Localization of a sound source: optimal positioning of sensors carried on autonomous surface vehicles

Bruno M. Ferreira, Aníbal C. Matos, Helder S. Campos, Nuno A. Cruz
INESC TEC, Faculty of Engineering, University of Porto
Rua Dr. Roberto Frias, 378
4200-465 Porto, Portugal
bm.ferreira@fe.up.pt

***Abstract*—This paper addresses the problem of optimal, three-dimensional, localization of an acoustic pinger. Making use of acoustic receivers, the time-of-arrivals are computed and fused with their positions to estimate the target position. Motivated by practical applications where autonomous surface vehicles are employed to carry sensing equipment, these receivers are constrained to lie in a plane. The optimal configuration of sensors is derived by maximizing the determinant of the Fisher information matrix. A method to track and estimate the position of the target is proposed and implemented in a formation of four vehicles. Experimental results show very motivating results with successful estimates of the target position.**

I. Introduction

Underwater sensors and vehicles usually require geolocation methods to determine their positions. In this work, special interest is given to the localization of mobile targets equipped with a simple and inexpensive acoustic pinger. A set of carefully placed receivers report the time-of-arrivals (TOAs) to a central position estimator that subsequently determines the position of the target. Complementing our previous work, the optimal placement of the acoustic receivers is derived for the plane-constrained case. This makes it possible to tackle tracking problems where the receivers are placed at the surface. Furthermore, autonomous surface vehicles (ASVs) (figure 1) can be used to carry the receivers over the region of interest and to *optimally* track the sound source (pinger) by placing themselves according to the pinger position estimate.

Similarly to this specific scenario, several works have stressed the importance of the relative positions of the emitters and of the sensors for range- and angle-based measurements [1]–[3]. In this paper, the optimal three-dimensional positioning of TOA sensors to track an underwater sound source is addressed. Although motivated by underwater applications, the work presented here has applications in general networks of distributed sensors that make use of TOA measurements to estimate the position of a source.

A. Placement of receivers to improve estimation

Transversal to many problems of localization is the impact of the relative positions of devices with regard to the beacons that provide useful informations to infer the position of the device. For example, the so-called dilution of precision in general positioning system (GPS) devices is the result of the configuration (relative positions) of the satellites. The confidence on the resulting estimate is generally sensitive to relative positions and certain configurations can lead to poor estimates. In opposition, dynamic positioning of sensors can bring several advantages over most of the underwater localization methods known, including static beacons used in LBL and USBL systems: firstly, the operation area becomes unbounded as the sensors can dynamically position themselves; secondly, optimal observability configurations can be obtained to ensure the best possible estimate.

This subject has recently attracted the attention of several researchers that have provided important results in optimal placement of sensors for different methods including range-only [2]–[5], received signal strength [4], bearing-only [3], [4] and TOA-only [3] localization.

In this work, similarly to [6], the Fisher information matrix (FIM) and its determinant are used to quantify the quality of the estimate, that is, the expected variance on the position estimate. The maximization of this latter is desired for theoretical optimal position of TOA sensors used in the estimation process. Two common approaches are considered in the literature: minimization of the average of the variances (A-optimality) or minimization of the volume of the confidence regions (D-optimality) (refer to [7] for further details). This

Fig. 1. Formation of four heterogeneous vehicles

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work builds upon D-optimality.

In our previous work [1], the following question has been posed: given a configuration of receivers, where should the emitter be so that its position can be optimally estimated? An analysis of the effect of the relative position of the receivers on the optimal position of the emitter has been conducted. Later in [6], the problem has been addressed in the opposite direction and the optimal positions of the receivers, given the position of the emitter, has been determined.

Motivated by navigation aid robot applications [8]–[10], recent results obtained in the context of three-dimensional localization of an acoustic target using TOA measurements are presented. In our previous work, it has been considered that the motions of the robots carrying the receivers are unconstrained in space. Hence, they were allowed to move to any *optimal position* in a three-dimensional space. Nevertheless, it is well known that precise positioning of underwater robots is a very challenging task when bounded errors are required and depends on relative measurements to auxiliary equipment such as acoustic beacons. As such, three-dimensional localization of an acoustic source using robots carrying receivers may be strongly affected by the uncertainties of the positions of the *listening robots* themselves. In contrast, our recent works have demonstrated that one may obtain very precise positioning of surface vehicles using GPS (see, for example, [11]). In addition to their precise positioning, small-sized surface vehicles are commonly easier to operate and are less expensive solutions. Therefore, the optimal positioning of sensors on the surface to estimate the three-dimensional position of the underwater target is explored. In this paper, the extension of our work to the plane constrained positions of receivers is addressed. By employing the determinant of the FIM, the optimal positions of the sensors are derived in order to minimize the uncertainty of the estimate.

