Random Linear Packet Coding For Fading Channels

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***Abstract*—Random linear packet coding is considered for use over underwater acoustic channels where long delays challenge the efficiency of standard ARQ. Adaptive power control and rate control are explored to overcome the effect of fading, while minimizing the average energy per successfully transmitted bit of information. Two extreme scenarios are considered: independent fading where the channel varies independently from one data packet to the next, and block fading where the channel stays fixed over a block of packets. For the case of block fading, we design a power (rate) adaptation strategy under an outage criterion. We show that there exists an optimal transmission power (number of coded packets) for which the average energy per bit is minimized. Finally, we quantify the benefits of adaptive power (rate) control using real channel data recorded over a mobile link in the 10kHz acoustic band. These results show that energy savings are available from adaptive power (rate) control strategies.**

***Index Terms*—Packet Coding, Underwater, Fading, Power Control, Rate Control.**

I. Introduction

Underwater acoustic channels are characterized by a long propagation delay which reduces the efficiency of traditional automatic repeat request (ARQ) schemes. To overcome this problem, we consider the use of random linear packet. By looking at packet-level coding (instead of bit-level coding) we are aiming to apply these techniques to the readily available acoustic modems in which the physical layer functions of modulation and coding cannot be modified.

Previous work in this field focused on non-fading channels, i.e. time-invariant channels in which the only source of packet loss is noise. For a link with no feedback, a scheme was proposed in [1], where the number of coded packets to be sent is determined based on a pre-defined reliability with which the original packets need to be recovered. For a link with feedback, [2] provided a scheme in which coded packets are transmitted in groups, each followed by feedback from the receiver, informing the transmitter of how many more coded packets need to be sent. The number of coded packets in each group can be designed to minimize the average transmission time or average energy consumption.

Adaptive power control and rate control have been extensively studied for wireless fading channels. In [3], the optimal allocation of redundancy between packet-level erasure coding and physical layer channel coding for a fading channel is considered. This work uses standard optimization techniques to determine the tradeoff between the erasure and channel codes. In [4], a study was performed to design power control that can enhance the performance of random network coding in wireless networks. A framework based on differential equations is utilized to analyze the throughput of random network coding, and to design a dynamic power control algorithm(s) that achieves higher multicast throughput. In [5] the tradeoff between channel coding and ARQ in Rayleigh block-fading channels is considered. That work compares the performance between a heavily coded system that leads to a low transmission rate but with fewer retransmissions, and a lightly coded system with higher transmission rate and more retransmissions. It shows that working with a larger error probability is advantageous since it leads to increased throughput. In [6], the authors employ network coding for broadcast in a cellular network. They propose a technique in which network coding and adaptive power control are used jointly to improve the network performance. Ref.[7] considers a cross layer scheme to optimize physical layer modulation and coding rate to maximize system throughput. It considers the use of Raptor codes and analyzes the performance over fast and slow fading channels. Ref.[8] also considers optimization of packet-level coding and bit-level coding. On channels that experience severe fading, performance is found to improve by adding more redundancy on erasure correction coding across packets.

In this paper, we extend the work of [1] to time-varying channels. On time-varying channels, the packet error rate depends on the channel state in addition to noise. The channel state is described by a fading coefficient G (the channel gain), which characterizes the received power in one data packet. We consider two cases: in the first case, the channel gain changes independently from one data packet to the next; in the second case, the channel gain stays fixed over a block of packets. We refer to these cases as independent fading and block fading, respectively, and regard them as two extremes between which a practical system is likely to operate.

The system model is described in Sec.II. Sec.III focuses on the analysis of independent fading, while Sec.IV focuses on the analysis of block fading. In Sec.V we use experimentally recoded values of the channel gain to emulate the system performance over a mobile acoustic link and quantify the benefits of power and rate control. Conclusions are summarized in Sec.VI.

