

Spectral Estimation of Cavitation Related Narrow-Band Ship Radiated Noise Based on Fractional Lower Order Statistics and Multiple Signal Classification

Umut Fırat^{1,2}, Tayfun Akgül¹

¹Department of Electronics and Communications Engineering,

Istanbul Technical University, Istanbul, Turkey

²TÜBİTAK BİLGEM, Gebze, Kocaeli, Turkey

{firatu, tayfunakgul}@itu.edu.tr

Abstract—Narrow-band spectral components of ship radiated noise are of crucial importance for sonar operators in order to detect and classify possible targets. Classical spectral estimation techniques have been widely used to fulfill the requirements for narrow-band signal detection. However, classical methods may not be sufficient when signal-to-noise ratio is relatively low. In this paper we propose to utilize the subspace based multiple signal classification for the cavitation related narrow-band ship radiated noise detection. In addition we propose the use of fractional lower order statistics instead of second order statistics when the ship radiated noise exhibits an impulsive behavior leading to drifting apart from Gaussianity. We present the results of a real ship noise data gathered from the Strait of Istanbul. We also investigate and present the $1/f$ -type noise characteristics of a sample ship noise segment and discuss how we can use this information for robust estimation of cavitation related narrow-band ship noise spectrum.

Keywords—Ship radiated noise, Multiple Signal Classification, α -stable processes, fractional lower order statistics, $1/f$ noise

I. INTRODUCTION

Underwater environment introduces acoustical variety due to several sound sources which can be anthropogenic, natural or biological. Typical anthropogenic sound sources are ship noises mainly originating from the propeller and the machinery. Narrow-band ship radiated noise has been very important for sonar operators in order to detect and classify possible targets. Among narrow-band ship radiated noise the cavitation related noise gives remarkable information about the propeller of the ship which can be utilized to classify the ship. This information includes the revolution rate of the propeller shaft and the propeller blades which may be extracted from the cavitation related narrow-band noise spectrum.

Spectral estimation of narrow-band ship radiated noise is traditionally performed by classical nonparametric spectral estimation techniques, namely the Fourier transform based periodogram and its derivatives [1]–[3]. A recent paper on the spectral estimation of ship noise suggests cyclostationarity in the cavitation related narrow-band noise which results in higher detection signal-to-noise ratio (SNR) over conventional methods [2]. Nevertheless nonparametric techniques may not be sufficient to estimate narrow-band characteristics of the ship

radiated noise in low SNR conditions. In this paper we propose to use an alternative spectral estimation approach based on subspace processing, i.e., multiple signal classification (MUSIC) where noisy signals are divided into signal and noise subspaces [4]. These subspace methods assume Gaussianity in the nature of the signal justifying the use of second order statistics (SOS). However ship radiated noise may exhibit impulsive behavior whose statistical characteristics can be modeled using a heavy tailed distribution. Thus we further propose to describe the ship radiated noise with an α -stable distribution which is often used to model heavy-tailed distributions. Since the second order moment of an α -stable distribution does not exist, fractional lower order moments (FLOMs) are used to represent its statistics [5]. This gives the opportunity to replace the inadequate SOS with the fractional lower order statistics (FLOS) in non-Gaussian signal processing. Thus we are able to show the superiority of FLOS-MUSIC proposed in [6] where the only difference with MUSIC is the use of the covariation coefficient matrix instead of the well-known covariance matrix. A good overview on stable signal processing is given in [5].

We also investigate a possible $1/f$ behavior of the ship radiated noise using the eigenvalues of the noise covariance matrix. This approach helps us distinguish the signal and noise spaces which then can be used in MUSIC algorithm to maximize the detection SNR. We show for a sample ship noise segment that the logarithmic eigenvalue distribution of the noise covariance matrix decreases almost linearly after an eigenvalue index which may be interpreted as the signal subspace dimension.

This paper is organized as follows. In Section II we briefly summarize FLOS. In Section III we give the definition of $1/f$ noise. Problem formulation is given in Section IV. In Section V we give the experimental results and conclude in Section VI.