B. Contributions

The main contributions of this work are the derivation of the optimal positions of the sensors to estimate the position of a target based on TOA measurements, as well as the positioning of the sensors making use of four coordinated ASVs. These are of special interest for the navigation of underwater vehicles and tracking of underwater sensors, resulting in a novel method enabling new capabilities in the localization domain. It should be stressed that, to our best knowledge, no similar results have been presented in the literature.

II. Optimal plane-constrained positions of sensors for 3D localization

As seen before, several methods can be used for locating a sound sources underwater. Particular interest is given to the TOA-based localization problem, for which a minimum set of four sensors is required to solve the problem of position estimation. Note that, in addition to the position, the time of emission and pinging period are unknown. Next, the main theoretical background is presented, including the measurement model and the specific FIM used in this paper to find the optimal relative positions of the sensors, with respect to the target, in order to obtain the best possible estimate, that is, with the least possible uncertainty. The model and the forthcoming derivations follow the work in [6] and [3].

A. TOA measurement model

The purpose of the acoustic sensors is to estimate the precise TOA of the signal sent by a simple and inexpensive acoustic pinger. One common assumption about the underwater acoustic (UWA) channel is that it exhibits such extreme micro-multipath that it can be modeled using a tapped delay line with gains modeled as complex Gaussian processes [12]. The resulting channel is the well known Rayleigh fading channel. In this work, the TOA estimation system was modeled assuming the channel to be Rayleigh fading and corrupted by additive white Gaussian noise (AWGN). These assumptions lead to an optimal maximum likelihood (ML) estimator that is called incoherent because it does not take into account the signal's phase information for narrow-band signals [13].

The most common inexpensive acoustic pingers transmit pure sine waves, changing only the signal duration, periodicity of the transitions and their frequency. To know what implications these signal parameters have in the delay estimation, the Cramer-Rao lower bound (CRLB) can be applied to the coherent version of the problem. Assuming AWGN with $N_0/2$ energy spectral density, the variance CRLB of the ML estimate of the non-random TOA τ of a known signal $s(t-\tau)$ is [13]:

$$\operatorname{Var}[\hat{\tau}-\tau] \geq \frac{N_0/2}{\int_{-\infty}^{\infty}\left[\frac{\partial s(t-\tau)}{\partial \tau}\right]^2 \mathrm{d}t}. \tag{1}$$

For pure sinusoidal waves with energy E at the receiver and angular frequency ω, the variance CRLB of the estimate becomes:

$$\operatorname{Var}[\hat{\tau}-\tau] \geq \frac{N_0}{2E\omega^2}, \tag{2}$$

this precision increase with frequency, for this particular problem, leads naturally to a choice of higher frequency sound sources. However, the channel path loss increases with frequency [14], making this frequency choice a compromise between precision and maximum range. The pulse duration does not influence the precision of the delay estimate provided that the energy remains the same. Note that, for incoherent receivers, this bound is too optimistic, even for large signal-to-noise ratios. In practice, any estimator based on these models have many more error sources and bias such as: multi-path, Doppler effect and mismatch between the received signal and the ideal one, created by the limited bandwidth of the transducers, which make the real error variance much higher than the expected by the CRLB. To simplify, all these effects will be accounted for in a single variance σ^2.

Consider a set of N sensors that are used to gather TOAs measurements to estimate the position and the time of emission of the target. The vector of observations is given by

$$\hat{\boldsymbol{\tau}} = \begin{bmatrix} \hat{\tau}_1 \\ \hat{\tau}_2 \\ \vdots \\ \hat{\tau}_N \end{bmatrix} = \boldsymbol{\tau} + \mathbf{w_s} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_N \end{bmatrix} + \begin{bmatrix} w_{s_1} \\ w_{s_2} \\ \vdots \\ w_{s_N} \end{bmatrix}, \tag{3}$$

where $\boldsymbol{\tau} \in \mathbb{R}^N$ is the vector of the TOAs, corrupted by a noise vector $\mathbf{w_s} = [w_{s_1}\ w_{s_2}\ \dots\ w_{s_N}]^T \in \mathbb{R}^N$.

