Path Planning of Autonomous Underwater Vehicle for Optimal Acoustic Tomography

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***Abstract*—Ocean acoustic tomography (OAT) can infer the state of the interior of the survey domain from measurements along the periphery. Traditionally, a network of mooring platforms with projector/hydrophone array is deployed, sometimes combined with moving vessel nodes. In this paper we look at the OAT problem with autonomous underwater vehicle (AUV) equipped with a hydrophone as a moving receiver and study the AUV path-planning techniques to optimize the tomographic measurements. The path planning problem is to find the optimal AUV depths that minimize the sound speed profile posterior uncertainty along its path under the AUV navigation constraint. Numeric simulation results are shown to confirm the effectiveness of the approach.**

***Keywords*—Ocean acoustic tomography; autonomous underwater vehicle; path planning; sound speed profile**

I. INTRODUCTION

Ocean acoustic tomography (OAT) has long been of great interests to both the oceanographic and acoustic research communities [1]. OAT can infer the state of the interior of the survey domain from measurements along the periphery. Traditionally, a network of mooring platforms with projector/hydrophone array is deployed, sometimes combined with moving vessel nodes. Recently, as the autonomous underwater vehicle (AUV) technology matures, it has started to change the way people used to probe the ocean [2] [3]. One of the most important advantages with AUV is the adaptivity. Using an AUV to carry the acoustic source or receiver has also been considered, aiming to achieving adaptive tomographic surveys [4].

In this paper we look at the OAT problem with AUV as a moving node and study the AUV path-planning techniques to optimize the tomographic measurements. Travel-time-based tomography is used to invert for the sound-speed perturbations of a background (reference) profile. To limit the degree of freedom, sound-speed perturbations are represented in terms of optimal orthogonal functions (OOFs) [5], which provides the smallest mean squared error in the final profile estimate by taking into account not only the statistics of the sound speed uncertainty but also the measurement noise.

It is preferred that the AUV acts as a moving receiver considering its limited energy capacity. Equipped with a hydrophone and a data acquisition system, the AUV is able to, along its path, capture multipath ray arrivals from a fixed source. As we know, the accuracy of the tomographic measurement depends on the number of the rays and how the various rays sample the region of interest in the environment. The characteristics of the rays the AUV captures would be different when the AUV is at different locations (range and depth). For a range-independent environment, it is assumed that the initial uncertainty of sound speed profile (SSP) is known from direct measurements of SSP. The path planning problem is then to find the optimal depths of AUV at different ranges to minimize the posterior uncertainty of SSP. Meanwhile, the navigation constraint shall be considered for the path planning problem.

The reminder of the paper is organized as follows. A theoretical review of travel-time based tomography is first introduced in Section II. Section III then formulates AUV path planning as an optimization problem. Section IV presents some numerical simulation results. Finally, Section V concludes the paper.

II. TRAVEL-TIME-BASED OAT

Assuming an AUV is at a fixed position (r_i, z_i), where r_i is the range from the source and z_i is its depth. As carried out classically in ocean tomography, the inversion for travel-time perturbations is linearized as [1]:

$$\mathbf{y}=\mathbf{C}\mathbf{x}+\mathbf{n}, \tag{1}$$

where $\mathbf{y}=[y_1, y_2, \cdots, y_m, \cdots y_M]^T$ is the data vector of travel-time perturbations, M is the number of eigenray, $\mathbf{x}=[x(\Delta z), x(2\Delta z), \cdots x(n\Delta z), \cdots, x(N\Delta z)]^T$ is the sound speed perturbation vector, Δz is the sampling interval of the SSP, $N\Delta z$ equals to the depth of the water column, $\mathbf{n}$ is the noise including both the true measurement noise and model error, and $\mathbf{C}$ is the measurement matrix also known as the kernel matrix. It is assumed that $\mathbf{x}$ and $\mathbf{n}$ are zero-mean Gaussian with prior known covariance matrices $\mathbf{P}=\mathrm{E}[\mathbf{x}\mathbf{x}^{\mathrm{T}}]$ and $\mathbf{R}=\mathrm{E}[\mathbf{n}\mathbf{n}^{\mathrm{T}}]$, respectively.