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II. System Model

On a time-invariant channel with gain G, the received signal power is related to the transmitted power by $P_R = GP_T$. The resulting SNR, $\gamma = P_R/P_N$, determines the packet erasure rate P_E. To reduce the effect of erasures, the transmitter takes a group of M original packets, encodes them into $N \geq M$ packets, and sends the coded packets over the link. The number of coded packets is determined such that the receiver can recover the original M packets with a pre-specified probability P_s^*. The solution, $N_M^*(P_E)$, is the smallest N for which [1]

$$P_s = \sum_{m=M}^{N} \binom{N}{m} (1 - P_E)^m P_E^{N-m} \geq P_s^* \tag{1}$$

When there is fading, G becomes a random variable. Consequently, the SNR γ becomes a random variable, and so do the packet erasure rate P_E and the success rate P_s.

To proceed with the analysis, one needs to know the distribution of the channel gain G. In [9], experimental results were presented which showed that a class of underwater channels can be modeled by a large-scale gain that obeys log-normal distribution. Following this results, we will assume that $g = 10\log_{10} G \sim \mathcal{N}(\bar{g}, \sigma^2)$.

III. Independent Fading

In the case of independent fading, the channel gain varies independently from one packet to the next. If an independent realization of the gain accompanies each packet, then a string of N packets is associated with a string of independent variables $G_1, G_2, \ldots G_N$. Assuming that these variables are identically distributed according to the p.d.f. $p_G(G)$, the *average* success rate is

$$\bar{P}_s = \sum_{m=M}^{N} \binom{N}{m} (1 - \bar{P}_E)^m \bar{P}_E^{N-m} \tag{2}$$

where $\bar{P}_E$ is the average probability of packet error,

$$\bar{P}_E = \int_0^{\infty} P_E(GP_T/P_N) p_G(G) dG \tag{3}$$

If we impose a requirement on the *average* probability of success, $\bar{P}_s = E\{P_s\} \geq \bar{P}_s^*$, the number of coded packets needed to satisfy this requirement is $N_M^*(\bar{P}_E)$, where $N_M^*(\cdot)$ is the function defined through Eq.(1).

The impact of fading is evident here in the different number of coded packets compared to the non-fading case with the same (average) success rate and the same transmitted power. The transmission power can be changed based on the statistics of fading. Increasing the transmitted power aims for shorter transmission, i.e. fewer coded packets. To account for this trade-off, we use the average energy per bit as a figure of merit. The average energy per successfully transmitted bit of information when the number of coded packets is chosen to maintain an average reliability P_s^* is

$$\bar{E}_b = \frac{N_M^*(\bar{P}_E) P_T}{M R_b} \tag{4}$$

where R_b is the bit rate. Substituting for $N_M^*(\bar{P}_E)$ quantifies the trade-off in choosing P_T. On the one hand, increasing P_T directly increases the transmitted energy per bit P_T/R_b, but on the other hand, it decreases $\bar{P}_E$ and consequently the number of coded packets. Hence, there exists an optimal choice of P_T that minimizes the average energy per bit. This fact is illustrated in Fig.1.

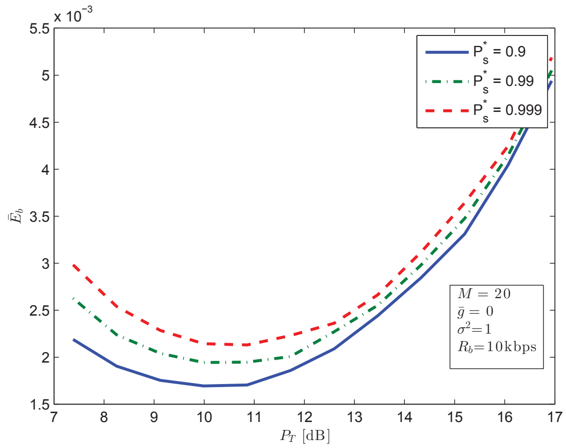


Figure 1. Relationship between the average energy per bit $\bar{E}_b$ and transmission power P_T for independent fading. The figure shows existence of optimal P_T for which the average energy per bit is minimized.