Note that w_{s_i} are variable and depend on several quantities such as the temperature along the water column, which also varies according to the horizontal position, and the positions of both emitter and receivers. Moreover, effects such as multipath or occlusions influence the distribution followed by w_{s_i}. In this paper, it is assumed that the noise variables are drawn from a normal distribution $w_{s_i} \sim \mathcal{N}(0, \sigma_i^2)$, where $\sigma_i^2 > 0$ is the variance. Moreover, it is also assumed that the w_{s_i} noise variables are uncorrelated.

Let us define the position of the target by the vector $\eta_t = [x_t\ y_t\ z_t]^T$ and the position of the i-th sensor placed on the robots as $\eta_{s_i} = [x_{s_i}\ y_{s_i}\ z_{s_i}]^T$, $i = 1, ..., N$, where $N \geq 4$ in the present case. Therefore, the relative positions, with regard to the target, are given by

$$\tilde{\eta}_i = [\tilde{x}_i\ \tilde{y}_i\ \tilde{z}_i]^T = \eta_{s_i} - \eta_t,\ i = 1, ..., N. \tag{4}$$

The entries of the vector $\boldsymbol{\tau}$ then result

$$\tau_i = ||\tilde{\eta}_i||/c_s + \tau_t. \tag{5}$$

where c_s is the sound speed, which is assumed to be constant here. Note that $\tilde{\eta}_i$ and τ_t, the time of emission, are unknown, hence the need for four sensors to determine the three-dimensional position of the target.

B. The FIM and the optimal estimation criterion

Based on [1], [6], for the measurement model (3), the FIM is given by

$$I(\eta_t) = \frac{1}{c_s^2} \begin{bmatrix} \sum_{i=1}^N \frac{\tilde{\eta}_i \tilde{\eta}_i^T}{\sigma_i^2 ||\tilde{\eta}_i||^2} & -c_s \sum_{i=1}^N \frac{\tilde{\eta}_i}{\sigma_i^2 ||\tilde{\eta}_i||} \\ -c_s \sum_{i=1}^N \frac{\tilde{\eta}_i^T}{\sigma_i^2 ||\tilde{\eta}_i||} & \sum_{i=1}^N \frac{c_s^2}{\sigma_i^2} \end{bmatrix}. \tag{6}$$

The reader is referred to [6] for the details of derivation of the FIM.

Expanding the matrix, using Cartesian coordinates, gives

$$I(\eta_t) = \frac{1}{c_s^2} \begin{bmatrix} I_1(\eta_t) & I_2(\eta_t) \\ I_3(\eta_t) & I_4 \end{bmatrix} \tag{7}$$

with

$$I_1(\eta_t) = \sum_{i=1}^N \frac{1}{\sigma_i^2 ||\tilde{\eta}_i||^2} \begin{bmatrix} \tilde{x}_i^2 & \tilde{x}_i \tilde{y}_i & \tilde{x}_i \tilde{z}_i \\ \tilde{x}_i \tilde{y}_i & \tilde{y}_i^2 & \tilde{y}_i \tilde{z}_i \\ \tilde{x}_i \tilde{z}_i & \tilde{y}_i \tilde{z}_i & \tilde{z}_i^2 \end{bmatrix},$$

$$I_2(\eta_t) = I_3^T(\eta_t) = -c_s \sum_{i=1}^N \frac{1}{\sigma_i^2 ||\tilde{\eta}_i||} \begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \\ \tilde{z}_i \end{bmatrix},$$

$$I_4 = c_s^2 \sum_{i=1}^N \frac{1}{\sigma_i^2}.$$

To find the best relative positions of the sensors, the D-optimality criterion is used. This criterion ensures that the solution gives the optimal positions of the sensors that minimize the volume of the confidence region (see [7]). Therefore, the determinant of the FIM is used as the metric

$$M(\eta_t) = \det\left(I(\eta_t)\right), \tag{8}$$

which has to be maximized.