II. FRACTIONAL LOWER ORDER STATISTICS

A. Stable Processes

The stable distribution is described by its characteristic function as [5]

$$\phi(t) = \exp(j\delta t - \gamma|t|^\alpha[1 + j\beta\text{signum}(t)w(t, \alpha)]) \quad (1)$$

where

$$w(t, \alpha) = \begin{cases} \tan \frac{\alpha\pi}{2} & \text{for } \alpha \neq 1 \\ \frac{2}{\pi} \log|t| & \text{for } \alpha = 1, \end{cases} \quad (2)$$

$$\text{signum}(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t = 0 \\ -1 & \text{for } t < 0 \end{cases} \quad (3)$$

and

$$-\infty < \delta < \infty, \gamma > 0, 0 < \alpha \leq 2, -1 \leq \beta \leq 1. \quad (4)$$

Thus, a stable distribution is completely determined by four parameters, the location parameter δ , the scale parameter γ , also called the dispersion, the index of skewness β and the characteristic exponent α . The characteristic exponent α is a shape parameter. It measures the "thickness" of the tails of the density function. A stable distribution with characteristic exponent α is called α -stable. When $\alpha = 2$, the relevant stable distribution is Gaussian [5]. Decreasing α suggests that the corresponding distribution has heavier tails.

These distributions have the convenient and appealing property that the sum of two independent stable random variables having the same characteristic exponent α is again α -stable. In addition, stable distributions are very flexible modeling tools and are able to describe a wide variety of non-Gaussian phenomena, from those which only slightly deviate from the Gaussian to those which are severely impulsive [5]. These properties make the stable distributions useful especially in applications which may exhibit outliers such as underwater acoustic signals.

B. Fractional Lower Order Moments

Although the second-order moment of a symmetric α -stable (S α S) random variable with $0 < \alpha < 2$ does not exist, all moments of order less than α do exist and are called the fractional lower order moments. In α -stable framework, a quantity called covariation plays an analogous role to the covariance for Gaussian random variables [5]. The covariation coefficient of jointly S α S random variables X and Y with $\alpha > 1$ is given by

$$\eta_{XY} = \frac{E(XY^{<p-1>})}{E(|Y|^p)}, \quad (5)$$

$$Y^{<p-1>} = |Y|^{p-1} \text{signum}(Y) \quad (6)$$

for any $1 \leq p < \alpha$, where p is the fractional moment order. Likewise the sample covariance function the covariation coefficient can be estimated from the data samples as

$$\hat{\eta}_{FLOM} = \frac{\sum_{i=1}^N X_i |Y_i|^{p-1} \text{signum}(Y_i)}{\sum_{i=1}^N |Y_i|^p} \quad (7)$$

for some $1 \leq p < \alpha$.

III. 1/f NOISE

The 1/f noise is a random process defined in terms of the shape of its power spectral density (PSD), $S(f)$. The power associated with the random process, measured in a narrow bandwidth, is roughly proportional to the reciprocal frequency [7],

$$S(f) = \frac{\text{constant}}{|f|^a}, \quad (8)$$

where a is known as the spectral exponent.

Another representation of 1/f noise can be the log-log eigenvalue distribution of the noise covariance matrix. Eigenvalues of this matrix are obtained by the well-known eigendecomposition as

$$\mathbf{C}\phi_i = \lambda_i \phi_i, \quad i = 1, \dots, N \quad (9)$$

where $\mathbf{C}$ is the N -by- N noise covariance matrix, ϕ_i are the eigenvectors and λ_i are the corresponding eigenvalues [8].

Like the slope of the PSD the slope of the eigenvalue distribution gives the 1/f behavior of the process. As shown in Section V ship radiated noise may follow a 1/f behavior after a certain eigenvalue index. This information can be used for robust estimation of cavitation related narrow-band ship noise spectrum.

IV. PROBLEM FORMULATION

A. Cavitation Related Narrow-Band Ship Radiated Noise

Dominant components of the narrow-band ship radiated noise originate from the propeller cavitation and the machinery [9]. Cavitation occurs when a ship's propeller revolution exceeds a certain threshold (at a given depth) which depends on the ship type and the propeller geometry. Cavitation leads to generation of bubbles around the propeller which collapse and form a broadband noise [10]. Low frequency signal caused by propeller revolution modulates this broadband signal which is usually modeled with the amplitude modulation. Amplitude modulated signal received at the hydrophone is traditionally demodulated with an envelope detector. This procedure is called Detection of Envelope Modulation on Noise (DEMON). Amplitude modulated signal can be modeled as

$$x[n] = (A + \mu m[n])c[n], \quad (10)$$

$$m[n] = \sum_{k=1}^K A_k \sin(w_k n) \quad (11)$$

where $m[n]$ represents the cavitation related narrow-band signal, $c[n]$ is the cavitation related broadband signal and μ is the modulation index. In this work we utilize the DEMON algorithm given in [3] to extract the cavitation related narrow-band signal, $m[n]$ which is subject to the spectral estimation.

