Robust Source Localization Using Predictable Mode Subspace in Uncertain Shallow Ocean Environment

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***Abstract*—Conventionally used matched-field source localization methods are sensitive to environmental parameter mismatch. In this paper, a new robust localization method is proposed. It is based on a decomposition of the field into predictable and unpredictable subspaces of the acoustic normal mode representation. The method uses the predictable subspace to reconstruct the replica vector and the orthogonality between the predictable and unpredictable subspaces is used to eliminate the impact of environmental uncertainty. The performance of the method is evaluated and compared to other matched-field methods using computer simulations. Results show that, in the presence of mismatches, the proposed method has superior probability of correct localization than the robust maximum-likelihood and the Bartlett methods.**

Keywords—uncertain ocean environment, robust localization, predictable mode subspace, reconstructed replica vector

I. Introduction

The spatial and temporal variations in underwater acoustic channel parameters, such as the water depth and the sound speed profile, make it difficult to characterize the channel accurately and resulting in an uncertain ocean acoustic environment. This uncertainty can cause severe performance degradation in sonar localization algorithms that rely on precise knowledge of the environmental parameters [1, 2].

Several localization algorithms that are robust to these uncertainties have been proposed, such as the matched mode processor (MMP) [3], the minimum variance beamformer with multiple neighboring location constrains (MV_NCL) [4], the optimum uncertain field processor (OUFP) [5-7], the maximum-likelihood (ML) estimator [8], the minimum variance beamformer with environmental perturbation constraints (MV_EPC) [9], the Minimax robust matched-field processor [10], and the convex optimization based matched-field processor [11]. MMP consists of decomposing the measured fields to obtain the excitations or amplitudes of constituent propagating modes, the source localization can then be estimated by matching these excitations with a grid of modeled replica excitations. Subsets of complete mode set can be considered to address problems such as environmental mismatch [12]. However, the modal decomposition itself can be ill-posed and unstable. The MV_NCL, the MV_EPC, the Minimax and the convex optimization processors all follow the idea that the robustness can be achieved by making constrains on the response of neighborhood points of the source location or the nominal values of the parameters. In the OUFP, prior statistical information about the source location, propagation, and array parameters is used in order to obtain optimal estimates of the source location parameters. This processor involves a computationally intensive integration over the model parameter space. In the ML estimator, a multidimensional search over the parameter space is required. In most situations the model parameter space is extremely large, thus solution via an exhaustive search is not a viable option. Furthermore, the objective function usually contains many local minima/maxima precluding the use of gradient descent methods. Therefore, this approach has been investigated mainly using simulated annealing and genetic algorithms.

Extending the idea of MMP, Tabrikian et al. [13] proposed a robust ML estimator, which decomposes the acoustic field into the predictable and unpredictable subspaces. In this estimator, the measurements are projected into the unpredictable null subspace before being matched to the predictable subspace. However, nulling the unpredictable modes is not perfect, since in constructing the orthogonal projection matrix the supposed modal depth eigenfunctions are different from the true ones because of ocean environmental mismatch, and this limits the localization performance. This paper proposed a new robust localization method in which the replica vector is reconstructed using the predictable mode subspace. The method uses the orthogonality between the predictable and unpredictable subspaces to eliminate the impact of environmental uncertainty. The performance of the proposed method is evaluated and compared to other existing methods by simulations. It is shown that the new method achieves higher probability of correct localization than the robust ML and conventionally used MFP methods.

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II. Robust Localization Scheme

A. Matched Mode Processor

In the far field, the normal mode representation of the acoustic pressure $p(r,z)$ at a receiver positioned $(r,\ z)$ for a source at depth z_{s} is

$$p(r,z)=\frac{ie^{-i\pi/4}}{\rho(z_s)\sqrt{8\pi r}}\sum_{m=1}^{M}\phi_m(z_{\mathrm{s}})\phi_m(z)\frac{e^{ik_m r}}{\sqrt{k_m}}, \quad (1)$$

where $\phi_m(\bullet)$ are the modal depth eigenfunctions, M is the number of the propagating modes in the channel, k_m denotes the horizontal wave number of the mth mode, and $\rho(z_{\mathrm{s}})$ is the water density at the source depth.

