The Subarray MVDR Beamformer for Active Littoral Sonar Systems

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***Abstract*—In the littoral ocean environment ambient noise makes passive detection difficult, placing increased demands on active sonar. For active sonar systems that use azimuthally omnidirectional transmitters the bottom, surface, shoreline reverberation and clutter can be quite challenging to overcome when using Doppler sensitive waveforms, especially for low velocity targets. Straightforward application of adaptive beamforming to increase resolution for active sonar is typically challenged by snapshot deficiency. This deficiency is caused by the conflicting requirements for high Doppler resolution for target detection and small time windows for building up a full-rank signal covariance estimates. It has been proposed recently to overcome these limitations through the use of reduced beamspace adaptive beamforming. [1] The Subarray MVDR beamformer is a method of adaptive beamforming that reduces the required degrees of freedom by splitting the array into sub arrays which are first conventionally beamformed. The subarray beams are then adaptively combined to form adaptive weights using the subarray centers as the element spacing. This approach can eliminate the snapshot deficiency problem while still outperforming a conventional beamformer in both Doppler and bearing resolution. This method also more robust to errors in the signal model that can lead to signal suppression.**

The Subarray MVDR beamformer is analyzed in this paper both against both simulated data and against experimental data collected during the GLINT/NGAS11 experiment conducted by the Centre for Maritime Research and Experimentation (CMRE) in 2011. This experiment consisted of towed arrays deployed by AUVs in a multistatic experiment where the detection of returns of Doppler sensitive waveforms from an Echo Repeater towed at low speed was being attempted in the presence heavy shoreline reverberation. Simulation results indicate that the Subarray MVDR beamformer shows promising rejection of interfering signals that are not effectively attenuated by conventional beamforming. The application of the Subarray MVDR beamformer to the experimental data then shows that the Doppler spread of the reverberation ridge is significantly reduced, and the bearing resolution significantly improved.

I. Introduction

A. Problem

Over the years submarines have grown quieter while the oceans have grown noisier. Sonar systems are continually challenged to keep pace with the threats. Active systems in littoral regions show promise, but are challenged by noise, reverberation, and clutter, especially for low Doppler targets. Theoretical developments in waveforms and adaptive processing, accurate timing, and the availability of low power processors with significant computational ability, make possible the application of the new approaches onto low-cost off-board bistatic sensing platforms such as AUVs. The theoretical developments in beamspace adaptive beamforming especially have made the application of high resolution processing feasible in active ASW systems, where before errors in the wave field estimation and snapshot deficiency limited acheivable performance.

B. Background

A monostatic active sonar system is one for which the source and receiver are co-located. When the receive array and source are spatially separated then the system is bistatic, and if there are multiple sources and/or receivers the system is multistatic. [2] A bistatic sonar system geometry is shown in Figure 1. In order to detect a target's bearing the array must be beamformed into candidate target directions. For active systems reverberation is one of the sources of noise against which a detector must work. Reverberation increases with source level, and as beamwidths increase and bandwidths decrease, reverberation increases quadratically, causing false alarms and burying the target in self noise. Reverberation comes from three sources; volume scattering, ocean surface scattering, and ocean floor scattering. In littoral environments all three scattering mechanisms play an important role, and along with ambient noise caused by distant and nearby shipping, limit the performance of active sonar.

In passive sonar adaptive beamforming algorithms have traditionally been used to improve beamforming performance. There are many forms of passive adaptive beamformers, but most of them build upon the Minimum Variance Distortionless Response beamformer(MVDR) or Capon beamformer. [3] The MVDR beamformer adaptively forms weights based on the signal+noise covariance estimated from recorded data. Nulls are steered in the direction of correlated noise that is arriving from undesired directions. Adaptive beamforming has not traditionally been performed in active sonar because the transmitted signal is generally transient, limiting the amount of time that can be used to estimate the signal covariance. Another problem with MVDR beamforming which is common to both passive and active processing is that the signal model must be quite accurate in order not to steer nulls in the direction of the main beam. There are many ways to make the MVDR