A. Measurement matrix

It can be shown on the basis of ray theory that the perturbation in travel time y_m along the mth eigenray can be written in a form of path integral [1]:

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$$y_m = \int_{\Gamma_m^{'}} \frac{ds}{c(z)} - \int_{\Gamma_m} \frac{ds}{c_0(z)}, \qquad (2)$$

where Γ_m and $\Gamma_m^{'}$ represent the eigenrays corresponding, respectively, to the mean and perturbed SSP $c_0(z)$ and $c(z) = c_0(z) + x(z)$ (z : depth). $c_0(z)$ is considered to be known from director measurement of SSP. For small perturbations of sound speed $x(z) \ll c_0(z)$, one can take $\Gamma_m^{'} = \Gamma_m$, so the previous equation becomes:

$$y_m = \int_{\Gamma_m} \frac{ds}{c(z)} - \int_{\Gamma_m} \frac{ds}{c_0(z)} \approx -\int_{\Gamma_m} \frac{x(z)}{c_0^2(z)} ds. \qquad (3)$$

Changing this path integral to an integral over depth between the starting depth z_b and the ending depth z_t of the ray segment, and adding a term $k_m(z)$ which is the number of times the mth ray passes the depth z , Eq. (3) becomes:

$$y_m = -\int_{z_b}^{z^t} \frac{x(z)k_m(z)}{c_0^2(z)\left|\sin(\theta(z))\right|} dz, \qquad (4)$$

where θ is the angle with respect to horizontal of the ray. Using Snell's law, and defining $c_{\max}$ as the sound speed for which the ray would become horizontal, Eq. (4) can be rewritten as:

$$y_m = -\int_{z_b}^{z_t} \frac{x(z)k_m(z)}{c_0^2(z)\sqrt{1 - \frac{c_0^2(z)}{c_{\max}^2}}} dz. \qquad (5)$$

The integral can be put into a finite difference form by letting the mth row of measurement matrix $\mathbf{C}$, which corresponds to the mth eigenray, be:

$$C_{mn} = \begin{cases} \dfrac{-k_m(n\Delta z)}{c_0^2(n\Delta z)\left|\sin(\theta(n\Delta z))\right|} \Delta z, \\ \qquad\qquad when\ c_0(n\Delta z) = c_0((n-1)\Delta z) \\ \dfrac{k_m(n\Delta z)}{\beta} \sqrt{1 - \dfrac{c_0^2}{c_{\max}^2}} \\ \times\left(\dfrac{1}{\min(c_{\max}, c_0(n\Delta z))} - \dfrac{1}{\min(c_{\max}, c_0((n-1)\Delta z))}\right), \\ \qquad\qquad when\ c_0(n\Delta z) \neq c_0((n-1)\Delta z) \end{cases} \qquad (6)$$

where $\beta = (c_0(n\Delta z) - c_0((n-1)\Delta z)) / \Delta z$, $n = 1, 2, \cdots, N$. Then the measurement matrix $\mathbf{C}$ can be written as:

$$\mathbf{C} = \begin{bmatrix} C_{11} & \cdots & C_{1n} & \cdots & C_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ C_{m1} & \cdots & C_{mn} & \cdots & C_{mN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ C_{M1} & \cdots & C_{Mn} & \cdots & C_{MN} \end{bmatrix}. \qquad (7)$$