Equation (1) can be expressed as

$$p(r,z)=\sum_{m=1}^{M}a_m(r,z_{\mathrm{s}})\phi_m(z), \quad (2)$$

where the mode-amplitude $a_m(r,z_{\mathrm{s}})$ is

$$a_m(r,z_{\mathrm{s}})=\frac{\sqrt{2\pi}e^{j\pi/4}}{\rho(z_{\mathrm{s}})}\frac{e^{jk_m r}}{\sqrt{k_m r}}\phi_m(z_{\mathrm{s}}). \quad (3)$$

Using (2), the signal received on an N-element VLA can be represented in vector notation, as

$$\mathbf{p}=\begin{bmatrix} p(r,z_1) \\ \vdots \\ p(r,z_N) \end{bmatrix}=\begin{bmatrix} \phi_1(z_1) & \cdots & \phi_M(z_1) \\ \vdots & \ddots & \vdots \\ \phi_1(z_N) & \cdots & \phi_M(z_N) \end{bmatrix}\begin{bmatrix} a_1 \\ \vdots \\ a_M \end{bmatrix}=\mathbf{Ea}, \quad (4)$$

where $\mathbf{E}$ is the $N\times M$ modal decomposition matrix and $\mathbf{a}$ is the $N\times 1$ mode-amplitude vector [14]. Assuming that there is only one radiating point source, the data snapshot vector including noise can then be written as

$$\mathbf{r}=a_0\mathbf{Ea}+\mathbf{n}_0, \quad (5)$$

where a_0 is the complex signal amplitude of the source and $\mathbf{n}_0$ is the temporally and spatially white noise.

In order to obtain the mode amplitudes we should perform modal decomposition on the measured phone-space data. The modal decomposition is a linear inverse problem

$$\tilde{\mathbf{a}}=\mathbf{Hr}, \quad (6)$$

where $\tilde{\mathbf{a}}$ represents the estimated mode-amplitude vector and $\mathbf{H}$ is the $M\times N$ mode inverse filter. Pseudo inverse method with truncated eigenvector decomposition (EVD) is often used for the mode inverse filter to solve the ill-posed inverse problem.

$$\mathbf{H}=\left(\mathbf{E}^{\mathrm{H}}\mathbf{E}\right)^{-1}\mathbf{E}^{\mathrm{H}}. \quad (7)$$

The present modal decomposition is performed with the cross-spectral density matrix (CSDM) instead of (6) to improve the signal-to-noise (SNR) and to reduce the signal variance, in which case the modal CSDM can be obtained as

$$\mathbf{R}_{\mathrm{M}}=\mathbf{HRH}^{\mathrm{H}}\simeq\langle\tilde{\mathbf{a}}\tilde{\mathbf{a}}^{\mathrm{H}}\rangle, \quad (8)$$

where $\mathbf{R}_{\mathrm{M}}$ represents the $M\times M$ transformed modal CSDM and $\mathbf{R}$ is the $N\times N$ sampled phone-space CSDM. The phone-space CSDM $\mathbf{R}$ is computed as

$$\mathbf{R}=\frac{1}{L}\sum_{l=1}^{L}\mathbf{r}_l\mathbf{r}_l^{\mathrm{H}}, \quad (9)$$

where $\mathbf{r}_l$ is the lth snapshot vector and L is the number of snapshots. Using (8), the basic Bartlett MMP is given by

$$B_{\mathrm{Bart}}\left(\tilde{\mathbf{S}}\right)=\mathbf{a}^{\mathrm{H}}\left(\tilde{\mathbf{S}}\right)\mathbf{R}_{\mathrm{M}}\mathbf{a}\left(\tilde{\mathbf{S}}\right), \quad (10)$$

where $\tilde{\mathbf{S}}=(\tilde{r},\tilde{z}_s)$ represents the source location search space, $\mathbf{a}\left(\tilde{\mathbf{S}}\right)$ is the normalized replica mode-amplitude vector on the location $\tilde{\mathbf{S}}$.