IV. Block Fading

If the channel gain does not change over the duration of a block of coded packets, then each packet in a block experiences the same G. The string of gains associated with the coded packets is now the same i.e. these variables are fully dependent. For a given block, we have $P_e = P_e(\gamma)$ with $\gamma = GP_T/P_N$, and the corresponding probability of success is given by

$$P_s(\gamma) = \sum_{m=M}^{N} \binom{N}{m} (1 - P_E(\gamma)^m P_E^{N-m}(\gamma) \tag{5}$$

The average probability of success is given by

$$\bar{P}_s = \int_0^{\infty} P_s(GP_T/P_N) p_G(G) dG \tag{6}$$

Since the channel is now slowly varying, it can be estimated at the receiver, and this information can be passed to the transmitter at the end of each block of packets. Once the channel is known at the transmitter, strategies can be devised to compensate for the fading. In the following sections, we focus on such strategies.

Specifically, one can let N vary with G, or let P_T vary with G. We call these strategies rate and power control, respectively. We want to compare these strategies in terms of the average energy per bit needed to achieve a given level of performance. Instead of the average success rate, we focus on the outage probability as a more appropriate measure of performance. The outage probability is defined as the probability that the success rate falls below a pre-specified threshold:

$$P_{out} = P\{P_s(GP_T/P_N) < P_s^*\} \tag{7}$$

As a benchmark, let us first consider the case in which there is no adaptive adjustment of either the transmission power or the number of coded packets. Setting their values to some fixed P_T and N, the outage probability is

$$P_{out} = P\{P_s < P_s^*\} = P\{\gamma < \gamma^*(N)\} = P\{G < G^*(P_T, N)\} \tag{8}$$

where $\gamma^*(N)$ is the SNR corresponding to the given P_s^*, and $G^*(P_T, N) = \gamma^*(N)P_N/P_T$. For a given distribution of the gain, the cumulative distribution function (8) is known. Specifically, if G is log-normally distributed, the outage probability can be obtained in the closed form,

$$P_{out} = P\{G < G^*\} = Q(\frac{\bar{g} - 10\log G^*}{\sigma}) \tag{9}$$

where Q denotes the Q-function, $Q(x) = 1/2erfc(x/\sqrt{2})$. This in turn yields

$$G^* = 10^{\bar{g} - \sigma Q^{-1}(P_{out})} \tag{10}$$

Hence, the functional dependence between P_{out} and P_s^* is known.

Fig.2 illustrates the relationship (5) between P_s and γ for different values of the number of coded packets N. For a specific value of the required success rate, i.e. reliability P_s^*, this figure implies the threshold SNR $\gamma^*(N)$. The threshold SNR decreases with increasing N.

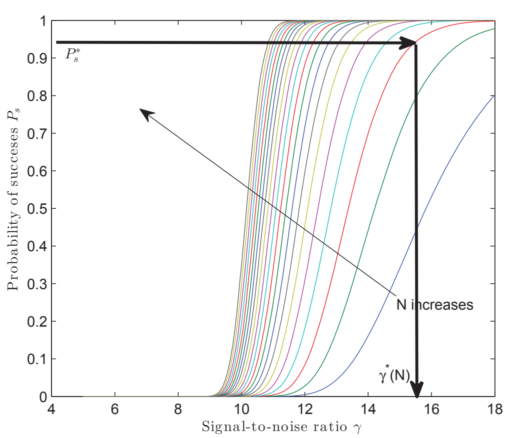


Figure 2. Relationship (5) between the probability of success P_s and the SNR γ.

Fig.3 shows the relationship between the outage probability and reliability. Given a design requirement (P_{out}, P_s^*), it points to the desired N, N^*. Note that this N^* corresponds to a given P_T. For a different P_T, $G^*(P_T, N)$ will be different, implying a different N^*.

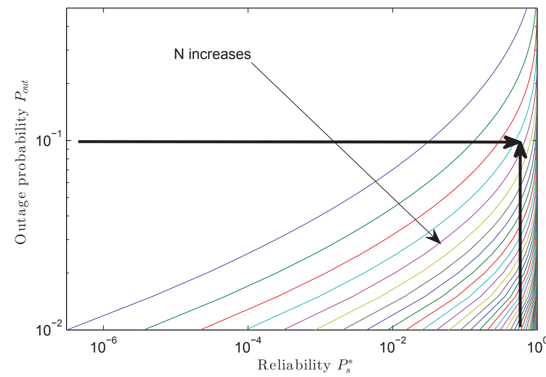


Figure 3. Relationship (8) between the outage probability P_{out} and the reliability P_s^*.