The problem can be formulated as follows: given the target position η_t, find the relative horizontal positions of the sensors $[\tilde{x}_i, \tilde{y}_i]$ that maximize the metric $M(\eta_t)$, or, mathematically,

$$\{\tilde{x}_i, \tilde{y}_i\}_{i=1}^N = \underset{\{\tilde{x}_i, \tilde{y}_i\}_{i=1}^N}{\operatorname{argmax}}\ M(\eta_t). \tag{9}$$

C. Plane constrained optimal positions

It is clear from (6) that the relative positions of the sensors with regard to the source impact on the FIM and consequently on the objective function M in (8). The idea is to find a set of conditions that ensure optimal positioning of the sensors for *optimal* estimation. In the forthcoming developments, the solution that maximizes M is derived. In what follows, it is considered that $\tilde{z}_i = z > 0$ for all $i = 1, ..., N$, that is, the sensors are at the same vertical position and are not coplanar with the target.

Consider (7) and note that I_4 is invertible. Applying the Schur complement to $I(\eta_t)$, as in [3] and [6], the objective function results into

$$M(\eta_t) = \frac{1}{c_s^8} \det(I_4) \det(I_1 - I_2 I_4^{-1} I_3), \tag{10}$$

where the argument of the matrix functions were dropped for clarity.

Note that I_4 is a scalar. Since $I_4 > 0$ and $I_2 = I_3^T$, it is easy to prove that $I_2 I_4^{-1} I_3$ is positive semi-definite. Moreover, by the matrix determinant lemma (see [15], for example), assuming that I_1 is invertible, it is possible to write

$$M(\eta_t) = \frac{1}{c_s^8} I_4 \left[(1 - I_4^{-1} I_3 I_1^{-1} I_2) \det(I_1) \right],$$

and it becomes clear that, in order to maximize M, the term $I_4^{-1} I_3 I_1^{-1} I_2$ has to be minimized. To this end, suppose that it is possible to set the horizontal position of the sensors at will and choose

$$\sum_{i=1}^N \frac{\tilde{x}_i}{\sigma_i^2 ||\tilde{\eta}_i||} = \sum_{i=1}^N \frac{\tilde{y}_i}{\sigma_i^2 ||\tilde{\eta}_i||} = 0. \tag{11}$$

It is important to note that the vertical position of the sensors with respect to the target can not be controlled and hence, unlike the work in [6], the last entry of the vectors I_2 and I_3 can not be set to zero.

Based on (10) and assuming that the condition in (11) is verified, define the matrix

$$\begin{aligned} A &= I_1 - I_2 I_4^{-1} I_3 \\ &= \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \end{aligned} \tag{12}$$

where

$$A_1 = \sum_{i=1}^{N} \frac{1}{\sigma_i^2 ||\tilde{\eta}_i||^2} \begin{bmatrix} \tilde{x}_i^2 & \tilde{x}_i \tilde{y}_i \\ \tilde{x}_i \tilde{y}_i & \tilde{y}_i^2 \end{bmatrix},$$

$$A_2 = A_3^T = \sum_{i=1}^{N} \frac{1}{\sigma_i^2 ||\tilde{\eta}_i||^2} \begin{bmatrix} \tilde{x}_i \tilde{z}_i \\ \tilde{y}_i \tilde{z}_i \end{bmatrix},$$

$$A_4 = \sum_{i=1}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} - \frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}} \left(\sum_{i=1}^{N} \frac{\tilde{z}_i}{\sigma_i^2 ||\tilde{\eta}_i||} \right)^2.$$