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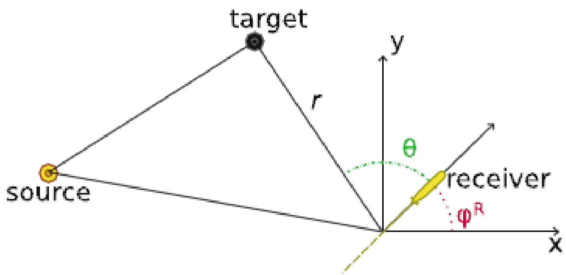

Fig. 1. A bistatic source-target-receiver example, where x and y are the coordinate system, r is the range between target and receiver, and Θ is the bearing between the array heading ϕ^R (as measured counterclockwise from the x axis) and the target [2]

beamformer robust against these errors such as; diagonal loading, beamspace processing, spatial averaging, etc. [3] The method that we have chosen to increase the robustness of the MVDR beamformer for active sonar is to pursue Subarray MVDR. Subarray MVDR conventionally beamforms smaller aperture arrays in the direction of interest and then adaptively combines the resulting beams to increase performance. This method of beamforming reduces the integration time needed for subarray covariance estimation (since there are generally many fewer subarrays than elements) and increases robustness by averaging out element level signal model errors.

C. Outline

This paper describes the first stage in an ongoing research project that started at CMRE (Formerly NATO Undersea Research Centre) in 2012 and will be continued at CMRE and at UCSD in the ECE Signal and Image Processing Department. The Subarray MVDR beamformer is compared against the conventional beamformer for simulated and experiment active sonar data. The experimental data were collected by a 32 element array towed by an AUV during the GLINT/NGAS11 Sea Trial. Similar (earlier) experiments are described in [2] and [4]. In Section II we describe the Subarray MVDR algorithm and show how robustness is achieved with a simulation example. In Section III we present the results obtained by analyzing the experimental dataset. We conclude and discuss future research directions in Section IV.

II. Method

A. Sonar Processing Chain

Mid-frequency active sonar systems are designed to localize and track submarines. In the GLINT/NGAS11 trials the goal was to multistatically detect and track an echo repeater in a near-shore littoral environment. A quasi-stationary source deployed from the NRV Alliance transmitted two different pulses: a continuous wave (CW) and a hyperbolic frequency modulated (HFM) chirp. The CW pulse is used to detect target velocity via the Doppler shift observed from the target return, as simulated by an echo repeater deployed from CMRE's coastal research vessel CRV Leonardo. The reason for using adaptive processing on the CW pulse only can be found in [5], but the bottom line is that single frequency Subarray MVDR is most easily implemented for narrowband signals. For this reason the Subarray MVDR algorithm is only applied to the CW waveform in this work. We note that the Subarray MVDR beamformer has been applied to other types of active sonar waveforms, including HFM pulses, as shown in [6]. While during GLINT/NGAS11 there were two AUVs towing arrays, only one was processed for this analysis. In the GLINT trial series the processing chain has consisted of: Fast Fourier Tranform (FFT)→beamform→Match Filter(MF)→IFFT→normalization→contact formation [2] [4]. In the work presented here the processing chain has been modified to: MF→FFT→subarray beamforming→subarray beam covariance estimation→adaptive weight calculation→adaptive beamforming→normalization→contact formation. The objective is to show the difference in beamformer performance so the peripheral steps prior to and following the beamformer are kept the same. The preprocessing and post processing steps are described below.

1) Preprocessing steps: The processing chain was altered to perform the subbeam covariance matrix estimation required to make the adaptive weights of the subarray beams. For this paper the MF stage was implemented by a FIR bandpass filter that was centered on the transmit pulse. This was used for simplicity and is sufficient for comparing the two beamformers for CW signals. An improved method would be to use a filter bank of exactly matched filters (CW with source waveform window weighting applied). Once the data is filtered recorded time segments are processed. The snapshots are created from 50% overlapping windows. The length of these windows was equal to the length of the transmitted pulse of 1 second. The window lengths were chosen to maximize the Doppler velocity resolution. Spectral smoothing was performed through the application of a Hamming window to the temporal data prior to the FFT. Trade offs between the window length and the total number of snapshots were performed with the final values given in Section III yielding the best results.

