Improving coherent tonal detection with phase interpolation and compensation

Chen Shaohua
Yichang Research Institute of Testing Technology
Yichang, China 443003
npuchen@163.com

***Abstract*—Tonal detection is very important in weak signal detection field, and phase coherent averaging tonal detector is superior to incoherent averaging power spectrum detector. For the purpose of resolving the problem of inaccurate phase compensation introduced by the existing coherent averaging tonal detector, the gain loss with inaccurate phase compensation is analyzed, and a modified detector using frequency domain interpolation to estimate the phase difference accurately is proposed. The three point interpolation in frequency domain is applied to obtain the accurate phase difference estimate when the averaging segment length is not exactly the integer times of the signal period, thus to avoid the gain loss introduced by averaging the segmented data not in-phase. Theoretical analysis shows that there will be a gain of 3.9dB at most with the modified detector. The detection performance of incoherent average power (AVGPR) detector, coherent generalized likelihood ratio test (CGLRT) detector, and the modified CGLRT detector is compared by computer simulations, and the results show that the modified detector operates better than the other two detectors especially with low SNR. Both theoretical analysis and simulations verify the effectiveness of the modified detector.**

Keywords—tonal detection; coherent averaging; phase compensation; phase interpolation;

I. INTRODUCTION

The detection of tonal buried in stochastic noise is widely used in communication, sonar, and ocean engineering. For tonal with unknown amplitude, frequency and phase, there are two kinds of tonal detection methods: the classical power spectrum estimation method and the modern spectrum estimation method [1,2]. The advantage of modern spectrum estimation lies in that it has higher resolution than the classical power spectrum estimation with limited number of observations, and it also improves the variance performance prominently. However, it has also some disadvantages such as requiring higher input SNR, and much more computations. In most engineering applications, the tonal to be detected is buried in heavy noise, and the performance of modern spectrum estimation deteriorates seriously. On the other hand, the power spectrum method based on FFT(fast Fourier transform) may remain fairly good detection performance with low SNR, so it's still paid much attention [3-6].

To detect weak tonals buried in heavy noise, the observation length should be increased to improve the processing gain of the FFT. However, the length for the FFT is limited by the following factors: firstly, the stationarity of the signal or noise background is limited; secondly, the real-time computation ability and storage capacity of signal processor is limited; thirdly, the variance of the spectrum estimation based on FFT would increase with the increase of observation length, which in turn increases the false alarm of tonal detection. Therefore, the segmented power spectrum estimation is usually used to avoid the above shortcomings, such as the Welch method [1]. When used in tonal detection, it's equalized to AVGPR detector [3].

The AVGPR detector is an incoherent detector since it only uses the amplitude information and gives up the phase information through squaring the amplitude of the DFT (discrete Fourier transform) of the observation. If the observed data are segmented into L segments of equal length, the averaging gain over stochastic noise is 5lgL dB. On the other hand, the coherent averaging method takes advantage of the phase information of every segments, and could reach the maximum processing gain of $10\lg L$ dB [7]. Therefore, coherent averaging detector would greatly improve the detection ability of weak tonals in noise. Since the coherent averaging requires that the length of segmented data is exactly the integer times of signal period, it's generally impossible when the tonal frequency is unknown. To deal with this problem, a CGLRT detector with phase compensation is proposed to make every segment of data to accumulate coherently [5]. However, there might be a shortcoming with the CGLRT detector since the phase resolution is discretized as the integer times of π/L, which would reduce the coherent effect and the detection performance in most cases.

We analyze the gain loss introduced by the phase difference between segments, and obtain a continual phase estimate with interpolation to prevent the effect of phase discretization by using FFT estimate. As a result, the phase difference between segments is compensated accurately, and a gain loss of at most 3.9dB is avoided. We also analyze the effect of frequency extension on the detection performance after long range underwater transmission. Computer simulations are carried out to compare the detection performance.

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II. SIGNAL MODEL

The observed tonal signal is described as

$$x(n)=s(n)+g(n)=A\exp[\mathrm{j}(\omega_0 n+\phi)]+g(n)\,,\quad n=0,1,\ldots,N-1 \qquad (1)$$

where $\omega_0=2\pi f_0/f_s$ is the normalized frequency, f_0 and f_s are the signal frequency and sampling frequency respectively, ϕ is the stochastic phase uniformly distributed in $[0,2\pi)$, $g(n)$ is stationary complex Gaussian noise with zero mean and variance ${\sigma_g}^2$, that's $g(n)\sim\mathcal{CN}(0,{\sigma_g}^2)$.