Following the same reasoning as above and using again the Schur complement, the objective function can be rewritten as

$$M(\eta_t) = \frac{1}{c_s^8} I_4 \det(A_4) \det(A_1 - A_2 A_4^{-1} A_3).$$

Since the product $A_2 A_4^{-1} A_3$ is positive semi-definite (note that supposing that A_4 is negative contradicts the fact that the determinant of the FIM is non-negative), it has to be minimized. Hence, suppose that it can be imposed that $A_2 = \mathbf{0}$, or equivalently

$$\sum_{i=1}^{N} \frac{\tilde{x}_i \tilde{z}_i}{\sigma_i^2 ||\tilde{\eta}_i||^2} = \sum_{i=1}^{N} \frac{\tilde{y}_i \tilde{z}_i}{\sigma_i^2 ||\tilde{\eta}_i||^2} = 0, \tag{13}$$

the metric results

$$M(\eta_t) = \frac{1}{c_s^8} I_4 A_4 \det(A_1).$$

The maximization of the metric proceeds with the maximization of the determinant of the matrix A_1. Thus, it is easy to verify that

$$\det(A_1) = \sum_{i=1}^{N} \frac{\tilde{x}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} \sum_{i=1}^{N} \frac{\tilde{y}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} - \left(\sum_{i=1}^{N} \frac{\tilde{x}_i \tilde{y}_i}{\sigma_i^2 ||\tilde{\eta}_i||^2} \right)^2, \tag{14}$$

which leads to an additional condition in order to maximize the metric: by verifying that the last term is non-negative, suppose that it can be imposed that

$$\sum_{i=1}^{N} \frac{\tilde{x}_i \tilde{y}_i}{\sigma_i^2 ||\tilde{\eta}_i||^2} = 0. \tag{15}$$

Additionally, by noting that

$$\sum_{i=1}^{N} \frac{\tilde{x}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} \sum_{i=1}^{N} \frac{\tilde{y}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} = \frac{1}{4} \left[\left(\sum_{i=1}^{N} \frac{\tilde{x}_i^2 + \tilde{y}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} \right)^2 - \left(\sum_{i=1}^{N} \frac{\tilde{x}_i^2 - \tilde{y}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} \right)^2 \right]$$

and imposing

$$\sum_{i=1}^{N} \frac{\tilde{x}_i^2 - \tilde{y}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} = 0 \tag{16}$$

gives

$$\begin{aligned} \det(A_1) &= \frac{1}{4} \left(\frac{\tilde{x}_i^2 + \tilde{y}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} \right)^2 \\ &= \frac{1}{4} \left(\sum_{i=1}^{N} \frac{1}{\sigma_i} - \frac{\tilde{z}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} \right)^2, \end{aligned}$$

where the relationship $\tilde{x}_i^2 + \tilde{y}_i^2 = ||\tilde{\eta}_i||^2 - \tilde{z}_i^2$ has been used.

Assuming that the conditions (11), (13), (15) and (16) hold, expanding the metric function results into

$$\begin{aligned} M(\eta_t) =& \frac{1}{c_s^8} c_s^2 \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \\ & \cdot \left(\sum_{i=1}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} - \frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}} \left(\sum_{i=1}^{N} \frac{\tilde{z}_i}{\sigma_i^2 ||\tilde{\eta}_i||} \right)^2 \right) \\ & \cdot \frac{1}{4} \left(\sum_{i=1}^{N} \frac{1}{\sigma_i^2} - \frac{\tilde{z}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} \right)^2 \\ =& \frac{1}{4 c_s^6} \cdot m_1(\tilde{\eta}_i) \cdot m_2(\tilde{\eta}_i) \end{aligned} \tag{17}$$

where

$$m_1(\tilde{\eta}_i) = \left(\sum_{i=1}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} \sum_{i=1}^{N} \frac{1}{\sigma_i^2} - \left(\sum_{i=1}^{N} \frac{\tilde{z}_i}{\sigma_i^2 ||\tilde{\eta}_i||} \right)^2 \right)$$

$$m_2(\tilde{\eta}_i) = \left(\sum_{i=1}^{N} \frac{1}{\sigma_i^2} - \sum_{i=1}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 ||\tilde{\eta}_i||^2} \right)^2.$$

