Experimental Study on Robust Supergain Beamforming for Conformal Vector Arrays

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***Abstract*—An experimental conformal array with vector sensors is designed and its supergain performance is studied in this paper. The robust supergain beamforming method is formulated as an optimization problem and it can be easily solved using the second-order cone programming. The manifold vector and the noise correlation matrix are both calculated using the real measured data which can alleviate the effects of the mismatches and make good use of the actual noise field information. The results show the robust supergain beamforming method possesses great potential for array gain enhancement and the sidelobe levels of the beampattern can be controlled properly which are superior to the conventional counterpart. Additionally, this supergain method can be implemented to arbitrary array (e.g. conformal vector array) with different channel response directivity, which shows good potential in practical engineering application.**

Keywords—conformal vector arrays; supergain; robustness; optimization; second-order cone programming

I. INTRODUCTION

The vector sensor technology has grown rapidly in recent decades and it has been applied in many fields such as communication, geo-acoustic inversion, underwater imaging and so on [1-4]. Distinguished with the scalar pressure sensors or hydrophones, vector sensors can measure the components of particle velocity and the acoustic pressure at the same point simultaneously [5], meaning they can make use of more available acoustic information. Consequently, the performance of a vector sensor array like directivity, localization accuracy etc. will be superior to the same aperture array consisting of scalar pressure sensors. Cray and Nuttall [6] gave a detailed comparison of vector-sensing and scalar-sensing linear arrays and revealed some unique features for linear vector arrays. A good introduction of the development of vector sensor theory can be found in [7]. As the signal processing techniques of the vector array can be included into the classical theoretical framework of scalar pressure sensor array, the existing beamforming methods and direction-of-arrival (DOA) estimation algorithms can be easily extended to vector arrays [8-12].

The supergain beamforming method (SGBF) can enhance the spatial gain without increasing the array size [13-16] which is attractive in sonar, radar, audio engineering and wireless telecommunication, etc. The knowledge of the correlation structure of the noise always plays an important role in supergain beamforming method and some models of the correlation structure under various environment assumptions have been developed for vector arrays, which are useful to analyze the system performance before deployment [9]. D'Spain et al. [12] also analyzed the optimal array gain and detection performance of vector sensor line arrays in a homogeneous, isotropic noise field. However, the assumptions of the environment noise are not always in accordance with practical situation and the theoretical calculation will be sometimes not accurate, even degrade the performance of supergain method. For a specific array, the experimental measurement is still an essential and direct way to evaluate its capability. In [17], the results of the minimum variance distortionless response (MVDR or Capon) method applying to process the actual data collected by the DIFAR array are presented. But these works are all limited to linear arrays and the researches on the other type of vector arrays seem to be not sufficient, except for Wettergren and Traweek's experimental studies on velocity sonar arrays conforming to hulls [18, 19].

As the vector sensor has the phenomenon of superdirectivity, the performance of robust supergain beamforming method applying to a conformal vector array will be more advantageous than scalar-sensing array with the same size. The purpose of this paper is to analyze it experimentally, with focus on the comparison with the delay-and-sum counterpart. The experimental conformal array is composed of 22 vector sensors and each vector sensor contains pressure and two orthogonal velocity channels. The manifold vector and the noise correlation matrix are both calculated from the real data acquired in a lake. The results were presented and analyzed to show the performance of robust supergain beamforming method.