B. *Optimal orthogonal functions*

Following [5], to limit the degrees of freedom, sound speed perturbation vector $\mathbf{x}$ can be approximated as a set of orthogonal functions:

$$\mathbf{x} \approx \mathbf{\Phi a}, \qquad (8)$$

where $\mathbf{a}$ is the weight vector and $\mathbf{\Phi}$ is a matrix that is composed of a small number of orthogonal basis vectors corresponding to the most important modes of variation in $\mathbf{x}$. The optimal choice of $\mathbf{a}$ and $\mathbf{\Phi}$ will be defined as the one which minimizes the expected mean squared error e in the resulting profile estimate:

$$e = \mathrm{E}[(\mathbf{x} - \mathbf{\Phi a})^{\mathrm{T}}(\mathbf{x} - \mathbf{\Phi a})]. \qquad (9)$$

This scalar error can be equivalently written as the trace of the covariance matrix:

$$e = tr\, \mathrm{E}[(\mathbf{x} - \mathbf{\Phi a})(\mathbf{x} - \mathbf{\Phi a})^{\mathrm{T}}]. \qquad (10)$$

Because the system is linear and the variables are Gaussian, with no loss of generality, it may be assumed that

$$\mathbf{a} = \mathbf{K y}. \qquad (11)$$

Substituting (1) and (11) into (10), the error objective for $\mathbf{K}$ to minimize is:

$$\begin{aligned} \mathrm{e} = \mathrm{tr}\,[&\mathbf{P} - \mathbf{\Phi K C P} - \mathbf{P C}^{\mathrm{T}}\mathbf{K}^{\mathrm{T}}\mathbf{\Phi}^{\mathrm{T}} + \\ &\mathbf{\Phi K}(\mathbf{C P C}^{\mathrm{T}} + \mathbf{R})\mathbf{K}^{\mathrm{T}}\mathbf{\Phi}^{\mathrm{T}}]. \end{aligned} \qquad (12)$$

Rearranging (12), and recalling that the columns of $\mathbf{\Phi}$ are orthonormal so that $\mathbf{\Phi}^T\mathbf{\Phi} = \mathbf{I}$, Eq. (12) becomes:

$$e = tr\left[\mathbf{P} - \mathbf{K C P \Phi} - \mathbf{\Phi}^T\mathbf{P C}^T\mathbf{K}^T + \mathbf{K}(\mathbf{C P C}^T + \mathbf{R})\mathbf{K}^T\right]. \qquad (13)$$

Taking the derivative of this trace with respect to the matrix $\mathbf{K}$, and setting the derivative to zero, we can get the optimal gain matrix:

$$\mathbf{K} = \mathbf{\Phi}^T\mathbf{P C}^T(\mathbf{C P C}^T + \mathbf{R})^{-1}. \qquad (14)$$

Substituting this value into (12), the error objective becomes:

$$e = tr\left[\mathbf{P} - \mathbf{\Phi}^T\mathbf{P C}^T(\mathbf{C P C}^T + \mathbf{R})^{-1}\mathbf{C P \Phi}\right]. \qquad (15)$$

It is clear that the function is minimized when the columns of $\mathbf{\Phi}$ are the eigenvectors corresponding to the largest eigenvalues of the matrix $\mathbf{P C}^T(\mathbf{C P C}^T + \mathbf{R})^{-1}\mathbf{C P}$. Those eigenvectors which comprise the columns of $\mathbf{\Phi}$ are the OOFs.

III. MODELING THE AUV PATH PLANNING PROBLEM

In this section we formulate AUV path planning as an optimization problem under the AUV navigation constraint. The objective function is the summation of the SSP posterior uncertainty in a range-independent environment.

A. *Objective function*

As the AUV is moving, the ith measurement point can be represented by its depth z_i and range r_i . The optimization problem is to find the optimal depth z_i^o when AUV is at range r_i to minimize the summation of SSP posterior uncertainty. According to (15), the objective is to minimize $e_i(z_i)$, which is equal to:

$$\max_{z_i \in \mathbf{Z}_i} tr(\mathbf{\Phi}_i^T \mathbf{P}_{i-1} \mathbf{C}_i^T (\mathbf{C}_i \mathbf{P}_{i-1} \mathbf{C}_i^T + \mathbf{R})^{-1} \mathbf{C}_i \mathbf{P}_{i-1} \mathbf{\Phi}_i), \qquad (16)$$

where $\mathbf{Z}_i$ denotes all feasible depths AUV can dive to at range r_i, $\mathbf{P}_{i-1}$ is the covariance matrix of the $(i-1)th$ measurement point. $\mathbf{\Phi}_i$ and $\mathbf{C}_i$ are the OOFs matrix and measurement matrix of the *ith* measurement point, respectively. Eq. (16) is obviously a nonlinear optimization problem. We will use an exhaustive searching algorithm to find the optimal depth z_i^o at various ranges.