The average energy per bit for this design is

$$\bar{E}_b = \frac{N^* P_T}{M R_b} \tag{11}$$

A. Power Control

Here, we allow the transmitted power to vary in response to G. For a *given*, known value of the gain, the transmitted power is set to some P_T, and N coded packets are sent. At the end of each block of coded packets, the transmitter learns a new value of G. The transmitter then calculates the new P_T, and so on.

A possible way to adjust P_T is so that the success rate P_s is kept constant at the desired level P_s^*, i.e. so that the SNR γ is kept constant at $\gamma^*(N)$. If it is possible to do so, there will be no outage. In doing this, however, there will be situations when the gain is very low (close to zero), which will demand a very large power (tending to infinity). Such a design is of no practical interest, since maximum transmission power cannot be unlimited. To overcome this problem, we turn to the allowable outage (the design value P_{out}). Specifically, we formulate the power adaptation strategy as follows:

$$P_T = \begin{cases} K/G, & G \geq G_{out} \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

where K is a constant, which is determined such that the desired success rate is met. Whenever $P_T \neq 0$, the SNR is constant, $\gamma = P_T G/P_N = K/P_N$. For a given N, the SNR needs to be equal to $\gamma^*(N)$. Hence,

$$K = \gamma^*(N)P_N \tag{13}$$

The transmitter is either on, in which case $P_s = P_s^*$, or off, in which case $P_s = 0$. The outage thus occurs when transmission is shut off. The probability of outage is

$$P_{out} = P\{G < G_{out}\} \tag{14}$$

i.e., the outage gain is $G_{out} = 10^{\bar{g} - \sigma Q^{-1}(P_{out})}$

The average energy per bit is

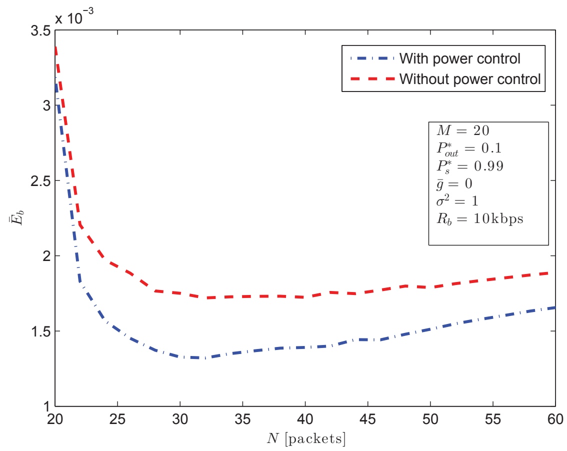
$$\bar{E}_b = \frac{N\bar{P}_T}{MR_b}$$

where

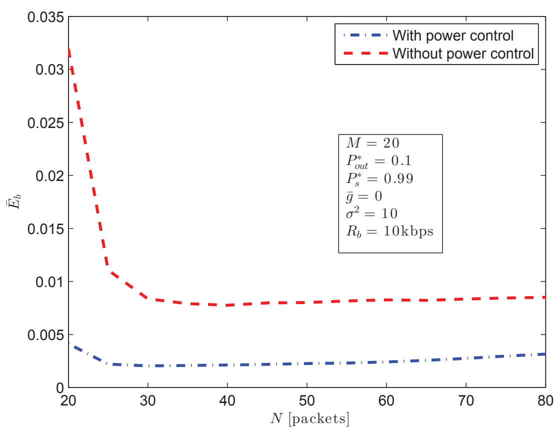
$$\bar{P}_T = \int_{G_{out}}^{\infty} \frac{\gamma^*(N) P_N}{G} p_G(G) dG \tag{15}$$

is the average transmitted power.