Tonal detection in Gaussian noise is a classical binary hypothesis test problem, and it's usually performed with the DFT.

The DFT of $x(n)$ is as follows

$$\begin{aligned}X(k)&=\sum_{n=0}^{N-1}x(n)\exp\left(-\mathrm{j}\frac{2\pi kn}{N}\right)\\&=A\sum_{n=0}^{N-1}\exp\left[\mathrm{j}\left(\omega_0 n-\frac{2\pi kn}{N}+\phi\right)\right]+\sum_{n=0}^{N-1}g(n)\exp\left(-\mathrm{j}\frac{2\pi kn}{N}\right)\\&\qquad k=0,1,\ldots,N-1\end{aligned} \qquad (2)$$

We can see that, the DFT is a coherent averaging detector for tonal component. On the other hand, it incoherently gather the noise energy when the noise observations are independent each other. The processing gain of the DFT tonal detector is 10lg$N$ dB when the independent observation length is N, which is the maximum gain might obtained by a linear processing system, therefore the DFT is an optimum tonal detector.

III. COHERENT AVERAGING TONAL DETECTOR

Though the DFT (so is also the FFT, fast Fourier transform) is an optimum tonal detector, for the reasons mentioned in section I, the observed data are usually segmented before the DFT. If the square of the amplitude of the DFT of every segment is averaged, the incoherent AVGPR detector is obtained. If the phase information of every segment is remained and synchronized, the result coherent detector like CGLRT does better than the incoherent AVGPR detector in tonal detection.

A. phase estimation for CGLRT tonal detector[5]

The observed data are divided into L segments of length N, and the lth segment is described as

$$x_l(n)=x(l\cdot R+n)\,,\quad n=0,\cdots,N-1\,;\ l=0,\cdots,L-1 \qquad (3)$$

Each segment of data can be overlapped or non-overlapped. If overlapped, the offset data length R should be greater than the correlation radius of noise $g(n)$. If non-overlapped, R=N. Therefore, Gaussian noise between segments are still independent each other.

For tonal frequency f_0, the corresponding digital frequency is $k_0^*=Nf_0/f_s$, and it is not exactly an integer in most cases. It can be described as the sum of an integer k_0 that is the closest to k_0^* and a decimal fraction α

$$k_0^*=k_0+\alpha \qquad (4)$$

where $|\alpha|\le 0.5$.

So the signal is written as

$$s_l(n)=A\exp(\mathrm{j}\phi)\exp\left[\mathrm{j}2\pi\frac{k_0^*}{N}(n+lR)\right] \qquad (5)$$

The DFT of $s_l(n)$ is

$$\begin{aligned}S_l(k)&=A\exp(j\phi)\exp\left(j2\pi\frac{k_0^*}{N}Rl\right)\\&\quad\times\sum_{n=0}^{N-1}\exp\left(j2\pi\frac{k_0^*-k}{N}n\right)\end{aligned} \qquad (6)$$

When $S_l(k), l=0,\cdots,L-1$ are averaged coherently, the component $\exp\left(j2\pi\frac{k_0^*}{N}Rl\right)$ are phase synchronous only if k_0^*R/N is an integer. Considering $k_0^*R/N=RT_s/T_0$ (where T_s, T_0 are sampling interval and signal period respectively), the DFT of every segment can be averaged coherently only if the offset signal length RT_s is exactly the integer times of signal period T_0.

If R=N, since $NT_s/T_0=Nf_0/f_s=k_0$, every segments can be averaged coherently at N discrete frequency $f_0=k_0f_s/N$ （k_0=0~N-1）. The segmented signals can not be averaged coherently at other frequencies, and phase compensation must be carried out.