B. MUSIC

Classical spectral estimation methods do not assume any parametric model on the data. One of them is the well-known periodogram,

$$\hat{P}_{PER}(f) = \frac{1}{N} \left| \sum_{n=0}^{N-1} x[n] \exp(-j2\pi f n) \right|^2 \quad (12)$$

where $x[n]$ is the signal sample and N is the signal length. Fourier based estimators like periodogram are only able to estimate broadband or low dynamic range PSDs. This reason gives us motivation to approach to narrow-band spectral estimation problem with more elaborate methods. Among many high resolution spectral estimation methods, subspace based methods are considered to be more suitable to narrow-band spectral estimation problem. These methods rely on the property that the noise subspace eigenvectors of a Toeplitz covariance matrix are orthogonal to the signal eigenvectors [4]. Thus, they are not only suitable for finding peaks in the spectrum but also separate the signal and noise spaces which yields easier extraction of the desired signal part. An example of such methods is MUSIC,

$$\hat{P}_{MUSIC}(f) = \frac{1}{\sum_{i=l+1}^M |\mathbf{e}^H \hat{\mathbf{v}}_i|^2} \quad (13)$$

where $\mathbf{e} = [1 \exp(j2\pi f) \exp(j4\pi f) \dots \exp(j2M\pi f)]^T$ is the frequency vector, $\hat{\mathbf{v}}_i$, are the eigenvectors of the covariance matrix and l is the signal subspace dimension [4]. Theoretically, when $f = f_i$, the i th sinusoidal frequency, so that $\mathbf{e} = \mathbf{e}_i$, $\hat{P}_{MUSIC}(f)$ goes to infinity. Due to estimation errors, a peak is exhibited at or near the sinusoidal frequency. The frequency estimates are found as the frequencies corresponding to the l largest peaks of (13) [4].

C. FLOS-MUSIC

Frequency estimates of the FLOS-based MUSIC can be obtained utilizing the sample covariation coefficient matrix contrary to the SOS-based MUSIC which uses the sample covariance matrix [6]. The components of the sample covariation coefficient matrix can be obtained using (7). However, the sample covariation coefficient matrix, as estimated by (7), is not symmetric and hence it has complex eigenvalues. A possible solution to this problem, as proposed by [11], can be performing the eigenvalue decomposition to the sum of the sample covariation coefficient matrix and its Hermitian transpose, thus obtaining real eigenvalues.

V. EXPERIMENTS

A. Dataset

In this work we have used the data recorded in the Strait of Istanbul in 2008 with the so-called ‘‘Bosporus Ambient Noise Acquisition System (BANAS)’’. Recording system consists of a vertical hydrophone array with eight linearly spaced hydrophones up to 30 meter [12]. Fig. 1 shows the measurement site where there is a heavy ship traffic with more than 50,000 ships passing a year [13]. Dataset consists of the records of the ships of opportunity with the time labels at the closest point of approach (CPA). We have used the data of a modern cargo ship among several ship classes because of the ease of access of her catalog information.

B. Experimental Results

Data analyzed consists of the records of two different hydrophones at the same time interval, one of them following a Gaussian-like distribution given in Fig. 2(a) while the other one being the record with an impulsive behavior as seen in



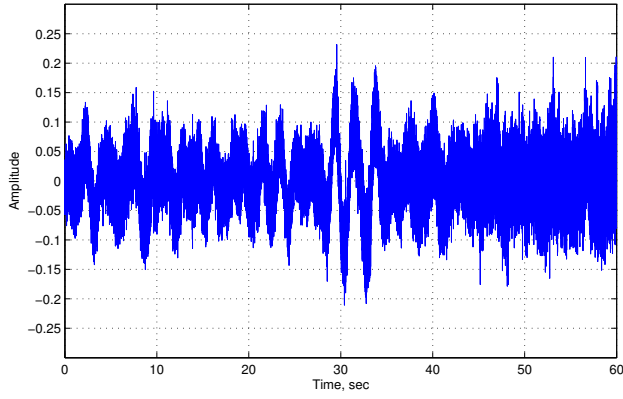
Fig. 1. Measurement site in the Strait of Istanbul (marked as BANAS) [12].