2) Postprocessing steps: Amplitude normalization was applied appropriately so that the output of the beamformer produced signals with physical units of sound pressure level in microPascals. The adaptive gain suppressed the noise from reverberation and ambient noise to very low levels. The data is arranged in a time-beam- Doppler cube that is searched by a beam and Doppler frequency domain target detector. The detection cube is essentially a three-dimensional split-window normalizer (with out guard band) that consists of an inner signal estimation cube surrounded by an outer window frame for noise estimation. This normalizer will detect a target by cueing on a SNR over a predetermined threshold. This detector is run every 200 msec to improve the temporal resolution fixed by the window length of the covariance estimator. The entire integration time is compressed to a single cell reciprocal to the overlap percentage. To quantify the results of the two beamformers a Doppler cube detector is manually placed on the target and the SNR improvement from the application of the Subarray MVDR Beamformer is presented. The effect on false alarm rate has not been studied at the time of this paper.

B. Conventional Beamforming and the Covariance Matrix

A conventional delay and sum beamformer will delay signal inputs in time so that an array of sensors can focus in a certain direction. The summation of the sensors creates gain against noise arriving from other directions. On a towed line array the directions are steered between 0 and 180 degrees from the bow of the array. A residual left-right ambiguity due to the omnidirectional nature of the hydrophones can cause shoreline or other undesireable clutter from the ambiguous direction to reduce the SNR of a detection on the target side of the array.

In an array of M elements each element records a signal $x(t)$ that contains the direct blast $s_0(t)$, the contact signal along with all reverberations signals $s_p(t)$ and the noise from each sensor $n(t)$. The small signal model [7] of each m^{th} sensor may be written as:

$$x_m(t) = \frac{1}{r_{m,0}} s_0(t) + \sum_{p=1}^{P} \frac{1}{r_{m,p}} s_p(t) * \delta(t - \tau_{m,p}) + n_m(t) \quad (1)$$

where $r_{m,p}$ is the distance from p reflection to m sensor, δ is an impulse function, $\tau_{m,p}$ is the delay in time from reflector p to sensor m, and $(*)$ represents convolution. There are P+1 reflections including the reflection off of the target. The entire wave field is represented as a vector of all $x_m(t)$ elements in a $Mx1$ vector $\mathbf{X}(t)$.

The frequency domain expression of the sum and delay beamformer is expressed as:

$$B(\omega) = \frac{1}{\alpha} \sum_{m=0}^{M-1} w_m e^{-j\omega\tau_m} x_m(\omega) = \mathbf{W}^H \mathbf{X}(\omega) \quad (2)$$

where w_m is the element weighting for the spatial window, α is the sum of the weights, ω is the angular frequency, $\mathbf{W}$ is the $Mx1$ steering vector with the desired weights and delay. The beamformer output power in the quadratic form is:

$$P(\omega) = E[|B(\omega)|^2] = \mathbf{W}^H \mathbf{R}(\omega) \mathbf{W} \quad (3)$$

where $\mathbf{R}(\omega)$ is the covariance matrix. In practice we take the FFT of the windows and average them over the number of snapshots to form a sample covariance matrix. The sampled covariance matrix is:

$$\hat{\mathbf{R}}(\omega) = \frac{1}{L} \sum_{l=0}^{L-1} \mathbf{X}_l(\omega) \mathbf{X}_l^H(\omega). \quad (4)$$

C. Adaptive Beamforming

Adaptive beamforming consists of changing the weights to be functions of the signal data. The MVDR beamformer forms weights in a way that will attempt to maintain the gain of the beamformer in the focused direction while steering nulls in the direction of other interference. In directions where there is more interference the beamformer will weight the sidelobes lower through the adaptive process. The MVDR beamformer computes the weights by minimizing the variance power of the beamformer output

$$P(\omega) = E[|B(\omega)|^2]. \quad (5)$$

The weights are calculated by solving the following minimization equation,

$$min \mathbf{W_o} [\mathbf{W}_o^H(\omega) \hat{\mathbf{R}}(\omega) \mathbf{W}_o(\omega)] \quad (6)$$

subjected to a unity gain constraint

$$\mathbf{W}_o^H(\omega) \mathbf{d} = 1 \quad (7)$$

where $\mathbf{d}$ is the steering vector that focuses the array beams and consist of

$$d_m = e^{-j\omega\tau_m}. \quad (8)$$

The above equations are solved by using the Lagrangian multiplier [3] and have the solution:

$$\mathbf{W}_o(\omega) = \frac{\hat{\mathbf{R}}^{-1}(\omega)\mathbf{d}}{\mathbf{d}^H \hat{\mathbf{R}}^{-1}(\omega)\mathbf{d}}. \quad (9)$$