It is clear that $m_2(\tilde{\eta}_i) \geq 0$ and $m_1(\tilde{\eta}_i) \geq 0$ since, from the definition of the metric, $M \geq 0$. From m_1 and m_2, it is possible to verify that the metric function only depends on the relative distances of the sensor with regard to the target, that is, the norm of the vectors $\tilde{\eta}_i$. In order to find a clearer representation and to maximize the objective function, let us expand the function m_1 as follows:

$$\begin{aligned} m_1(\tilde{\eta}_i) =& \sum_{i=1}^{N} \frac{\tilde{z}_i^2}{\sigma_i^4 ||\tilde{\eta}_i||^2} + \sum_{i=1}^{N} \sum_{j \neq i}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 \sigma_j^2 ||\tilde{\eta}_i||^2} - \sum_{i=1}^{N} \frac{\tilde{z}_i^2}{\sigma_i^4 ||\tilde{\eta}_i||^2} \\ & - \sum_{i=1}^{N} \sum_{j \neq i}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 \sigma_j^2 ||\tilde{\eta}_i|| ||\tilde{\eta}_j||} \\ =& \sum_{i=1}^{N} \sum_{j \neq i}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 \sigma_j^2 ||\tilde{\eta}_i||^2} - \sum_{i=1}^{N} \sum_{j \neq i}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 \sigma_j^2 ||\tilde{\eta}_i|| ||\tilde{\eta}_j||} \end{aligned}$$

For each sensor i, define the set of the remaining sensors that are at the same distance from the target, that is, $\Omega_i = \{ j = 1, 2, ..., N, \tilde{\eta}_j \in \mathbb{R}^3 : j \neq i, ||\tilde{\eta}_j|| = ||\tilde{\eta}_i|| \}$. Similarly, for each sensor i, its complement is denoted $\bar{\Omega}_i$. Hence, m_1 can be decomposed and simplified as follows:

$$\begin{aligned} m_1(\tilde{\eta}_i) =& \sum_{i=1}^{N} \sum_{j \in \bar{\Omega}_i}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 \sigma_j^2 ||\tilde{\eta}_i||^2} + \sum_{i=1}^{N} \sum_{j \in \Omega_i}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 \sigma_j^2 ||\tilde{\eta}_i||^2} \\ & - \sum_{i=1}^{N} \sum_{j \in \bar{\Omega}_i}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 \sigma_j^2 ||\tilde{\eta}_i|| ||\tilde{\eta}_j||} - \sum_{i=1}^{N} \sum_{j \in \Omega_i}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 \sigma_j^2 ||\tilde{\eta}_i||^2} \\ =& \sum_{i=1}^{N} \sum_{j \in \bar{\Omega}_i}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 \sigma_j^2 ||\tilde{\eta}_i||^2} - \sum_{i=1}^{N} \sum_{j \in \bar{\Omega}_i}^{N} \frac{\tilde{z}_i^2}{\sigma_i^2 \sigma_j^2 ||\tilde{\eta}_i|| ||\tilde{\eta}_j||} \end{aligned} \tag{18}$$

Some conclusions can be drawn at this point: 1) sensors that are equally distant from the target do not impact on the m_1 function; 2) the complement of Ω_i, $\bar{\Omega}_i$, can not be null, otherwise m_1 is null and so is the objective function M, in other words, there must be at least one sensor whose distance from the target must be different from the others; 3) since m_2 is non-negative, the second term of m_1 in (18) has to be minimized in order to maximize the objective function.

The change of variable $\gamma_i = \frac{\tilde{z}_i}{||\tilde{\eta}_i||}$ and $\lambda_i = \frac{1}{\sigma_i^2}$ is now adopted. Henceforth, by noting that $\tilde{z}_i = \tilde{z}_j = z \; \forall \; i, j = 1, ..., N$, the function m_1 can be rewritten as

$$m_1(\tilde{\eta}_i) = \sum_{i=1}^{N} \sum_{j \in \bar{\Omega}_i}^{N} \lambda_i \lambda_j \gamma_i^2 - \sum_{i=1}^{N} \sum_{j \in \bar{\Omega}_i}^{N} \lambda_i \lambda_j \gamma_i \gamma_j. \tag{19}$$

It is important to observe that $\gamma_i = 1$ implies that $||\tilde{\eta}_i|| = \tilde{z}_i$ and $\gamma_i \to 0$ implies $\tilde{\eta}_i \to \infty$. Assume that the distance to the target can be made so large that $\gamma_i \in [0, 1]$. According to the expression of m_1, it is desired to minimize the second term. Without loss of generality, suppose that the sensors can be organized in two complementary sets according to their distance to the target: the set Γ_0 of the sensors that are at an infinite distance $\Gamma_0 = \{i = 1, ..., N, \gamma_i \in [0, 1] : \gamma_i = 0\}$ and $\bar{\Gamma}_0 = \{i = 1, ..., N, \gamma_i \in [0, 1] : \gamma_i \neq 0\}$, which is its complement.