II. BASIC FORMULATION

The vector sensors simultaneously measure the components of pressure and two or three orthogonal particle velocities at the same point, which can be written as

$$\boldsymbol{p}_m = \exp(ik\boldsymbol{r}_m^{\mathrm{T}}\boldsymbol{u})\begin{bmatrix} 1 \\ Q^{\mathrm{T}}\boldsymbol{u} \end{bmatrix} \tag{1}$$

where $\boldsymbol{r}_m$ is the position of the mth-sensor, $\boldsymbol{u}$ is the unit-length direction vector representation of an incident plane wave and Q is an orthogonal matrix which describes the orientation of the velocity channels. The wavenumber is $k = 2\pi/\lambda$, λ denoting the wavelength. The manifold vector will be

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$$\boldsymbol{P}(\theta,\phi)=\begin{bmatrix}\boldsymbol{p}_1\\ \boldsymbol{p}_2\\ \vdots\\ p_M\end{bmatrix} \tag{2}$$

In the following experiment, only two horizontal velocity channels are considered. All of the pressure channels are arranged together and so are the velocity 1 and velocity 2 channels. This is a little different from (2), but the final results are not influenced.

The robust supergain beamforming method can be formulated as an optimization problem which is [15, 20]

$$\begin{aligned}&\min_{w} \quad \boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_n\boldsymbol{w},\\ &\text{s.t.} \quad \boldsymbol{P}^{\mathrm{H}}(\theta_0,\phi_0)\boldsymbol{w}=1,\\ &\qquad \|\boldsymbol{w}\|\le\sigma,\\ &\qquad \left|\boldsymbol{P}^{\mathrm{H}}(\theta_0,\phi_j)\boldsymbol{w}\right|\le\delta_j,\\ &\qquad (\phi_j\in\phi_{SL},\ j=1,\cdots,N_{SL})\end{aligned} \tag{3}$$

where $\boldsymbol{w}$ is the weighting vector, $\|\bullet\|$ denotes the Euclidean norm. (θ_0,ϕ_0) is the steering direction. σ is the specified upper bound of the weighting vector norm, and δ_j is the prescribed sidelobe peak value. ϕ_j is the discrete N_{SL} directions in the sidelobe areas. Only the horizontal beampattern $(\theta_0=90^\circ)$ is considered here.

In practice, the theoretical manifold vector will not be accurate due to the channel gain and phase mismatch, element position perturbations and structural scattering etc. The real manifold vector which is calculated using measured data will be appropriate to alleviate the effects of these mismatches and enhance the final performance. The details can be found in [21]. The noise correlation matrix is also estimated by the received noise data which is

$$\boldsymbol{R}_n=\frac{1}{L}\sum_{l=1}^{L}\left[\boldsymbol{n}(l)\boldsymbol{n}^{\mathrm{H}}(l)\right] \tag{4}$$

where L is the number of training snapshots. It is useful to take full advantage of the actual noise field information.

The weighting vector can be computed easily using numerical methods, e.g. second-order cone programming (SOCP) [22] and it is robust due to the white noise gain constraint (weighting vector norm constraint). The sidelobe levels will be also controlled properly. This beamforming method can give flexible supergain results and it can be applied to arbitrary array. Additionally, it also does not concern with the directivity response of the sensor element.

III. Experimental Results

The experimental conformal array shown in Fig. 1 consists of 22 vector sensors and each vector sensor contains three channels: pressure and two orthogonal velocities in horizontal plane. Unfortunately, three sensors (which are red in Fig.1) are not working in the experiment and only the data received by the remaining 19 vector sensors (57 channels) are used.

The experiment was carried out in a lake located at Zhejiang Province of China. As shown in Fig. 2, the conformal vector array and the projector were both put at 9.4m below the lake surface. To obtain the real manifold vector, the signal transmitted by the projector was rectangular single frequency continuous wave (CW). The frequency is 3kHz and the impulse repetition period was 10ms.

In the experiment, the array was rotated from 0° to 360° with step 3° in the horizontal plane to receive the signals, which is equivalent to the situation that the signals were incident from these different directions. After the pre-processing, the received signals were sampled by A/D converters and the sample frequency was 16kHz. The data from A/D converters were saved to PC and processed off-line. Using the methods presented in [21], the real manifold vector is obtained and it will be used to compute the weighting vector.