The resultant ideal adaptive beamformer power output is:

$$P_o(\omega) = \mathbf{W}_o^H(\omega) \hat{\mathbf{R}}(\omega) \mathbf{W}_o(\omega). \quad (10)$$

This is the pure form MVDR or Capon beamformer. There are a few problems with this method. Nulls will be created in the direction of any correlated noise that is not in the direction of the steered beam. If phase errors exist in the calculation of the steered beam the MVDR beamformer will then null out the main lobe instead of the off-beam signal. There are several methods for making the beamformer less sensitive to signal model errors. See Figure 2 which illustrates the nulls created by the MVDR method and increased bearing resolution of the power output. [3]

The top panel in Figure 2 shows the eigenvalues for a 16 element array that contains 3 separate signals with a SNR of 10 dB, 0 dB, and -10 dB. The middle panel shows the output power vs beam angle, and the bottom panel shows the adaptively formed weights. The nulls formed at the higher SNR signals are very narrow and can attenuate the signal by a larger margin. This makes it very susceptible to phase errors caused by poor array element localization, waveguide effects, or electronic errors. [8] Another problem with the classic MVDR is the number of snapshots required to create a full rank covariance matrix. [3] A common rule of thumb is the number of snapshots should be equal to or greater than two times the number of degrees of freedom, i.e. the number of array elements. In the active sonar scenario we desire to maximize Doppler resolution which requires large window sizes. If we choose too many snapshots (i.e. partially overlapping windows) for the estimation of the array covariance we will begin to integrate over reverberation without a target return, reducing the gain of the beamformer. If we choose a window size that is too short we will surrender Doppler resolution, which likewise impacts the target SNR. A solution to these problems comes through the use of the Subarray MVDR Beamformer. [1]

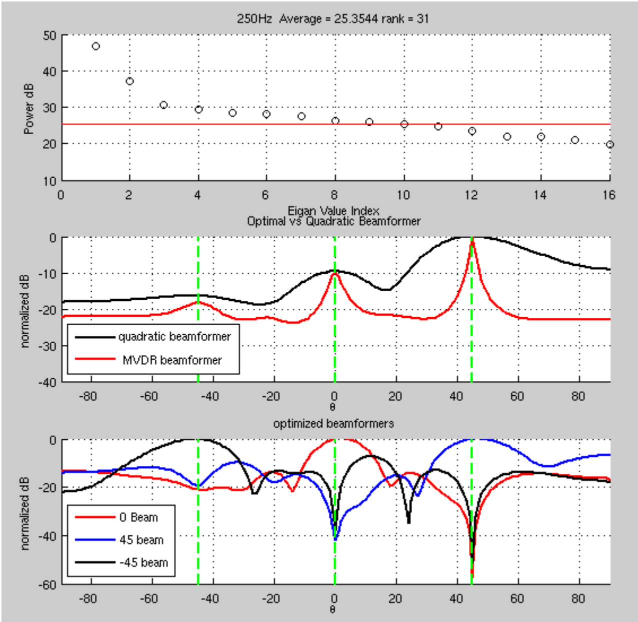

Fig. 2. MVDR beamformer simulation

D. Subarray MVDR Beamforming

Subarray adaptive beamforming will reduce number of degrees of freedom by sectioning the array into subarrays. These subarrays are first conventionally beamformed. The beams for all subarrays in the direction of interest are then adaptively combined. The subarrays can be spatially shaded to prevent spectral leakage. A 50% overlap can be applied to prevent spatial aliasing. Shading and overlap only really apply to the line array scenario and are not a requirement for valid results. N is the number of subarrays of K elements each, with n the subarray index and k the subarray element. The corresponding subarray beamformer output is,

$$B_n(\omega) = \frac{1}{\alpha} \sum_{k=0}^{K-1} w_k e^{-j\omega\tau_k} x_k(\omega) = \mathbf{W}_n^H \mathbf{X}_n(\omega). \quad (11)$$

