}_{k+1}=\mathbf{P}^{\mathbf{xz}}_{k+1}(\mathbf{P}^{\mathbf{z}}_{k+1})^{-1} \tag{38}$$

$$\hat{\mathbf{x}}^+_{k+1}=\hat{\mathbf{x}}^-_{k+1}+\boldsymbol{\kappa}_{k+1}(\mathbf{z}_{k+1}-\hat{\mathbf{z}}^-_{k+1}) \tag{39}$$

$$\mathbf{P}^+_{k+1}=\mathbf{P}^-_{k+1}-\boldsymbol{\kappa}_{k+1}\mathbf{P}^{\mathbf{z}}_{k+1}\boldsymbol{\kappa}^T_{k+1} \tag{40}$$

The above derived the algorithm for BOTT based on DDF2, fig.2 represented the algorithm by the block diagram.

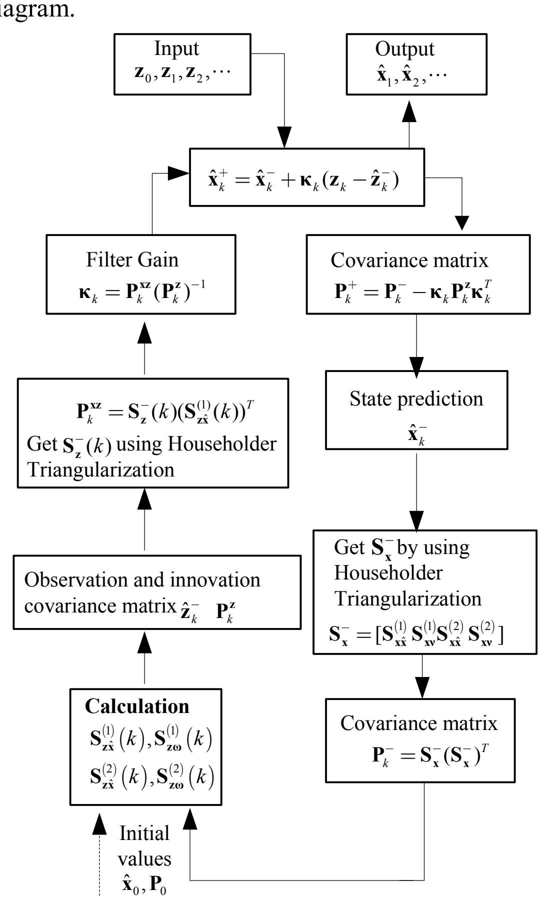

Fig.2 Block diagram of DDF2

4. SIMULATION RESULTS

We assume that the UUV performs a 360 degree turn, and the target performs a constant velocity motion. The simulation step T is set to 1 sec. Simulation time is set to 1000s. The initial condition of the target and UUV is as shown in table 1.

Table.1 target and UUV state initial value

	Underwater Target	UUV
Initial Position(m)	(500,3000)	(500,2000)
Initial Velocity(m/s)	3.6	2.5
Initial Course (°)	135	30
Maneuver Strategy	linear motion	360 degree turn

The initial relative distance of the UUV and the target r_0 is assumed to be known, while velocity of the target is unknown, the initial bearing measured by UUV's passive sonar is $\mathbf{z}_0$. Let $M=r_0^2$, then the filter initial state and initial covariance matrix are listed as the following form:

$$\mathbf{x}_0=\begin{bmatrix}r_0\times\sin\mathbf{z}_0\\ r_0\times\cos\mathbf{z}_0\\ -1.5\times\sin(\pi/6)\\ -1.5\times\cos(\pi/6)\end{bmatrix} \tag{41}$$

$$\mathbf{P}_0=0.01\times\begin{bmatrix}M&0&0&0\\0&M&0&0\\0&0&100&0\\0&0&0&100\end{bmatrix} \tag{42}$$

The proposed DDF2 algorithm with previously mentioned DDF1 and EKF algorithm are simulated by 100 times Monte Carlo simulation experiments with the same initial conditions. Fig. 3, fig. 4 show the position, heading and relative distance simulation results, as can be seen from the figures, the proposed method can fast convergence to the target's real value. , Fig.5, fig.6 give the position and velocity tracking errors of the proposed methods, as can be seen from the figures, the errors of the DDF2 is smallest and the tracking performance is the best compares with that of DDF1 and EKF. All of the three methods have a large deviation in the initial period, which is mainly due to the fact that the initial velocity of the target is unknown.