Substituting equ. (4) into (6), we have

$$S_l(k_0)=A\exp(j\phi)\exp\left(j2\pi\frac{k_0+\alpha}{N}Rl\right)\sum_{n=0}^{N-1}\exp\left(j2\pi\frac{\alpha n}{N}\right) \qquad (7)$$

Since k_0, N, R and l are all known, the component $\exp\left(-j2\pi\frac{k_0}{N}Rl\right)$ can be multiplied at the two sides of $X_l(k_0)$ simultaneously to eliminate phase difference introduced by that component, where $X_l(k_0)$ is the FFT of $x_l(n)$. The result is

$$Z_l(k_0)=X_l(k_0)\exp\left(-j2\pi\frac{k_0}{N}Rl\right)$$

$$= A\exp(j\phi)\exp\left(j2\pi\frac{\alpha}{N}Rl\right)\sum_{n=0}^{N-1}\exp\left(j2\pi\frac{\alpha n}{N}\right)+G_l^{'}(k_0) \quad (8)$$

We can see that, for a certain frequency bin k_0, the array $Z_l(k_0), l=0,\cdots,L-1$ is a complex sinusoidal function of the unknown parameter α. Performing the FFT with array $Z_l(k_0), l=0,\cdots,L-1$, we can obtain the amplitude to form an array of $U(k)$ with the length of L. α is estimated with k_{max} corresponding to the peak of $U(k)$.

$$\hat{\alpha}=\frac{Nk_{\max}}{RL} \quad (9)$$

Then we have

$$y_l(k_0)=Z_l(k_0)\exp\left(-j2\pi\frac{\hat{\alpha}}{N}Rl\right)$$

$$= A\exp(j\phi)\sum_{n=0}^{N-1}\exp\left(j2\pi\frac{\hat{\alpha}n}{N}\right)+G_l^{''}(k_0) \quad (10)$$

So $y_l(k_0)$ are phase synchronous for every frequency bin of k_0 with k_0=0~N-1. The phase difference is eliminated between data segments, and the CGLRT detector can be obtained (see Appendix for the formulations).

However, the phase estimate is discretized from equ.(9). We'll analyze the averaging gain loss introduced by the phase discretization.

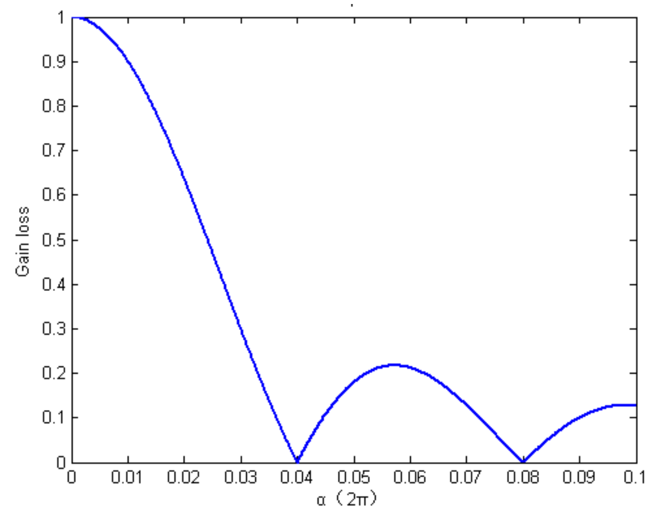
B. Gain loss caused by phase difference

When the phase difference α between segments in equ. (9) is not compensated, there will be a gain loss introduced by averaging L segments of $y_l(k_0)$ in (10).

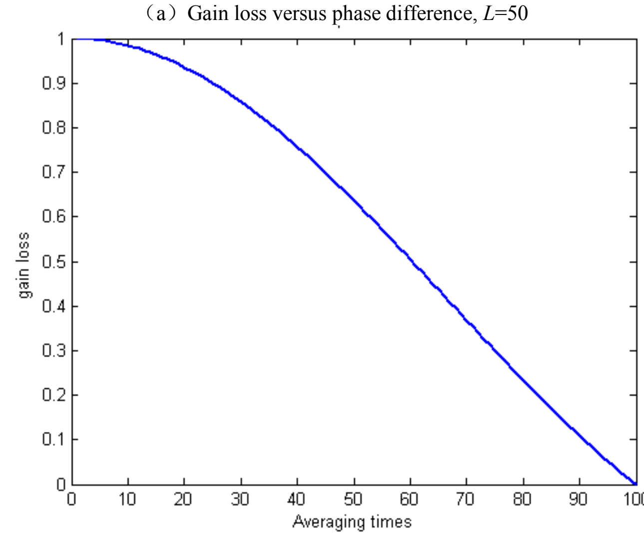
$$G_p=\frac{1}{L}\left|\sum_{l=0}^{L-1}\exp\left(j2\pi\frac{\alpha}{N}Rl\right)\right|=\frac{1}{L}\left|\frac{\sin(\pi R\alpha L/N)}{\sin(\pi R\alpha/N)}\right| \quad (11)$$