Fig. 2(b). This is evident in Fig. 3(a) and 3(b) where the histograms of one-minute-long records of non-impulsive and impulsive signals are given, respectively. While the reason for this impulsive behavior is unknown to us it is presumably because of a random transient noise near that hydrophone. This behavior can be explicitly seen from 3(c) where α -stable distribution fits of these two signals obtained using the code in [14] are presented. The distribution corresponding to the impulsive data (blue/solid curve) has heavier tails compared to the distribution corresponding to the non-impulsive data (red/dashed curve). This is explicitly supported by the parameters of the distribution fits where the impulsive noise has a characteristic exponent $\alpha = 1.7289$, while the non-impulsive one has $\alpha = 1.9627$. This behavior gives us motivation to approach to the spectral estimation problem of the impulsive noise using FLOS instead of SOS.

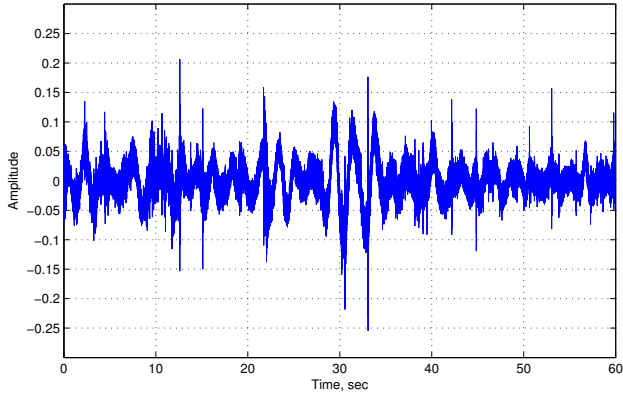
Estimated DEMON spectra of the non-impulsive data using the periodogram, MUSIC and FLOS-MUSIC can be seen in Fig. 4. Since the periodogram is suitable to estimate broadband or low dynamic range PSDs it fails to estimate the blade rate (BR) of the ship’s propeller or any of its harmonics sufficiently as shown in Fig. 4(a). We can hardly observe the BR and a few harmonics which have very low detection SNRs. However, MUSIC and FLOS-MUSIC capture the narrow-band spectra well as given in Fig. 4(b) and Fig. 4(c), respectively. SOS- and FLOS-based methods are comparable due to the fact that the signal exhibits a Gaussian-like distribution with a characteristic exponent close to 2. Both of the methods estimated the BR at 6.15 Hz and its five harmonics.

Superiority of the FLOS-MUSIC comes in the spectral estimation of the impulsive noise with a characteristic exponent $\alpha = 1.7289$ given in Fig. 5. While the periodogram shown in Fig. 5(a) and MUSIC in Fig. 5(b) fail to estimate any component of the spectra, FLOS-MUSIC succeeds to represent the BR at 6.15 Hz as shown in Fig. 5(c).

We have determined the signal subspace dimension in MUSIC-based methods empirically and obtained the best results with a dimension between 30-40. One can observe similar results in Fig. 6 presenting the eigenvalue distribution of the covariance matrix where an approximately linear decrease is seen after the 38th eigenvalue. This $1/f$ behavior may imply that determining the signal subspace dimension can



(a)



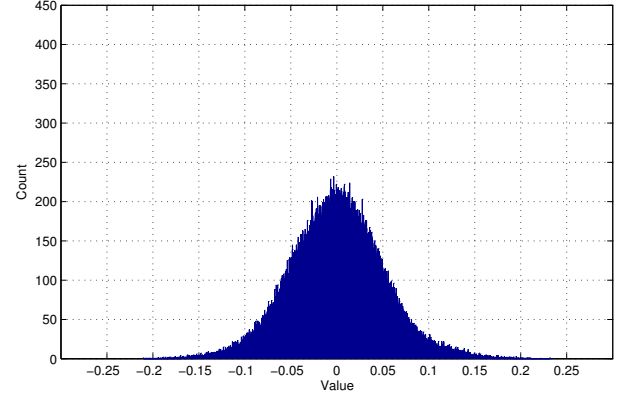
(b)

Fig. 2. Time signals analyzed, a) Non-impulsive signal, b) Impulsive signal.