The covariance matrix $\mathbf{P}_i$ is updated at every step, when at range r_i it can be expressed as:

$$\mathbf{P}_i = \mathbf{P}_{i-1} - \mathbf{\Phi}_i \mathbf{\Phi}_i^T \mathbf{P}_{i-1} \mathbf{C}_i^T (\mathbf{C}_i \mathbf{P}_{i-1} \mathbf{C}_i^T + \mathbf{R})^{-1} \mathbf{C}_i \mathbf{P}_{i-1}, \qquad (17)$$

It is assumed that the initial SSP covariance matrix $\mathbf{P}_0$ is known from direct measurements of SSP.

B. Path representations

To define AUV paths, we have to discretize the two-dimensional (2D) plane as shown in Fig. 1, where (r_i, z_i) stands for the position of the *ith* measurement point, Δr is the horizontal measurement step and Δz^s is the vertical search step.

For the sake of safety, an upper bound and a lower bound on the depth of the AUV are set up to avoid hitting the surface vessels or bottom. Besides, the pitch constraint must be considered since the AUV can not dive with an overly large pitch angle. We denote the maximum pitch angle as $\varphi_{\max}$, then the maximum vertical distance the AUV can go from range r_{i-1} to r_i is $\Delta z_{\max} = \Delta r \tan(\varphi_{\max})$. Also, there is a limit on the horizontal distance – the maximum range. Since if AUV goes too far in the horizontal, the communication and control may get lost; what's more, if AUV is too far away from the source, the receive signal strength will be too weak to detect. According to the sonar equation [6], the approximate relationship between signal-to-noise ratio (SNR) and range from the AUV to the source is shown in Fig. 2. It is assumed that the transmitting transducer's source level is 190 dB, and it has a 2 kHz center frequency with 500 Hz bandwidth.

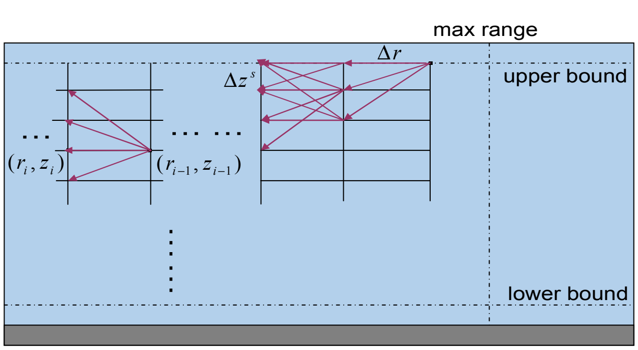

Figure 1. Illustration of ocean discretization for AUV path.

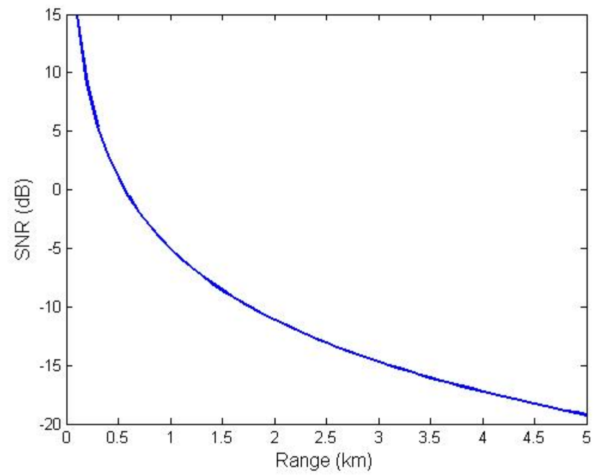

Figure 2. SNR of the received signal as a function of ranges from the source to the AUV.