With equ.(11), the relationships between the gain loss and the phase difference, as well as the averaging times is demonstrated in Fig.1, where R=N/2, that's 50% overlap. We can see that, the more the phase difference and the more the averaging times, the more the gain loss. Only if the phase difference and averaging times are limited to a certain range, a small gain loss may be reached, while it's very difficult when the tonal frequency is unknown.

For equ.(9), assuming that N=512, R=N/2, L=50, the maximum estimation error of α is $0.5N/(RL)$=0.02. It can be seen from Fig.1, the gain loss is 20lg（-0.6367）= -3.9dB. In the case of R=N/2, that's 50% overlap usually used, the -3.9dB gain loss will not varying with L. The reason is that, when L increases, though the estimation error of α becomes small in (9), the gain loss in Fig. 1(b) increases, and the two tendencies would cancel each other. The analogous phenomenon would happen when L decreases.



（a）Gain loss versus phase difference, L=50



（b）Gain loss versus averaging times, α=0.02

Fig.1 Gain loss versus phase difference and averaging times

The above analysis means that a gain loss of 3.9dB would happen by the discretized estimation of α, which may be very important for weak tonal detection, and this gain loss may be avoided by modifying the phase estimation.

C. Modify the coherent detector with phase interpolation

The three point interpolation algorithm in time delay estimation [8] is used to estimate the digital frequency k_{max} accurately in equ. (9), and its time delay estimation precision is about $10^{-2}T_s$, where T_s is the sampling interval. If $r_2=U(k_{\max})$, $r_1=U(k_{\max}-1)$, $r_3=U(k_{\max}+1)$, the decimation fraction of k_{max} is

$$\Delta k=\frac{r_3-r_1}{2(2r_2-(r_3+r_1))} \quad (12)$$

Replace k_{max} in equ. (9) with $k_{max}+\Delta k$, the precision of the phase estimate α will be greatly improved, and reach to $0.01N/(RL)$=4×10^{-4}. We can see in Fig.1(a), the gain loss is very small in this case, that's, smaller than 0.02dB, which may be ignored. There are still more accurate time delay estimation algorithms, such as the modified three point interpolation algorithm[9], and it may reach a high precision time delay estimation of $10^{-3}T_s$. But it's enough to use the simple three point interpolation algorithm in our application. Besides, zero padding to the end of a time array before the FFT is also feasible[10], which means interpolation in frequency domain, but it need more computations.

IV. COMPUTER SIMULATIONS

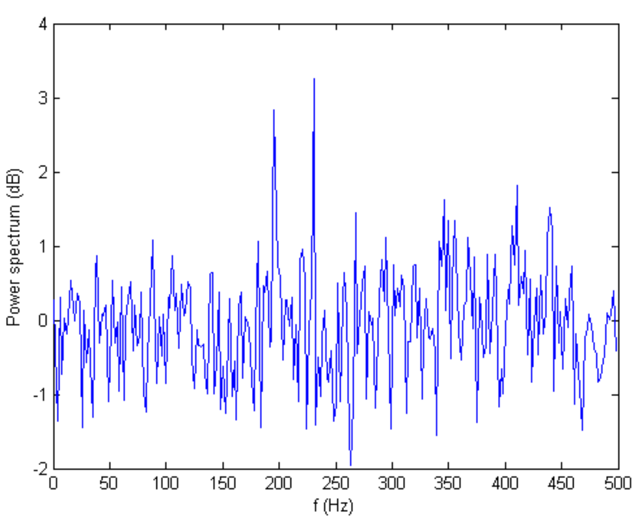
The detection performance of the AVGPR detector, the CGLRT detector and the modified CGLRT detector is compared with each other by computer simulations. The total length of observation data is 12000, the sample frequency f_s=1000Hz, so the time length is totally 12 seconds. The length of each segment N=512, 50% overlap, so there are totally L=45 segments. There are two tonal components with their frequencies f_1=195Hz and f_2=230Hz respectively. The frequencies of the two tonals can be set arbitrarily. The number of tonals are also not limited.