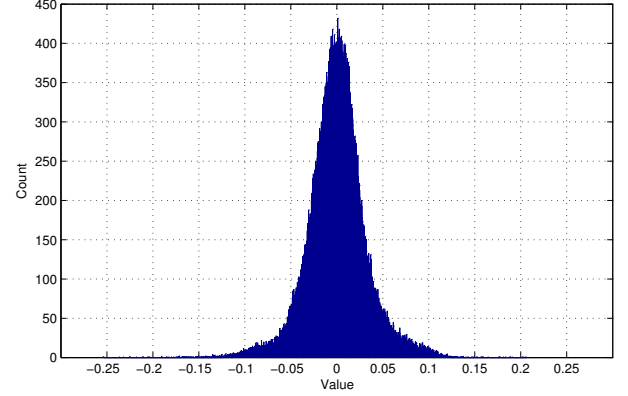
be accomplished by utilizing the eigenvalue distribution of the covariance matrix. Assuming the part of the eigenvalue distribution with “lower energy” represents the noise subspace a line can be fit to this part of the distribution. The eigenvalue index where the eigenvalue distribution is separable with this line can be employed as a signal subspace dimension which then can be used in MUSIC.

VI. CONCLUSION

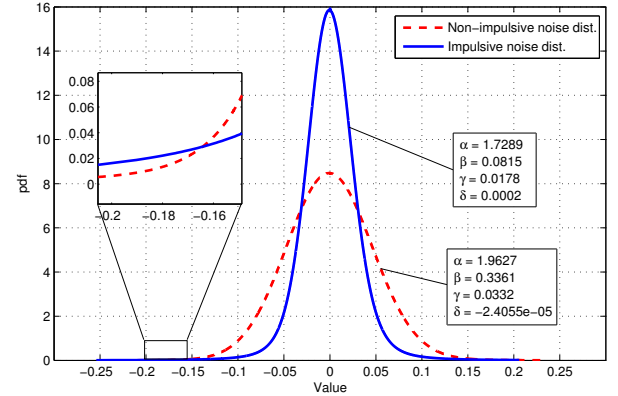
We have investigated the use of MUSIC and FLOS-MUSIC in the spectral estimation of the cavitation related narrow-band ship radiated noise. It is clear that when the classical spectral estimation methods cannot answer to the narrow-band spectral estimation of the signals with low SNRs we can make use of the subspace based high resolution methods. We also conclude that exploiting the distribution of the impulsive signals and modeling them as α -stable processes lets us use FLOS instead of “almost compulsory” SOS. The sample ship noise studied in this work exhibits an α -stable distribution which gives us motivation to further investigate the stable properties of ship-radiated noise in general and come up with a well-defined process yielding to even better estimates. Finally we have shown the $1/f$ -type behavior of the cavitation related narrow-band radiated noise of a sample ship which we have been able to use for the determination of the signal subspace dimension.



(a)



(b)



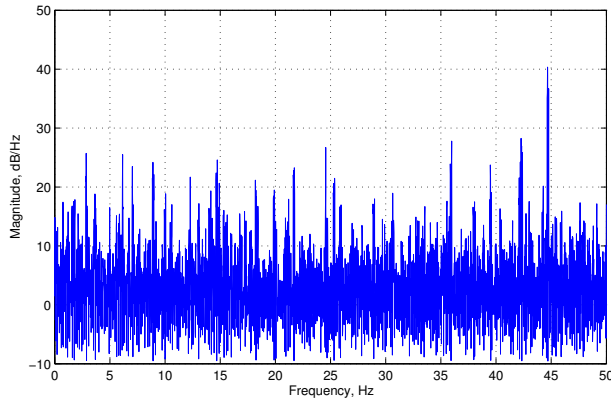
(c)

Fig. 3. Histograms of the signals with α -stable distribution fits, a) Histogram of the non-impulsive signal, b) Histogram of the impulsive signal, c) α -stable distribution fits with the blue one being the fit to the impulsive signal.