We can also get the Capon form MMP

$$B_{\mathrm{MV}}\left(\tilde{\mathbf{S}}\right)=\frac{1}{\mathbf{a}^{\mathrm{H}}\left(\tilde{\mathbf{S}}\right)\mathbf{R}_{\mathrm{M}}^{-1}\mathbf{a}\left(\tilde{\mathbf{S}}\right)}. \quad (11)$$

B. Robust Localization using the Reconstructed Replica Vector

In the presence of environmental uncertainties, modal eigenfunctions are not deterministic anymore. Thus, the MMP method denoted by (10) and (11) will encounter a mismatch problem which results in decrease of the localization performance.

Environmental uncertainties have different impact on the modal eigenfunction variations. Some modal eigenfunctions remain more correlated than others [13]. One of the larger uncertainties in acoustic propagation modeling is the effect of boundary interactions. Typically, higher-order modes have more significant coupling to the boundary and are thus less predictable. In typical shallow-water channels, the higher-order modes are associated with rays that have a higher number of reflections from the boundaries and larger group delays in the medium. Therefore, these modes are often affected by medium variations more than the lower-order modes. In other words, uncertainties in the channel parameters cause the higher-order modes to be less correlated than the lower-order modes. This means that the available information on the source location is weaker in the higher modes than the lower modes.

Consequently, the mode space can be decomposed into two subspaces, one is predictable and another is unpredictable.

Assume that M_0 modes, constructing the space $\mathbf{\Omega}_p$, are predictable while the M-M_0 modes in the space $\mathbf{\Omega}_u$ are unpredictable. Then the field can be written as a sum of predictable and unpredictable components

$$\mathbf{r} = a_0 \mathbf{E}_{\Omega_p} \mathbf{a}_{\Omega_p}(r, z_s) + a_0 \mathbf{E}_{\Omega_u} \mathbf{a}_{\Omega_u}(r, z_s) + \mathbf{n}_0 , \tag{12}$$

where a_0 is the unknown complex source amplitude. $\mathbf{E}_{\Omega_p}$ and $\mathbf{E}_{\Omega_u}$ are the predictable and unpredictable modal matrices, respectively. $\mathbf{a}_{\Omega_p}$ and $\mathbf{a}_{\Omega_u}$ are corresponding mode-amplitude vectors, where the sub-subscript p denotes the predictable parts and u denotes the unpredictable ones. $\mathbf{n}_0$ is the environmental noise. Since $a_0 \mathbf{E}_{\Omega_u} \mathbf{a}_{\Omega_u}(r, z_s)$ is unpredictable, it can be modeled as generalized environmental noise.

Let $\mathbf{n}_g = a_0 \mathbf{E}_{\Omega_u} \mathbf{a}_{\Omega_u}(r, z_s) + \mathbf{n}_0$ be the total noise and $\mathbf{A}_{\Omega_p} = \mathbf{E}_{\Omega_p} \mathbf{a}_{\Omega_p}(r, z_s)$ be the reconstructed replica vector. Then the filed can be rewritten as

$$\mathbf{r} = a_0 \mathbf{A}_{\Omega_p}(r, z_s) + \mathbf{n}_g . \tag{13}$$

$\mathbf{n}_g$ is generally not white noise. According to the central limit theorem, assuming $\mathbf{n}_g$ is white has the lowest risk. So, the Bartlett form of the proposed method is

$$B_{\text{Bart}}(\tilde{\mathbf{S}}) = \mathbf{A}_{\Omega_p}^{\text{H}}(\tilde{\mathbf{S}}) \mathbf{R} \mathbf{A}_{\Omega_p}(\tilde{\mathbf{S}}) . \tag{14}$$

The Capon form is

$$B_{\text{MV}}(\tilde{\mathbf{S}}) = \frac{1}{\mathbf{A}_{\Omega_p}^{\text{H}}(\tilde{\mathbf{S}}) \mathbf{R}^{-1} \mathbf{A}_{\Omega_p}(\tilde{\mathbf{S}})} . \tag{15}$$

Conventional MMP can use a subset of complete mode to localize the source robustly in uncertain ocean environment too. But the modal decomposition should be firstly performed. The estimation error of modal amplitudes under environmental uncertainty will lower the localization performance, especially at low SNRs. It can be seen from equations (14) and (15) that the mode filtering can be avoided in the proposed method. The method used the orthogonality between the predictable and unpredictable subspaces to eliminate the impact of environmental uncertainty. Thus, the high localization performance can be expected.