The tonal detection results of the three detectors with SNR=-25dB are shown in Fig.2. The input SNR is defined as

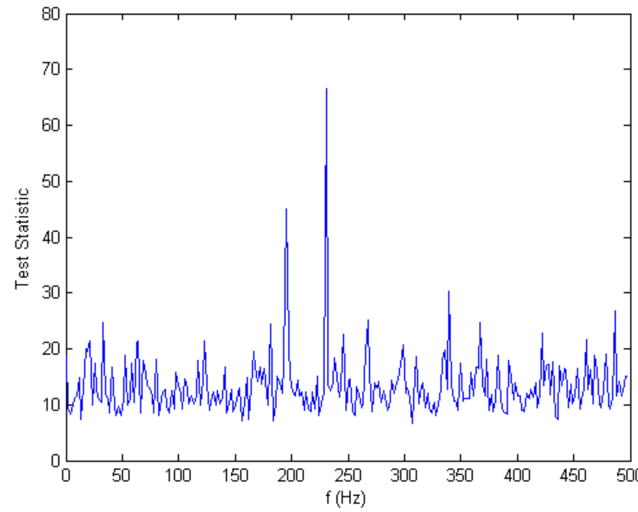
$$\text{SNR}=10\lg\left(A^2/\sigma_g^2\right)$$

The noise added to the tonal signal is stationary complex Gaussian noise with zero mean and variance depending on the SNR. We can see that the incoherent AVGPR detector works not well with the low SNR of -25dB, and the height of the tonal jumping off the continuous spectrum background is not obvious. On the other hand, the tonal detected with the CGLRT detector is obvious, and the modified detector works the best.

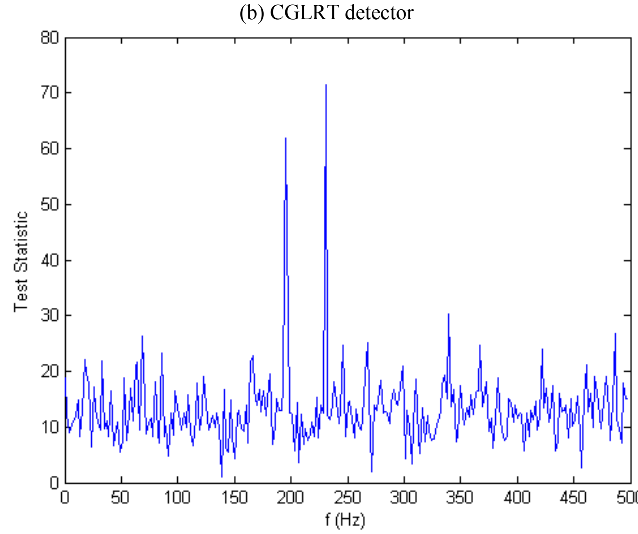
The tonal detection results of the three detectors with SNR=-20dB are shown in Fig.3. We can see that the three detectors all show obvious tonals, while the performance of the modified detector is the best.