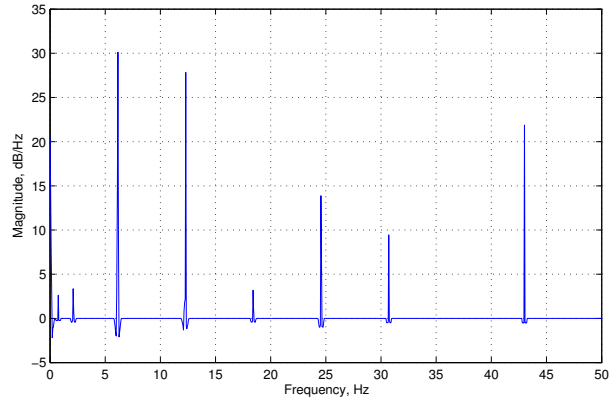
Thus, second future work will be the investigation of the $1/f$ -type behavior of the ship radiated noise in general.

REFERENCES

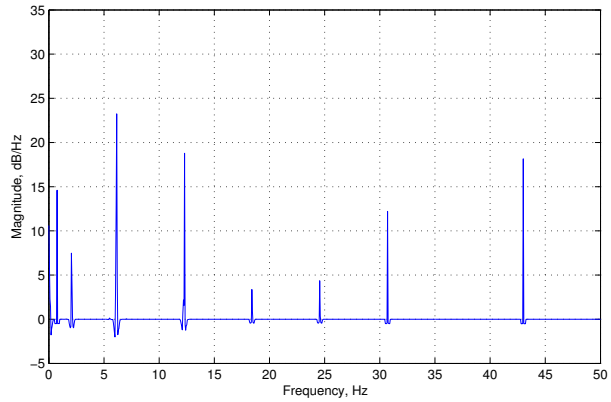
- [1] P. T. Arveson and D. J. Vendittis, “Radiated noise characteristics of a modern cargo ship,” *The Journal of the Acoustical Society of America*, vol. 107, p. 118, 2000.
- [2] J. Antoni and D. Hanson, “Detection of surface ships from interception of cyclostationary signature with the cyclic modulation coherence,”



(a)

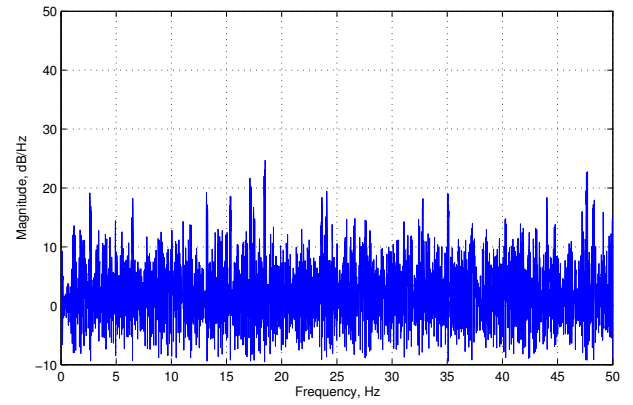


(b)

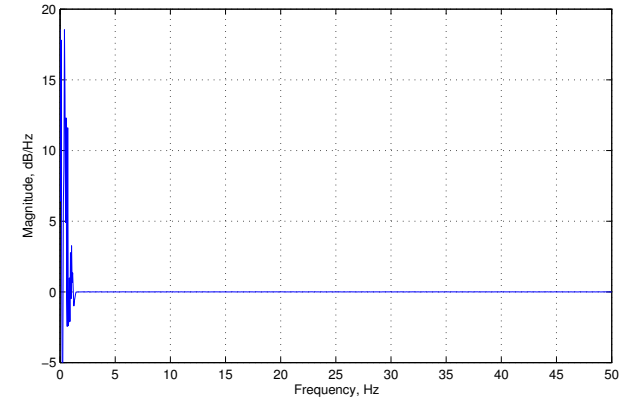


(c)

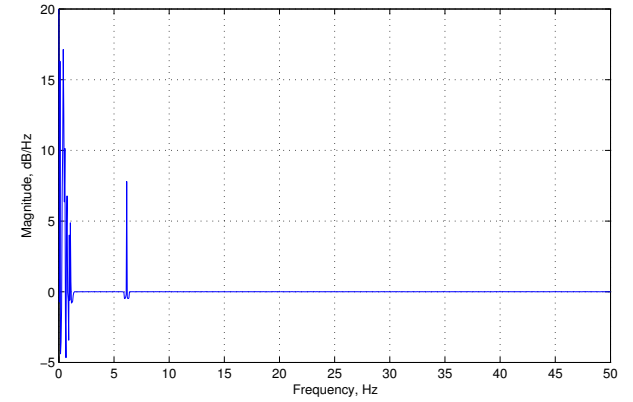
Fig. 4. DEMON Spectra of the non-impulsive signal, a) Periodogram with Hamming window, b) MUSIC with the signal subspace dimension of 34, c) FLOS-MUSIC with the signal subspace dimension of 34 and the FLOM order of 1.9627.