Suppose that the placement of the sensors verifies the condition

$$\sum_{i=1}^{N} \sum_{j \in \bar{\Omega}_i}^{N} \lambda_i \lambda_j \gamma_i \gamma_j = 0. \tag{20}$$

Hence, using (17), (19) and (20), the metric finally results

$$M(\eta_t) = \frac{1}{4c_s^6} \left(\sum_{i \in \bar{\Gamma}_0} \lambda_i \gamma_i \sum_{j \in \bar{\Omega}_i} \lambda_j \right) \left(\sum_{i=1}^{N} \lambda_i - \sum_{i \in \bar{\Gamma}_0} \lambda_i \gamma_i^2 \right)^2. \tag{21}$$

The maximization of this last expression is not trivial since it admits several local maxima and the different combinations in the organizations of the sets Γ_0 and $\bar{\Gamma}_0$ may lead to different discrete solutions that have to be individually analyzed in practice. Hereafter, the case with four sensors is tackled.

D. Special case with four sensors and same noise variance

In order to exemplify and to assess the optimal sensor configuration, the problem considering four sensors is now addressed. Recall that this is the minimum number of sensor required to estimate the position of a target whose time of emission is unknown. Consider that $\lambda_i = \lambda, \; \forall \; i = 1, ..., 4$ and that the sensor $\{1\}$ belong to the set $\bar{\Gamma}_0$, that is, its distance from the target is finite, while the remaining sensors, $\{2, 3, 4\}$ belong to Γ_0. Therefore, it is easy to verify that the metric function becomes

$$\begin{aligned} M(\eta_t) =& \frac{1}{4c_s^6} \left(\lambda \gamma_1^2 \cdot 3\lambda \right) \left(4\lambda - \lambda \gamma_i^2 \right)^2 \\ =& \frac{3}{4c_s^6} \lambda^3 \gamma_1^2 \left(4 - \gamma_1^2 \right)^2. \end{aligned}$$

Interestingly, for the domain of γ_1, the maximum of the metric is reached when $\gamma_1 = 1$, that is, when the sensor $\{1\}$ is above the target.

Hence, setting

$$\begin{aligned} \tilde{\eta}_1 =& [0, 0, \tilde{z}_i]^T \\ \tilde{\eta}_i =& \begin{bmatrix} \rho \cos \left(\frac{2\pi(i-2)}{3} + \psi_0 \right) \\ \rho \sin \left(\frac{2\pi(i-2)}{3} + \psi_0 \right) \\ \tilde{z}_i \end{bmatrix}, \; i = 2, ..., 4 \end{aligned} \tag{22}$$

with $\rho = \infty$ and ψ_0 being any angular offset, verifies the conditions (11), (13), (15), (16) and (20). Therefore, the metric M is maximized over the horizontal positions of the sensors. The offset ψ_0 was added to emphasize the independence of the metric with regard to the rotation of the overall configuration of sensors.

The results presented here are in agreement with the work in [16], where the difference TOA problem was considered. Through genetic algorithm simulation, the results have provided a similar configuration with a central receiver and three other placed on the vertices of a equilateral triangle. Nonetheless, in [16] the receivers were constrained to move in a circle with bounded radius.

III. COORDINATED PLACEMENT OF SENSORS

Given the target location, the optimal positions of the sensors and hence of the vehicles are uniquely determined up to a formation rotation. Using this important feature, the formation of vehicles can be guided so that the observation of the target is maintained optimal even under motion of the target. A two-step iterative procedure contemplating estimation and positioning processes is implemented, similar to the one that has been proposed in [6]. For formation keeping, a centralized coordination scheme whose principles were partially exposed in [17] will be used. A brief explanation about the technology and methods used is now provided.

A. Acoustic receiver

The acoustic sensor (incoherent receiver) was implemented on a small field programmable gate array (FPGA) to minimize energy consumption. The receiver can be configured on the fly to a broad range of pinger signals with different frequencies and pulse widths, adapting itself to different pingers. The FPGA maintains an internal clock synchronized with the vehicle's GPS through its pulse per second (PPS) and NMEA 0183 standard outputs, which provide information about the position and current time over a serial port. This way, all TOAs share the same time-base across the different vehicles with a precision equal to the GPS time synchronization, which is usually below hundreds of nanoseconds, way bellow the expected CRLB for the estimate variance.