(a) AVGPR detector

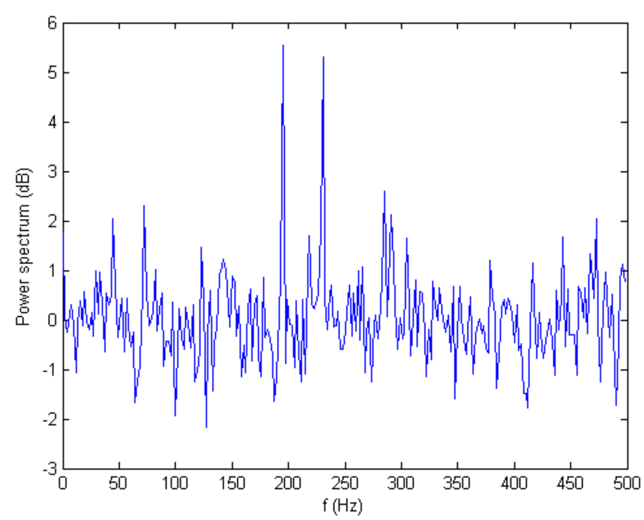


(b) CGLRT detector

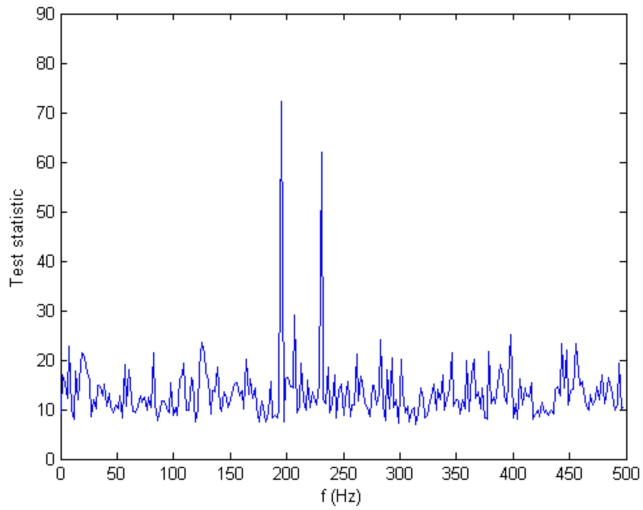


(c) Modified CGLRT detector

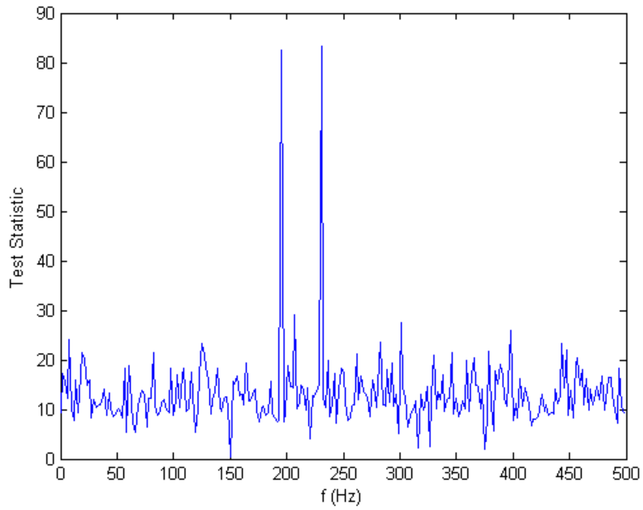
Fig.2 Performance comparison of tonal detection with the three detectors, N=512, L=45, 50% overlap, SNR=-25dB



(a) AVGPR detector



(b) CGLRT detector



(c) Modified CGLRT detector

Fig.3 Performance comparison of tonal detection with the three detectors, N=512, L=45, 50% overlap, SNR=−20dB

V. CONCLUSIONS

Coherent detector with phase synchronization improves greatly the processing gain and the tonal detection performance, especially with low SNR. The phase compensation is not very accurate with the CGLRT detector since the phase is estimated and discretized with FFT, which results in a gain loss of 3.9dB at most.

We improve the precision of the phase estimate several tens times with frequency interpolation to avoid the gain loss. Computer simulations are carried out to compare the detection performance of the incoherent AVGPR detector, the CGLRT detector, and the modified CGLRT detector. Both theoretical analysis and computer simulations show that the modified detector is superior to the CGLRT detector, and the two coherent detectors do better than the incoherent AVGPR detector.

APPENDIX

The AVGPR detector is defined as [3]

$$\mathrm{AVGPR}(\boldsymbol{X}) = \sum_{l=0}^{L-1} \left| X_l(k) \right|^2$$

where $X_l(k)$ is the FFT of $x_l(n)$ in equ. (4).

The CGLRT detector is defined as [5]

$$\ln \Lambda(\boldsymbol{X}) = -L \ln \frac{\hat{v}_1}{\hat{v}_0}$$

where

$$\hat{v}_0 = \frac{1}{L} \sum_{l=0}^{L-1} \left| X_l(k) \right|^2$$

$$\hat{v}_1 = \frac{1}{L} \sum_{l=0}^{L-1} \left| X_l(k) - \hat{\mu} \right|^2$$

$$\hat{\mu} = \frac{1}{L} \sum_{l=0}^{L-1} X_l(k)$$

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