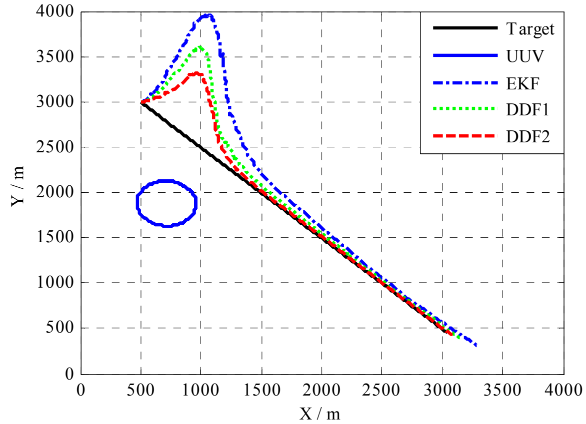

Fig.3 Position tracking based on DDF2, DDF1 and EKF

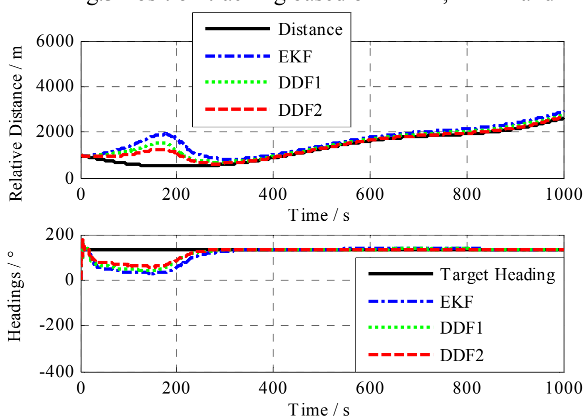

Fig.4 Relative distance and heading tracking based on DDF2, DDF1 and EKF

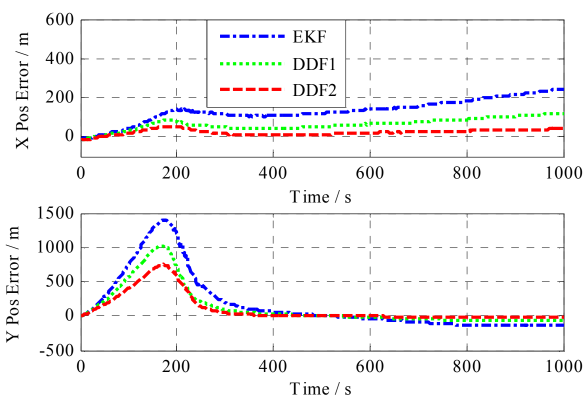

Fig.5 Position errors of DDF2, DDF1 and EKF

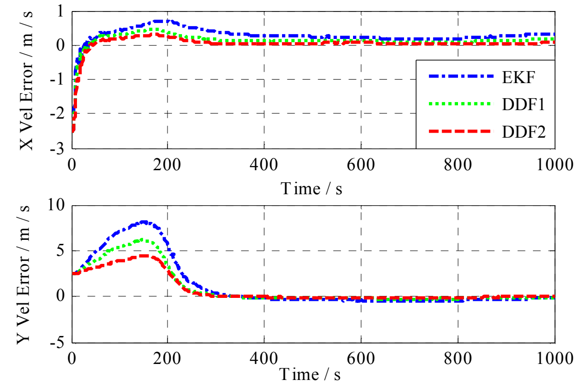

Fig.6 Velocity errors for DDF2, DDF1 and EKF

In order to further analysis and comparison the algorithms performance between the proposed method DDF2 and DDF1, EKF, the statistic characteristics of root mean square error (RMSE) is used. Fig.7, fig 8 give the RMSE of target's position and velocity. As can be seen, the BOTT based on DDF2 has higher estimation accuracy, and EKF has the minimum.

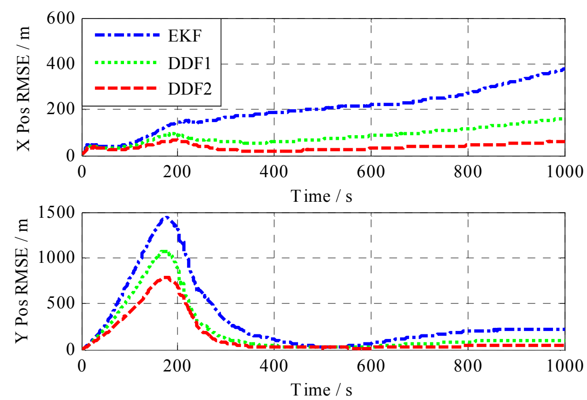

Fig.7 Position RMSE based on DDF2, DDF1 and EKF

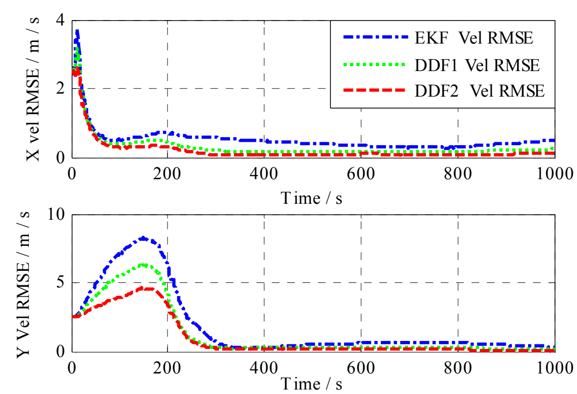

Fig.8 Velocity RMSE based on DDF, DDF1 and EKF

5. CONCLUSION

The proposed BOTT algorithm based on DDF2 can improve the target tracking performance compared with the DDF1 and EKF. The DDF2 algorithm linearizes the nonlinear terms of the target tracking system model by second order Stirling's interpolation and gives the detailed designing steps of the algorithm. The method not only simplified the calculating process of target tracking, but also has higher tracking performance. The 100 times Monte Carlo simulation also proved the conclusion. This paper has supposed that the noise statistical characteristic has known, which is in contrast to the real instance, in the following step, we will focus on adaptive method to solve the problem above.

6. ACKNOWLEDGE

This research work is supported by the National Natural Science Foundation of China (Grant No.E091002/ 50979017), Ph.D. Programs Foundation of Ministry of Education of China and Basic Technology (Grant No. 20092304110008), Harbin Science and Technology Innovation Talents of Special Fund Project (Outstanding Subject Leaders) (No. 2012RFXXG083).

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