Fig.4 shows the energy per bit as a function of N. This result indicates that there is an advantage in terms of the average energy per bit when power control strategy is used. It can be seen that higher energy savings are available when the variance of the gain is higher. It also shows the existence of the optimal number of coded packets for which the average energy per bit is minimized.



(a) $\sigma^2 = 1$



(b) $\sigma^2 = 10$

Figure 4. Relationship between $\bar{E}_b$ and N for the system with power control (15) and the system that uses fixed power (11). The fixed power is equal to the average power consumed by the adaptive system. It can be seen that higher energy savings are possible when the variance of the gain is higher.

Fig.5 shows the minimum energy per bit that is required when the optimum N is used in each transmission. The energy is plotted as a function of the block size M. It can be seen that larger block size tend to use lower energy per bit.

B. Rate Control

Here, the transmission power is fixed at P_T and N varies in response to the gain. Similarly as before, we adjust N so

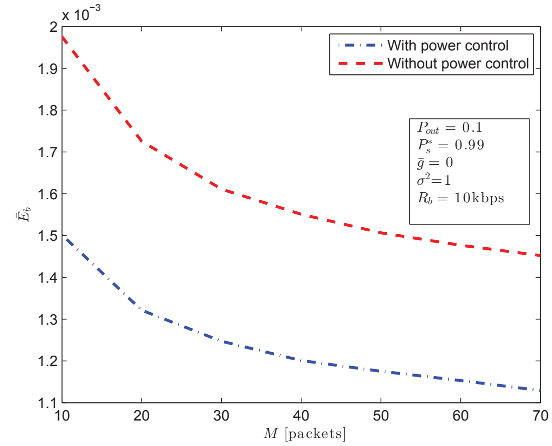


Figure 5. Minimum energy per bit for different block sizes using the power control algorithm.

that the success rate P_s is kept constant at the desired level of reliability P_s^*. However, if the gain is very low, a very high number of coded packets will be needed, driving the energy per bit to zero and possibly violating the assumption that the gain is constant over the entire long block of packets. To overcome this problem, we turn to the allowable outage, and formulate the adaptation strategy as follows:

$$N = \begin{cases} N_M^*(P_E(P_T G/P_N)) & G \geq G_{out} \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

where $N_M^*(\cdot)$ is the function defined through Eq.(1). Whenever $N \neq 0$, its value is determined such that the desired success rate is met for a given P_T. The transmitter is either on, in which case $P_s = P_s^*$, or off, in which case $P_s = 0$. The outage thus occurs when transmission is shut off. The probability of outage is

$$P_{out} = P\{G < G_{out}\} \tag{17}$$

The average energy per bit is

$$\bar{E}_b = \frac{\bar{N} P_T}{M R_b}$$

where

$$\bar{N} = \int_{G_{out}}^{\infty} N_M^*(P_E(G P_T/P_N)) p_G(G) dG \tag{18}$$

is the average number of coded packets per block. The average energy per bit thus depends on the chosen value of P_T, as illustrated in Fig.6. We see that there exists a choice of P_T for which the average energy per bit is minimized.

Fig.7 shows the minimum energy per bit that is required when the optimum P_T is used in each transmission. The energy is plotted as a function of M. Again, it can be seen that larger block sizes are favorable.

V. Experimental Results

In this section we present the performance of power control and rate control strategies by using the experimentally

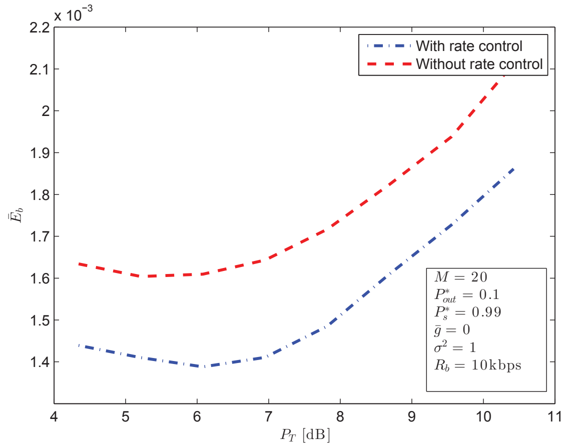


Figure 6. Relationship between $\bar{E}_b$ and P_T for the system with rate control (18) and the system that uses fixed number of coded packets (11). The fixed number of coded packets is equal to the average number of coded packets required by the adaptive system.