(a)



(b)



(c)

Fig. 5. DEMON Spectra of the impulsive signal, a) Periodogram with Hamming window, b) MUSIC with the signal subspace dimension of 34, c) FLOS-MUSIC with the signal subspace dimension of 34 and the FLOM order of 1.7289.

Oceanic Engineering, IEEE Journal of, vol. 37, no. 3, pp. 478–493, 2012.

- [3] A. Kummert, “Fuzzy technology implemented in sonar systems,” *Oceanic Engineering, IEEE Journal of*, vol. 18, no. 4, pp. 483–490, 1993.
- [4] S. Kay, *Modern Spectral Estimation: Theory and Application*. Englewood Cliffs, NJ: Prentice Hall, 1988.
- [5] M. Shao and C. Nikias, “Signal processing with fractional lower order moments: stable processes and their applications,” *Proceedings of the*

IEEE, vol. 81, no. 7, pp. 986–1010, 1993.

- [6] M. A. Altunkaya, H. Deliç, B. Sankur, and E. Anarım, “Subspace-based frequency estimation of sinusoidal signals in alpha-stable noise,” *Signal Processing*, vol. 82, no. 12, pp. 1807 – 1827, 2002. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0165168402003134>
- [7] M. S. Keshner, “1/f noise,” *Proceedings of the IEEE*, vol. 70, no. 3, pp. 212–218, 1982.
- [8] T. Ozkurt, T. Akgul, and S. Baykut, “Principal component analysis of the fractional brownian motion for $0 < H < 0.5$,” in *Acoustics, Speech*

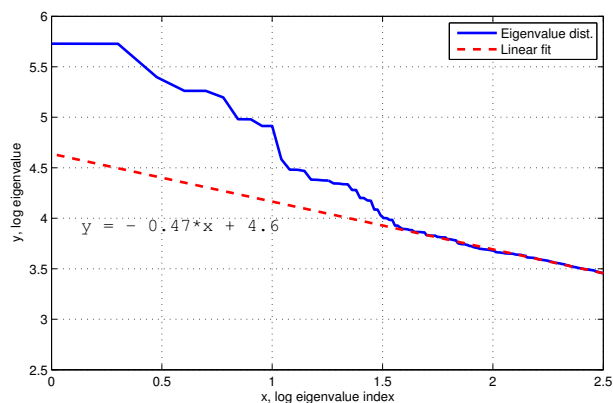


Fig. 6. Log-log eigenvalue distribution with a linear fit to the distribution.

and Signal Processing, 2006. ICASSP 2006 Proceedings. 2006 IEEE International Conference on, vol. 3, 2006, pp. III–III.

- [9] R. J. Urick, *Principles of underwater sound*, 3rd ed. New York: McGraw-Hill, 1983.
- [10] D. Ross, *Mechanics of underwater noise*. Los Altos, California: Peninsula Publishing, 1987.
- [11] P. Tsakalides and C. Nikias, “The robust covariation-based MUSIC (ROC-MUSIC) algorithm for bearing estimation in impulsive noise environments,” *Signal Processing, IEEE Transactions on*, vol. 44, no. 7, pp. 1623–1633, 1996.
- [12] U. Ulug, T. Akgul, and C. Gezer, “Ambient noise measurements in the strait of istanbul,” in *Proceedings of the Institute of Acoustics, Conference on Underwater Noise Measurement, Impact and Mitigation*, vol. 30, oct 2008, pp. 35–42.
- [13] A. Isabekov, S. Baykut, and T. Akgul, “Underwater ambient noise analysis using wavelet transform and empirical mode decomposition methods,” in *European Conference on Noise Control, EURONOISE-2009*, Edinburgh, Scotland, Oct 2009.
- [14] M. Veillette. (2012, Jul.) Stbl: Alpha stable distributions for MATLAB. [Online]. Available: <http://www.mathworks.com/matlabcentral/fileexchange/37514>