C. Identification of the Predicable Modes

Environmental mismatch causes errors in both the modal horizontal wave numbers and eigenfunctions. To identify the predictable modes, we treat the uncertain environmental parameters as random variables. Under this assumption, the horizontal wave numbers and modal eigenfunctions in the propagation model are also random. Tabrikian [13] point out that the correlation of the modes in the presence of environmental uncertainty can be evaluated by computing the second order statistics of the horizontal wave numbers. Here we briefly review the method.

Assume a candidate subset of modes with horizontal wave number vector, $\mathbf{k}$, of dimension $M_0 \times 1$. Let k_l be the lth realization of the vector k. The covariance matrix of k can be estimated by

$$\Gamma = \operatorname{cov}(\mathbf{k}) \approx \frac{1}{L} \sum_{l=1}^{L} \left(\mathbf{k}_l - \hat{\mathbf{k}}\right)\left(\mathbf{k}_l - \hat{\mathbf{k}}\right)^{\text{H}} , \tag{16}$$

where $\hat{\mathbf{k}}$ is the unbiased estimator of $\bar{\mathbf{k}} \triangleq \text{E}(\mathbf{k})$

$$\hat{\mathbf{k}} = \frac{1}{L} \sum_{l=1}^{L} \mathbf{k}_l . \tag{17}$$

Decompose the horizontal wave numbers by letting

$$\mathbf{k} = \bar{\mathbf{k}} + \Delta\mathbf{k} = \bar{\mathbf{k}} + \Delta\mathbf{k}^{\perp} + \varepsilon_1 \bar{\mathbf{k}} + \varepsilon_2 \mathbf{1} , \tag{18}$$

where $\mathbf{1}$ is a vector of ones. $\Delta\mathbf{k}^{\perp}$ is given by

$$\Delta\mathbf{k}^{\perp} = \Delta\mathbf{k} - \mathbf{F}\left(\mathbf{F}^{\text{H}}\mathbf{F}\right)^{-1} \mathbf{F}^{\text{H}} \Delta\mathbf{k} = \mathbf{P}_{\mathbf{F}}^{\perp} \Delta\mathbf{k} , \tag{19}$$

where

$$\mathbf{P}_{\mathbf{F}}^{\perp} = \mathbf{I} - \mathbf{F}\left(\mathbf{F}^{\text{H}}\mathbf{F}\right)^{-1} \mathbf{F}^{\text{H}} \Delta\mathbf{k} . \tag{20}$$

It was found that the horizontal wave number errors collinear with the vector $\mathbf{1}$ result in an error in the estimate of the source complex amplitude, whereas horizontal wave number errors collinear with the vector $\bar{\mathbf{k}}$ result in a bias in the source range estimate. Horizontal wave number errors in other directions (i.e. the space defined by $\Delta\mathbf{k}^{\perp}$) result in distortions of the ambiguity/likelihood function.

In order to exclude only unpredictable modes that result in distortions of the ambiguity/likelihood function, take the projection of the estimated covariance matrix of the horizontal wave number Γ into the null space of the matrix **F**, i.e., let

$$\Gamma_{\text{proj}} = \mathbf{P}_{\mathbf{F}}^{\perp} \Gamma \mathbf{P}_{\mathbf{F}}^{\perp} . \tag{21}$$

Let S_{M_0} be the space of all M_0 -sized subsets of the M modes, and let $\Gamma_{\text{proj}}(\mathbf{\Omega}_c)$ denote the covariance matrix Γ_{proj} calculated for a particular subset selection, $\mathbf{\Omega}_c$, of M_0 modes. Then the most predictable selection of M_0 modes, $\mathbf{\Omega}_p$, is determined as that subset which minimizes the trace of $\Gamma_{\text{proj}}(\mathbf{\Omega}_c)$

$$\Omega_p = \arg\min_{\Omega_c \in S_{M_0}} \operatorname{trace}\left(\Gamma_{\mathrm{proj}}(\Omega_c)\right). \tag{22}$$

In practice, a method for determining M_0 is required. Here we use Tabrikian's method [13], that is using values of M_0 which were found to give the best localization performance.