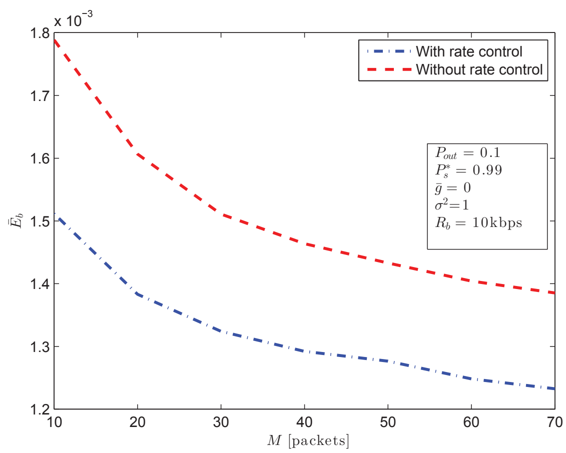


Figure 7. Minimum energy per bit for different block sizes using the rate control algorithm.

recorded gain values from the Pacific Storm experiment [9]. The Pacific Storm experiment was conducted on the submerged portion of the San Andreas fault, off the coast of Northern California in September 2010. The transmitter was an autonomous underwater vehicle (AUV), moving at about 3m above the ocean floor at an approximate depth of 125m. The receiver was suspended from a surface ship at a depth of about 3m. The AUV was repeatedly transmitting signals with a bandwidth of 4kHz centered around 10kHz. The distance between the transmitter and the receiver varied between 200m and 1km. The gain values, recorded once every 5 seconds, are shown in Fig.8. These values were used to emulate the system performance by feeding them to the adaptive control algorithm (12) and (17). The time series of the resulting P_T and N were calculated and used to compute the average energy per bit.

For purposes of evaluating the performance of the power control and rate control algorithms, we assume the following

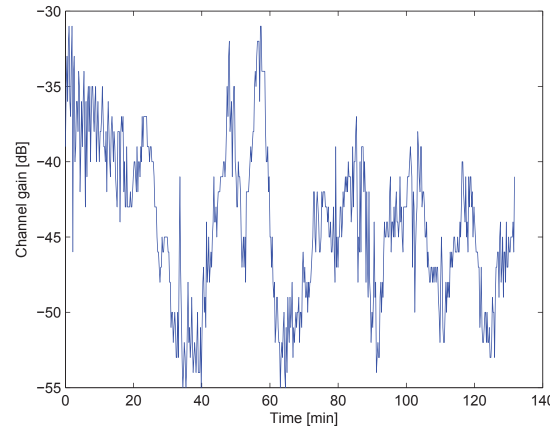


Figure 8. Channel gain values from the Pacific Storm experiment.

parameters: $M = 10$ original packets, $N_b = 100$ bits per packets, $R_b = 10$kbps, $P_{out} = 0.1$ and $P_s^* = 0.99$. The experimentally recorded gain values show that the channel remains stable for a few seconds. The transmission time of several packets is thus within the channel coherence time, i.e. the block-fading model applies.

Fig.9 shows the results for power control, while Fig.10 shows the results for the rate control. As seen from the figures, energy savings are available by employing the power and rate control algorithms while maintaining the reliability at a desired value P_s^*.

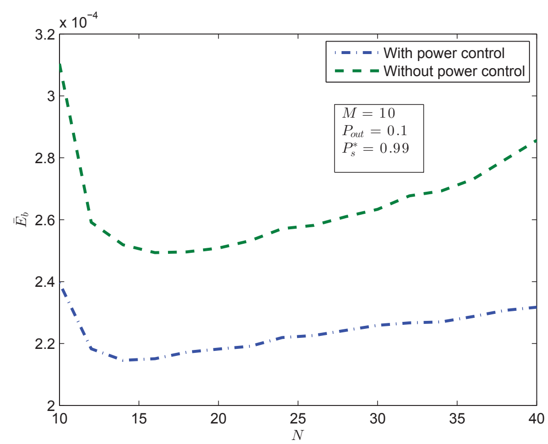


Figure 9. Average energy per bit for the power control algorithm operating on the channel of Fig.8.