The amplitude responses of the 8th-sensor which is arbitrarily selected for example at frequency 3kHz is shown in Fig. 3. Theoretically, the pressure channel is omnidirectional and the two orthogonal velocity channels are both the same with the dipole contour. The practical directivity responses are roughly in accordance with the ideal ones and the distortions are mainly caused by mismatches and errors which are mentioned previously. The amplitude responses of the other sensors also have the same properties which are not shown here for compactness.

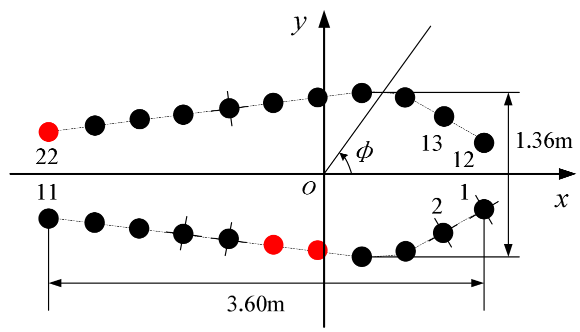


Fig. 1. Experimental conformal vector array

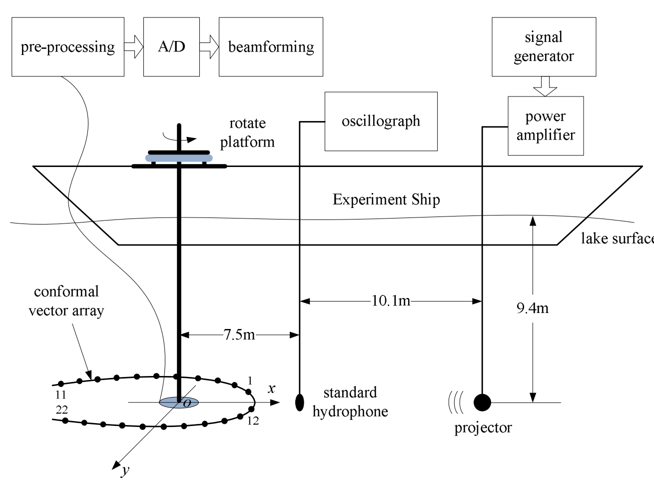
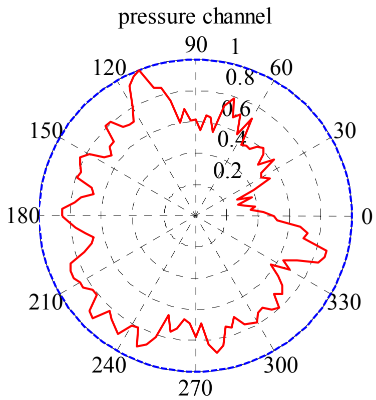
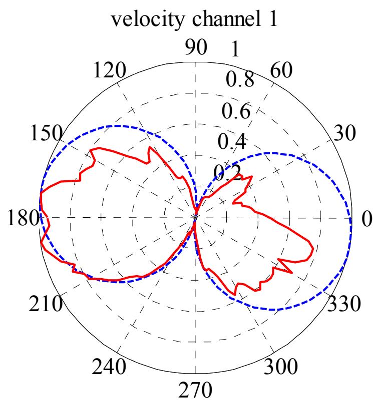


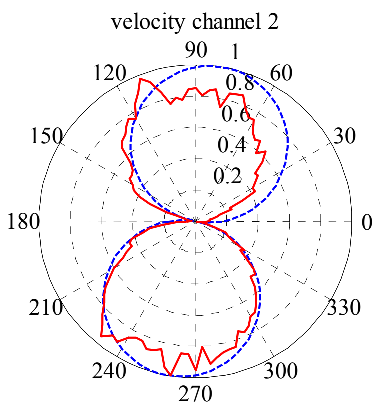
Fig. 2. Configuration of Experiment



(a)



(b)



(c)