III. SIMULATION AND ANALYSIS

Simulation scenario extended from the general mismatched benchmark model [15] was used to check the localization performance of the proposed method. Results of the matched field processor (MFP), MMP and the robust ML estimator are also given here for performance comparison.

Fig. 1 reviews the model, where the vertical line array contains 50 hydrophones spaced 2 m apart ranging from 2m to 100m in depth. The nominal ocean depth is 102.5m, the nominal value of sound speed is shown by the solid line, with the dotted line representing the maximum and minimum values of the sound speed. The nominal values of the sediment density and sound absorption ratio are $1.7\mathrm{g}/\mathrm{cm}^3$ and $0.35\mathrm{dB}/\lambda$, respectively. The point source is narrow band at frequency 250Hz.

The predictable modes were identified according to the variance of the horizontal wave numbers. One hundred independent realizations of the environment were used in order to estimate the variance of k_m. The number of predictable modes used was $M_0 = 7$. The estimated horizontal wave number standard deviation of the modes is shown in Fig. 2. Fig. 3 depicts the wave number standard deviation of the selected mode numbers 3 through 9 after rejection of the components collinear with the vectors $\bar{\mathbf{k}}$ and $\mathbf{1}$.

The probability of correct localization (PCL) is used as the performance metric. PCL is defined as the fraction of realizations in which the source is localized to within an acceptable range and depth error about the true location. For the PCL results given in the simulation, the acceptable localization errors are taken to be 400m in range and 2m in depth. The PCL itself has a standard deviation estimate of

$$\sigma = \sqrt{\frac{\mathrm{PCL}(1-\mathrm{PCL})}{n}}, \tag{23}$$

where n is the total trials number. In this paper, n=2500 is used. As an upper bound, the largest standard deviation occurs for PCL=0.5; for this value and n=2500 the standard deviation is $\sigma = 0.01$, which is approximately the plotted line thickness for the PCL figure shown later.

In the simulation, the array spans the entire water column, so the MFP and MMP have identical localization performance [12]. Fig.4 gives the PCL of the proposed method, the robust ML and the MFP/MMP under different SNRs (array level SNR, including array gain). It can be seen that the conventional MMF/MMP estimators provide PCL that is less than 0.6 for any SNR. The localization performance of the robust ML estimator is better than MMF/MMP. The proposed method has the best localization performance. One of the reasons that the PCLs of the robust ML and the proposed method do not increase beyond a threshold SNR (about 30dB in this simulation) is the effect of errors in the eigenfunctions at the receivers. These two methods both make the assumption that the eigenfunctions and the corresponding horizontal wave numbers are steady in the predictable mode subspace. This is not the truth in the uncertain ocean environment. The little errors in the predictable mode subspace become dominant at high SNRs. So even the SNR is very high, the proposed method cannot localize the target correctly with probability of 1.

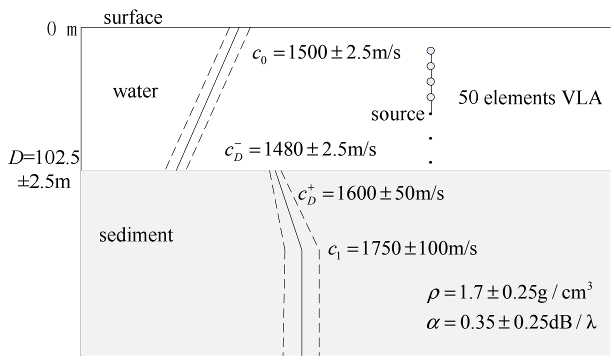

Fig.1. Benchmark ocean environment

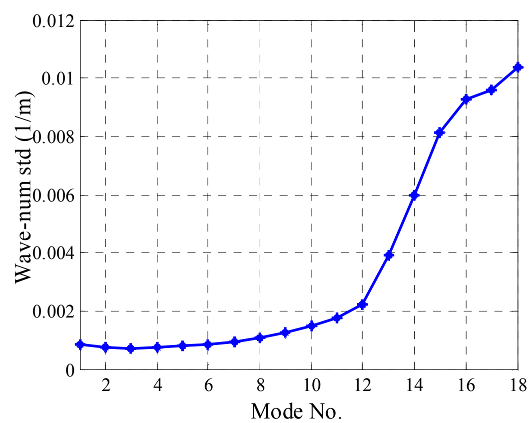

Fig.2. The horizontal wave number standard deviations of all modes without projection