IV. Simulation Results

The configuration of the simulation environment is shown in Fig. 3. A point source at a center frequency of 2 kHz with 500 Hz bandwidth is considered, located at 18 m depth. The depth upper bound $UB = 4\,\mathrm{m}$, the lower bound $LB = 84\,\mathrm{m}$, and the maximum pitch angel $\varphi_{\max} = 10^o$. Since $\Delta z_{\max} = \Delta r \tan(\varphi_{\max})$, if we chose $\Delta z_{\max} = LB - UB = 80\,\mathrm{m}$, then Δr should be not less than 453 m. Here we chose $\Delta r = 500\,\mathrm{m}$, and the vertical search step $\Delta z^s = 4\,\mathrm{m}$.

As shown in Fig. 2, SNR is adequate for identification of multipath signal arrivals when the AUV is 5 km away from the source. The AUV is assumed to move toward the source for better signal receiving since the hydrophone is mounted on the head. That is, the AUV starts to move from a location close to the surface and does measurements from range 5km to 1km as shown in Fig. 3. The noise vector in (1) is zero-mean Gaussian with covariance matrix $\mathbf{R} = \sigma^2 \mathbf{I}$, where $\sigma = 0.001$ seconds. The initial SSP statistics comes from the shelf break PRIMER experiment [7]. The total initial uncertainty is 1541.7 $(\mathrm{m/s})^2$.

In case 1, we are to find a path that minimizes the summation of SSP posterior uncertainty of the entire water column. Fig. 4 shows the simulation results. The top graph

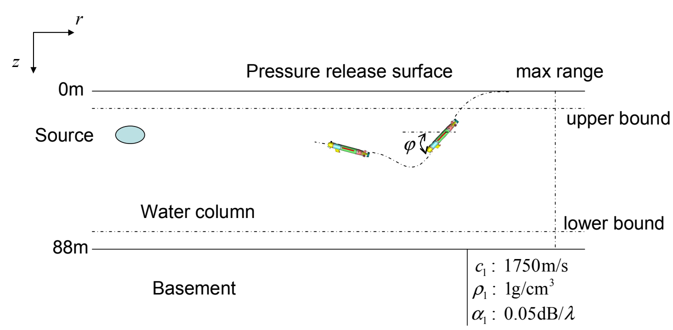

Figure 3. Configuration of the simulation environment. The AUV is moving toward the source.

shows the optimal path (the red line) from range 5 km to 1 km as well as the residual uncertainty of sound speed at various depths. The bottom graph shows the remaining summation of SSP posterior uncertainty at various ranges. After the first measurement at range of 5 km, the total uncertainty reductions are about 1450 $(m/s)^2$. Then the uncertainty reaches the steady state gradually.

In case 2, we would focus on some particular regions, for example, from depth 20 m to 60 m, where the uncertainty is larger than the other regions. Then the objective function will be the summation of sound speed uncertainty from depth 20 m to 60 m. Fig. 5 shows the simulation results. The top graph shows the optimal path (the red line) from range 5 km to 1 km as well as the residual uncertainty of sound speed from depth 20 m to 60 m. The bottom graph shows the remaining summation of sound speed uncertainty at every step. After the first measurement at range of 5 km, the total uncertainty reductions are about 956 $(m/s)^2$. Then the uncertainty reaches the steady state gradually.