Taking the output of the conventional beamformers we shift the beams into the steer direction with respect to the entire array. The shifting is taken from each subarray center to make a new steering vector consisting of N beams. The subarray steering vector element is described in [1] as:

$$c_n = e^{-j\pi\omega\tau_n}. \quad (12)$$

The covariance matrix is formed with the subarray outputs, creating a reduced beamspace covariance matrix of NxN elements. As before the total integration time is divided into overlapping windows creating L snapshots which are averaged to create the subbeam covariance estimate.

$$\hat{\mathbf{\Gamma}}(\omega) = \frac{1}{L} \sum_{l=0}^{L-1} \mathbf{B}_l(\omega)\mathbf{B}_l^H(\omega) \quad (13)$$

This subbeam covariance matrix is being adaptively processed using the MVDR method for the steering vector $\mathbf{c}$. The Subarray MVDR algorithm creates a beamformer more robust to signal model errors. This is due to the spatial averaging that occurs in the conventional subarray beamforming step. When the conventional beamformer is very long and the MVDR covariance matrix is very short in comparison this may be the only constraint needed. As N increases and K decreases the beamformer becomes more susceptible to phase errors and may need to add diagonal loading or some other constraint. In the case of active sonar the rank was kept so low that no additional constraint was required to produce the results presented here.

Additionally, as stated previously, the Subarray MVDR does not need as long an integration time to obtain enough snapshots to form a full rank covariance matrix. The choice of the number of subarrays determines the number of snapshots. A detailed analysis of this process and the statistical significance can be found in [1]. The adaptive weights now become

$$\mathbf{H}_o(\omega) = \frac{\hat{\mathbf{\Gamma}}^{-1}(\omega)\mathbf{c}}{\mathbf{c}^H\hat{\mathbf{\Gamma}}^{-1}(\omega)\mathbf{c}}, \quad (14)$$

which are illustrated in Figure 3.

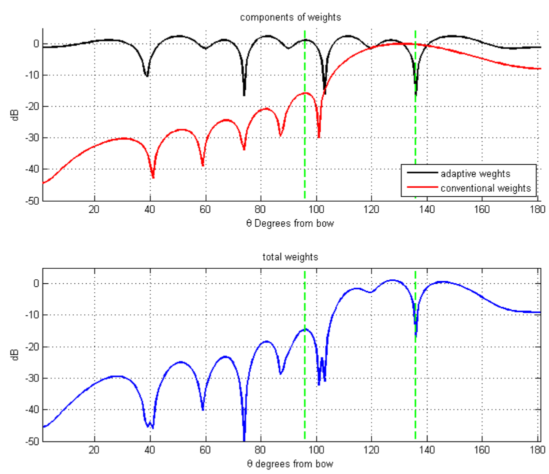

Fig. 3. Conventional and adaptive weights

Figure 3 shows the two stages of weights as components and their combined weighting. There are 2 interferers in this simulation to show that the null will typically form inside the main beam unless the sidelobes of the subarray can not filter out the reverberation on their own. The conventional weights $\mathbf{W_n}$ are shown in red and the adaptive weights $\mathbf{H_o}$ are shown in black in the upper panel, while in the lower panel the subarray combined weights are shown in blue.

Figure 4 shows a comparison between the beamformer output of a conventional beamformer with Hamming weights and the Subarray MVDR beamformer. The simulation consists of a 32 element line array spaced at $\lambda/2$ with 3 subarrays of length 16, 50% overlapping with Hamming windows on