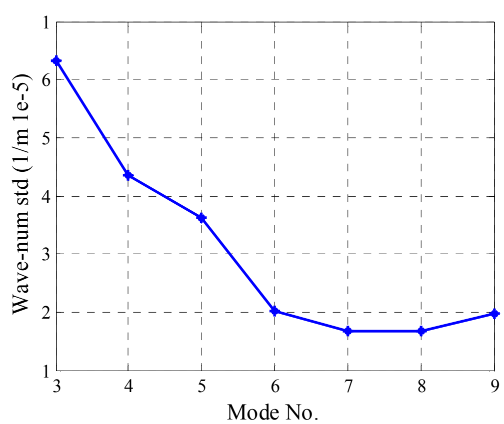

Fig.3. The horizontal wave number standard deviations of selected modes with projection

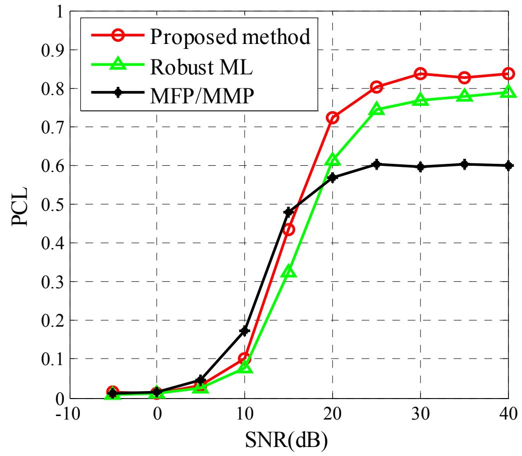

Fig.4. PCL of three methods (Array level SNR)

Fig.5 gives the localization surfaces of MFP and proposed method when SNR=25dB. White circles mark the true source location and black circles mark the maximum point of the corresponding localization surface. The proposed method has a range estimation error of 150m while the error of MFP is 1450m. Note that at this SNR, the MFP/MMP achieves PCL's of 0.6, whereas the proposed estimator provides a PCL of 0.8.

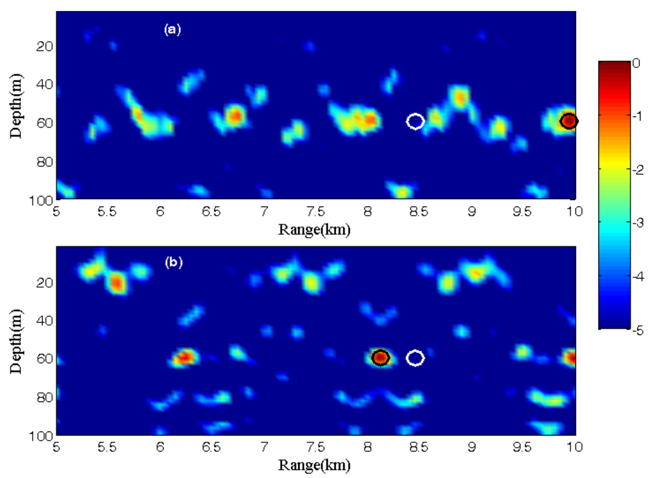

Fig. 5. The localization surfaces (dB) of MFP (a) and proposed method (b).

IV. CONCLUSIONS

This paper proposed a robust localization method for uncertain shallow ocean environment. The method uses the fact that there exists a predictable mode subspace in such an environment, and this subspace is used to reconstruct the replica vector to make the localization more robust. Results from simulations show the effectiveness of the proposed method.

In this work, the orthogonality between the predictable and unpredictable subspaces is used to eliminate the impact of environmental uncertainty. At high SNRs, the little error becomes dominant in the predictable mode subspace, and this prevents the PCL further increase above the threshold SNR.

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