Fig. 3. Theoretical (dotted line) and practical (solid line) amplitude response of the 8th-sensor

A sample of the magnitude of the channel-to-channel noise cross correlation at frequency 3kHz is shown in Fig. 4. The values are all normalized by the maximum one. Note that the numbers 1-19 represent the pressure channels, 20-38 the velocity 1 channels and 39-57 velocity 2 channels. It is found the noise correlations among some channels are obvious even though the distances of the channels are larger than one-half wavelength. The noise field is assumed to be ergodic in a relatively long time, but the structure of the cross correlation will be different from Fig. 4 when the noise data are not collected in the desired time. Fortunately, some channels still show apparent noise correlation in the other time which is essential to the supergain beamforming method.

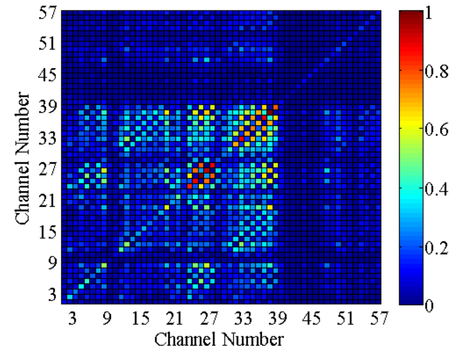
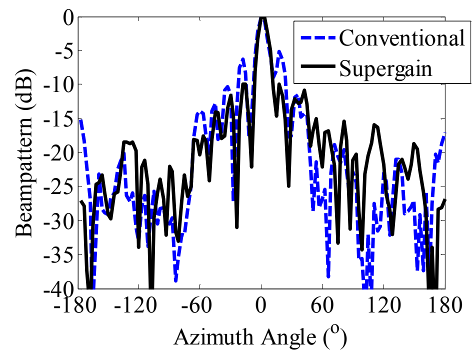
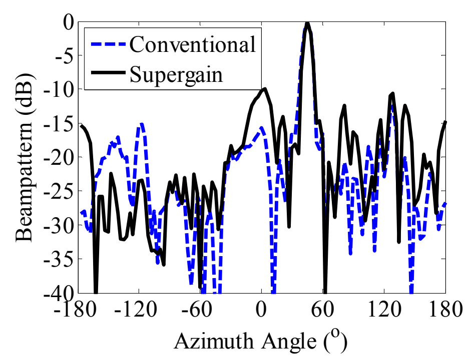


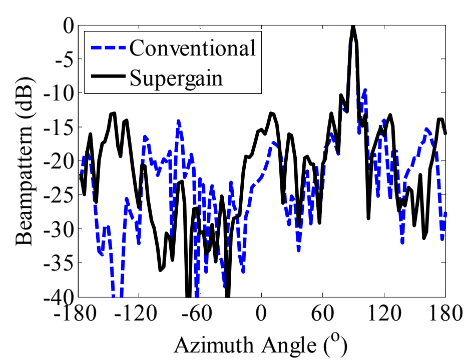
Fig. 4. Magnitude of the sensor noise response cross correlation at frequency 3kHz