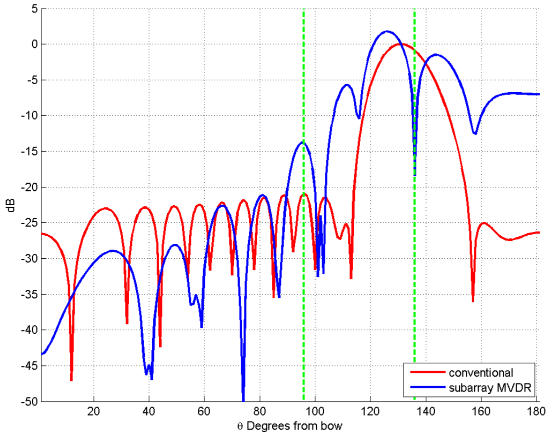

Fig. 4. Sub Array MVDR beamformer compared to a hamming weighted conventional beamformer

the subarray weights. The snapshot amount was 10. This simulation was performed purely in the frequency domain. The look direction was 130 degrees and jamming signals arriving at 95 and 135 degrees were added to demonstrate the null forming ability. The SNR of the interferers were 10dB above the ambient Gaussian noise background. The power output

$$P_s(\omega) = \mathbf{H_o}^H(\omega)\hat{\mathbf{\Gamma}}(\omega)\mathbf{H_o}(\omega) = (\mathbf{c}^H\hat{\mathbf{\Gamma}}^{-1}(\omega)\mathbf{c})^{-1} \quad (15)$$

comparison is shown in Figure 5. This figure shows the vast improvement in resolution that can be gained from the Subarray MVDR beamformer.

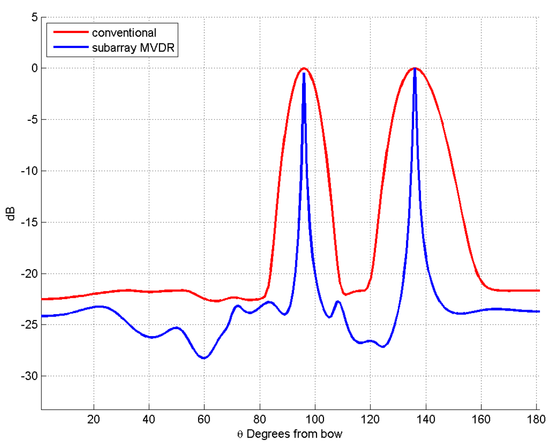

Fig. 5. Simulation output

III. Results

A. Experiment

In September of 2011 the GLINT/NGAS11 multistatic sonar experiment was conducted by CMRE (NURC at the time) in littoral Italian waters. The experiment consisted of a sound source deployed from the NRV Alliance in stationkeeping mode, an echo repeater deployed from the CRV Leonardo, and two towed arrays deployed from AUVs. The data analyzed in this section comes from only one of the two receive arrays. During this run the receiver was subjected to back-beam reverberation from a nearby shoreline. This caused detections of E/R returns of the Doppler sensitive CW signal (1s) to become engulfed by clutter from the backlobe and therefore undetectable in many instances. A total of 15 transmissions were analyzed for these results. The data was taken as the receiver moved closer to shore, experiencing high backbeam reverberation along the way.

The receive array was 32 elements; one element was used for timing and was excluded from this analysis. The CW waveform was a 1 second shaded pulse. The data files included the direct blast and the echo return from a towed repeater. This dataset proved to be a good foundation for testing the Subarray MVDR beamformer performance.

B. Parameter Choices

The subbarray number was chosen to be N = 2, and the subarray element count K=16 to ensure we need the fewest amount of snapshots to estimate the subarray beam covariance. If there are more than one source of interference N=2 will not have the degrees of freedom to suppress all the interference and diffuse reverberation. However this implementation is robust. Since there was no overlapping of the elements there was no spatial weighting in the conventional beamforming. To obtain the Doppler resolution needed, an FFT length or snapshot size of 1 second was used. Five snapshots were used to estimate the 2x2 covariance. 50% temporal overlapping and Hamming shading was used.

C. Geo-referenced Transmission Visualizations

Figures 6 and 7 show geo-referenced sound pressure levels of the direct blast and other returns. The range resolution is quite poor due to the 1 s CW waveform used. While the range resolution of a CW is poor we note that a second HFM waveform, broadcast simultaneously with the CW, is used to estimate range of contacts first detected by the CW processing. The direct blast and the target return are indicated by a circle and plus marker at approximately [11000 4500] meters. A distinct increase of bearing resolution with the Subarray MVDR can be easily seen by comparing these two plots. Note also the proximity of the shoreline on the backbeam.