VI. Conclusion

Random linear packet coding is a simple and effective technique that can be used in place of traditional ARQ to overcome the latency of acoustic links. We have investigated system design for time-varying channels, in light of meeting pre-specified decoding success and outage requirements.

For fast-varying channels, we show that an optimal transmission power exists for which the average energy per bit

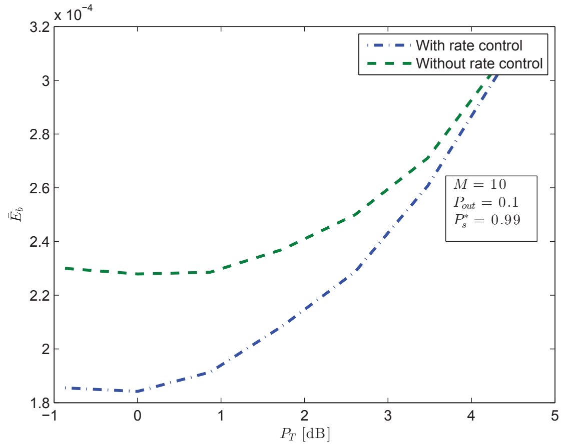


Figure 10. Average energy per bit for the rate control algorithm operating on the channel of Fig.8.

is minimized. For slowly-varying channels with feedback, we propose two control strategies, one for adjusting the transmission power, and another for adjusting the coding rate. Using a log-normal model for the channel gain, as well as a series of experimentally recorded gains, we quantify the energy savings available from the two strategies.

Random linear packed coding is particularly beneficial in multicast scenarios, where waiting for acknowledgments from multiple receivers aggravates latency. Future work will concentrate on multicast scenarios.

Acknowledgments

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References

[1] R. Ahmed and M. Stojanovic, "Random linear packet coding for high speed acoustic communication: An experimental analysis," in *OCEANS, 2012 - Yeosu*, May 2012, pp. 1–7.

[2] D. Lucani, M. Medard, and M. Stojanovic, "On coding for delay; network coding for time-division duplexing," *IEEE Trans. on Information Theory*, vol. 58, pp. 2330–2348, 2012.

[3] T. Courtade and R. Wesel, "Optimal allocation of redundancy between packet-level erasure coding and physical-layer channel coding in fading channels," *IEEE Trans. on Communications*, vol. 59, no. 8, pp. 2101–2109, August.

[4] K. Su, D. Zhang, and N. Mandayam, "Network coding aware power control in wireless networks," in *46th Annual Conference on Information Sciences and Systems (CISS)*, 2012, pp. 1–5.

[5] P. Wu and N. Jindal, "Coding versus arq in fading channels: How reliable should the phy be?" *IEEE Trans on Communications*, vol. 59, no. 12, pp. 3363–3374, December.

[6] T. Tran, D. Nguyen, T. Nguyen, and D. Tran, "Joint network coding and power control for cellular radio networks," in *Second International Conference on Communications and Electronics, ICCE(2008)*, 2008, pp. 109–114.

[7] Y. Cao and S. Blostein, "Cross-layer optimization of rateless coding over wireless fading channels," in *25th Biennial Symposium on Communications (QBSC), 2010*, 2010, pp. 144–149.

[8] C. Berger, S. Zhou, Y. Wen, P. Willett, and K. Pattipati, "Optimizing joint erasure- and error-correction coding for wireless packet transmissions," *IEEE Trans. on Wireless Communications*, vol. 7, no. 11, pp. 4586–4595, November.

[9] G. Hollinger, S. Choudhary, P. Qarabaqi, C. Murphy, U. Mitra, G. Sukhatme, M. Stojanovic, H. Singh, and F. Hover, "Underwater data collection using robotic sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 5, pp. 899–911, 2012.