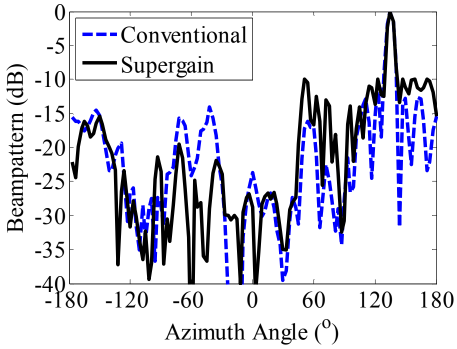
(a) $\phi_0 = 0^\circ$



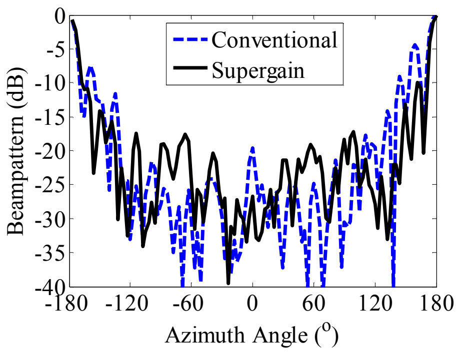
(b) $\phi_0 = 45^\circ$



(c) $\phi_0 = 90^\circ$



(d) $\phi_0 = 135^{\circ}$



(e) $\phi_0 = 180^{\circ}$

Fig. 5. Practical beampatterns

The practical beampatterns at frequency 3kHz are shown in Fig. 5 and the steering azimuth direction is ranging from 0° to 180° where the step is 45°. The noise correlation matrix is recalculated as the steering direction is changed. Obviously, the sidelobe levels (SLs) of the supergain beampatterns are controlled under the desired level -10dB ($\delta_j = 10^{-10/20} = 0.3162$), where the white noise gain constraint is set to be 0dB ($\sigma = 10^{0/20} = 1$). By comparison, the SLs of the conventional beampatterns shown in Fig. 5(a) and 5(e) are unacceptably high, but in Fig. 5(b~d) the SLs are also under -10dB level. The main beamwidths of the conventional beampatterns are almost the same as the supergain beampatterns.

TABLE I. ARRAY GAIN (dB)

Steering direction (°)	0	45	90	135	180
Signal 1					
CBF	18.7	17.7	18.2	16.4	17.9
SGBF	22.3	22.1	25.5	21.9	19.7
Improvement	3.6	4.4	7.3	5.5	1.8
Signal 2					
CBF	19.1	17.8	18.0	16.9	18.0
SGBF	23.0	23.3	26.2	21.7	21.1
Improvement	3.9	5.5	8.2	4.8	3.1
Signal 3					
CBF	17.8	19.7	18.4	18.2	17.2
SGBF	23.7	24.7	27.7	23.5	20.9
Improvement	5.9	5.0	9.3	5.3	3.7

The array gain (AG) is defined as the difference between signal-to-noise ratio (SNR) of the output signal and SNR of the input signal of an array, which is an important performance measure of the array. To evaluate the potential for AG enhancement of this conformal vector array using the supergain method, the LFM signals with three input SNRs (which are spaced by 10dB in turn) are incident from 0° to 180° with step 45° which are the same with the steering directions in this experiment. Since the center frequency is 2.9kHz and bandwidth is 200Hz, it can be regarded as a narrow band signal. The beams can be formed using the weighting vectors calculated above at frequency 3kHz. Although this frequency is a little different from the signal center frequency, the effects are so small that can be neglected. After the beamforming procedure, the output SNRs of the supergain and conventional methods can be calculated and the final AGs are all listed in Table I. Since the SNRs of the channels are different from each other, the reference input SNR to compute the AG is the average value of these SNRs. As shown in Table I, the AGs of the conventional method are all about 18dB and the ones of supergain method are almost larger than 20dB. Obviously, the supergain method is superior to the conventional counterpart in improving the AG and it is enhanced by 5.2dB averagely. The AG improvement is changed with the steering direction and it is the largest when the steering direction is 90° showing the best detection performance.

IV. CONCLUSION

An experimental study on robust supergain beamforming method for conformal vector array is presented. The manifold vector is calculated from the real measured data which will alleviate the effects of the mismatches and enhance the final performance. It is not only very useful to the supergain method but also to the conventional one. The noise cross correlation matrix is also estimated using the practical data and it is beneficial to make use of the actual noise field information sufficiently. The results show the robust supergain beamforming method applying to the conformal vector array can provide higher AG than conventional method and the SL can be controlled properly to avoid high false-alarm probability. Additionally, this supergain method can be applied to arbitrary array no matter what the channel response directivity is omnidirectional or not, which shows good potential in practical engineering application.

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