D. Doppler Cube Processing

Doppler data cube consists of bearing, range and Doppler velocity data information. Evaluating performance enhancements of reverberation rejection is not a trivial task due to the fact that the target and reverberation will change as a function of relative speed difference and bearing. Two main criteria that this paper presents as improvements are the visual reduction of the reverberation ridge seen in Figures 8 and 9 and the SNR increase of the Doppler search frame when placed on the target contact. The Doppler search frame consists of an inner and outer cube that estimates the signal using the inner

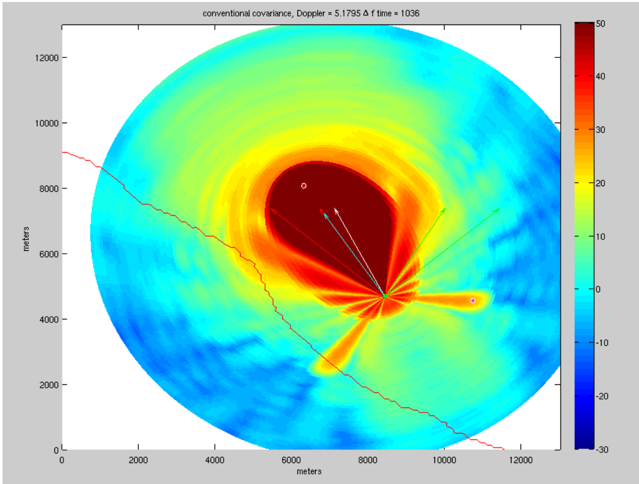

Fig. 6. Conventional Beamforming Results

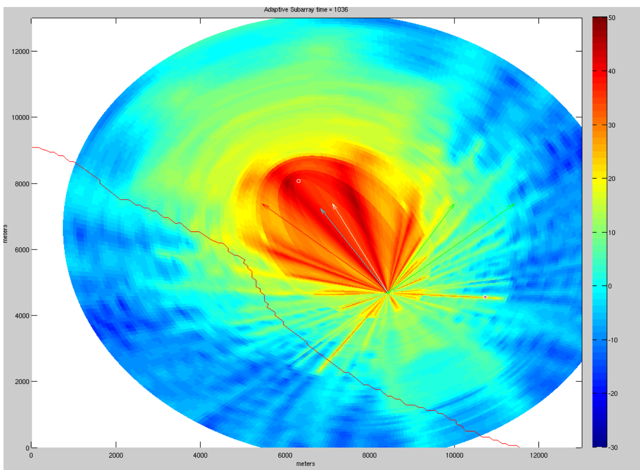

Fig. 7. adaptive beamforming

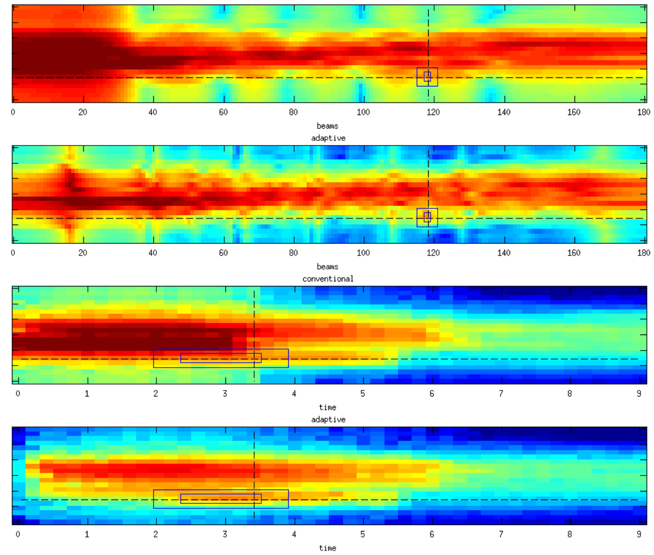

Fig. 8. Doppler cube output

cube and the noise using the outer frame. If the mean of the amplitude squared of the inner cube is greater than the mean of the amplitude squared of the outer frame the SNR is positive and vice versa. In this analysis the Doppler cube size was set to match the length of the transmission pulse in time (1 s) and corresponding Doppler frequency resolutions (1 Hz). The bearing dimension size was chosen based on the improved resolution that the Subarray MVDR beamformer provides. These levels can be interpreted as thresholds that the Doppler cube would analyze when it searches the entire space for target contacts. A low SNR would result in many false alarms and higher probability of detection while a high threshold would simultaneously decrease the probability of detection and false alarm.