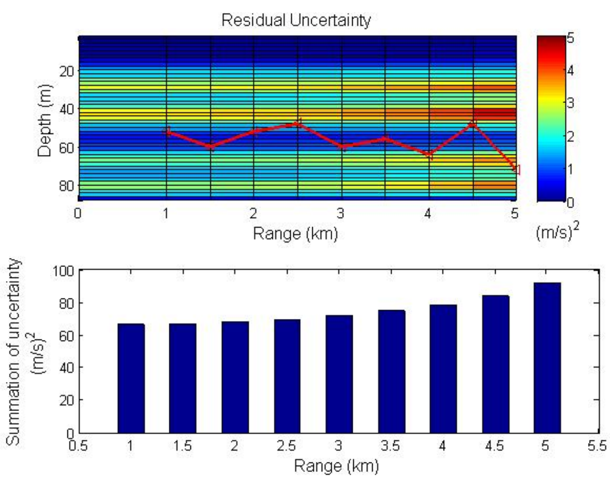

Figure 4. Optimal path and residual uncertainty of the entire water column.

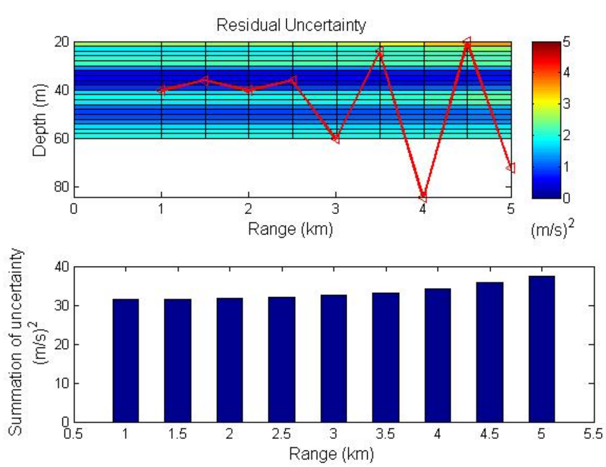

Figure 5. Optimal path and residual uncertainty of the interested region from depth 20 m to 60 m.

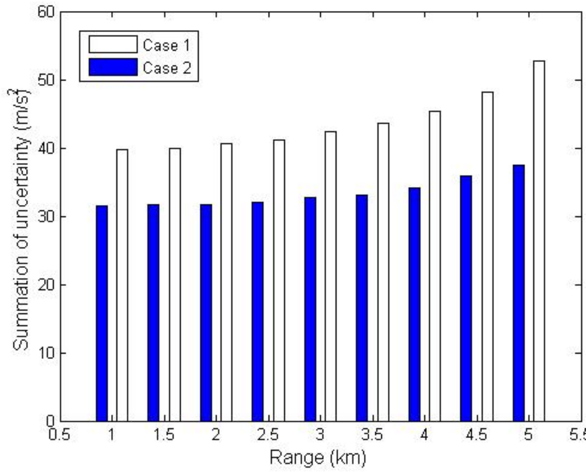

Figure 6. Comparison of summation of sound speed residual uncertainty from depth 20 m to 60 m between two cases.

From Fig. 4 and Fig. 5 we can see that the residual uncertainty of sound speed is smaller in case 2 than in case 1, especially in the region from depth 20 m to 40 m. A comparison of summation of sound speed residual uncertainty from depth 20 m to 60 m between the two cases is shown in Fig. 6. It shows that in case 2 the total uncertainty of the region of interest has reduced more effectively than in case 1.

V. CONCLUSIONS

In this paper, a novel path planning strategy for AUV to reduce uncertainty of SSP is developed based on OAT. Using travel-time-based OAT and OOFs we formulate the objective function under the AUV navigation constraint. The objective function used is the summation of posterior sound speed uncertainty, which is updated from the prior estimation uncertainty. Then we use an exhaustive searching algorithm to find the optimal path of the AUV. The sound speed uncertainty is rapidly decreased after the first step measurement, and then the uncertainty reaches the steady state gradually. If we just focus on some regions with high initial uncertainty, the simulation results show that the residual uncertainty in these regions are smaller than the results if we pay attention to the entire water column.

For future effort, we would consider the range-dependent and time-varying case. One way to deal with range-dependent case is to divide the environment into multiple range-independent regions, and then optimize the AUV path as a whole.

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