In [6] the authors divide the processed Doppler cube dataset into regions and measure the reverberation in different zones defined by the spread of the reverberation in range and bearing for specific receiver speeds. Both the target Doppler velocity and the reverberation Doppler velocity is a conversion of the change in frequency given in the equation:

$$f = f_o(1 + 2(v/c)cos\theta). \tag{16}$$

Where v is the relative velocity between the ship and target or ship alone when referring to reverberation. The v and θ create the top two images in Figure 8. The top panel shows the conventional beamformer and the one below shows the Subarray MVDR Beamformer. The two bottom panels show the Doppler frequency vs time to display the range dimension. In Figure 8 the reverberation ridge is clearly reduced in Doppler spread allowing for much of the area becoming closer to being noise limited instead of reverberation limited. A deeper analysis of the reverberation patterns will be performed in the future to better understand the total improvement caused by using adaptive beamforming to reject the shoreline reverberation. In this study we take the visual results as an indication that the beamformer is reducing the reverberation and at the same time we analytically evaluate the target SNR gain.

The Subarray MVDR beamformer was applied to 15 consecutive transmissions from the GLINT/NGAS11 trial data for a scenario when the towed array and the E/R moved close to the shore. The average SNR for the conventional beamformer in this case was 2.6 dB, while the subarray adaptive beamformer provided an average SNR of 8 dB, indicating an improvement of 5.4 dB.

Figure 8 displays the entire search space for one transmission. Figure 9 a close up of the Doppler cube signal and noise area is displayed for the same transmission. The dashed lines is the estimation of the target from non-acoustic sensors and the rectangles represent the Doppler cube placement. As stated before the inner cube is used to estimate the signal and the outer cube is used to estimate the noise.

IV. Conclusions and Future Work

The Subarray MVDR beamformer shows potential to raise the probability of detection in a littoral environment for a

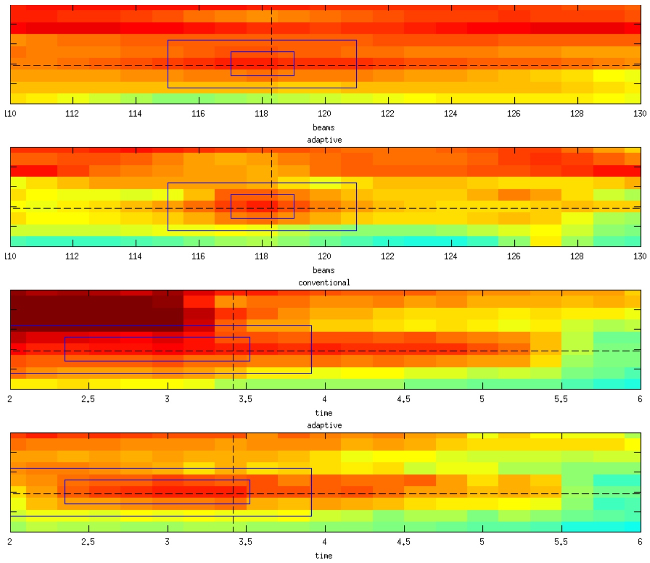

Fig. 9. Doppler cube output zoomed into signal

given probability of false alarm when using Doppler sensitive waveforms in active sonar. Using a CW pulse the Subarray MVDR converged much faster than an element level MVDR beamformer would require, allowing the use of shorter waveforms. The beamformer successfully increased the bearing resolution and reduced the area over which the backlobe reverberation interference was integrated. Results were shown both for simulation and for experimental data collected during CMRE's GLINT/NGAS11 sea trial.

Continued work to include the HFM signal in the processing is underway. A full receiver operating characterization of the Subarray MVDR beamformer for finding the E/R during GLINT11 is also planned. In this context an in-depth look at the inhomogeneous shoreline clutter on the backbeam of the towed array will be undertaken. Finally to complete the analysis the receive array deployed from the second AUV will be processed in a like manner to increase the statistical significance of the studies findings.

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