B. Coordination of autonomous vehicles

Coherent motion and formation keeping of vehicles has been achieved by means of a centralized coordination algorithm that guides the vehicles. This centralized algorithm has been developed based on [18] and posteriorly modified to ensure robustness under communication constraints. The reader is referred to [11] for a brief description of the control method and an analysis of the performances when using low rate, intermittent communications. The detailed explanation of the method is beyond the scope of this paper. Therefore, a brief description is provided.

The method assumes the existence of a virtual leader and of several followers. The virtual leader position, along with relative positions of the references of the vehicles with regard to this latter, uniquely define the formation. The inclusion of a virtual leader is advantageous in this scenarios since it does not suffer from external disturbances, measurement or positioning errors, in opposition to real vehicles, and thus avoids the propagation of these undesired effects on the formation. The followers are the real vehicles that position themselves according to the virtual leader and relative position references.

According to the optimal configuration (22), the formation is set so that it satisfies the final angular configuration, that is, vehicle 1 on the center of a circle of radius ρ_f where the remaining three vehicles are placed so that

$$\eta_{s_i} - \eta_{s_1} = \begin{bmatrix} \rho_f \cos\left(\frac{2\pi(i-2)}{3} + \psi_0\right) \\ \rho_f \sin\left(\frac{2\pi(i-2)}{3} + \psi_0\right) \\ 0 \end{bmatrix}, \quad i = 2, ..., 4$$

The radius of the formation ρ_f is varied according to the behavior adopted: during homing, that is, the approach to the target, the formation is maintained *small* and thus ρ_f is small. After having approached the target, the formation must grow to improve the estimate confidence and thus ρ_f is made larger.

C. Homing to target

As previously seen in [1], and strengthened in this paper, the relative positions of the sensors have a strong impact on the expected variance of the estimate. Therefore, it might be insensate to position the formation of vehicles equipped with the sensors based on any estimate, regardless of the observability of the target. Therefore, alternative behaviors should be adopted to drive the vehicles to a near optimal configuration to subsequently start positioning based on the estimate of the target. Hereafter, the positions of the sensors are considered to be coincident with the vehicles that carry them.

Using the same principle as in USBL systems, it is possible to determine the direction of the target with a relatively *tight* and carefully placed set of sensors. By making use of the formation of vehicles already in their final angular configuration, at each epoch, the direction is determined and subsequently used to drive the formation towards the target. Let $\bar{\eta} = \frac{1}{N}\sum_{i=1}^{N} \eta_{s_i}$ be the mean position of the sensors and $\bar{\tau} = \frac{1}{N}\sum_{i=1}^{N} \hat{\tau}_i$ be the mean TOA. The direction to which the formation is driven is determined as

$$v = -\frac{\sum_{i=1}^{N} (\eta_{s_i} - \bar{\eta})(\hat{\tau}_i - \bar{\tau})}{\left|\left| \sum_{i=1}^{N} (\eta_{s_i} - \bar{\eta})(\hat{\tau}_i - \bar{\tau}) \right|\right|}. \quad (23)$$

This direction is provided to the virtual leader, which generates the path of the formation.

D. Estimation-based positioning

1) Estimation: The position of the target is estimated whenever a new set of coherent measurements has been reported by the vehicles to a central estimation algorithm. It is considered that coherent measurements are those that, given the distances between the vehicles in the formation, do not differ more than a given user-defined threshold. By combining the positions of the sensors to the TOAs, the position of the target is estimated using a Newton's method applied to multidimensional nonlinear equations. The reader is referred to [6] for a detailed derivation of the algorithm.

2) Positioning: As stated earlier, positioning the formation based on the estimate may lead to poor performances. Two criteria have been adopted to switch the behavior of the formation from homing to estimation-based positioning:

1) The Newton's algorithm converges in a limited number of iterations;
2) The horizontal position estimate is sufficiently close to the horizontal center of mass (mean position) of the formation.

Given a successful estimate of the target position, the virtual leader drives the formation towards the target at each epoch. The second criterion ensures that the formation is in an appropriate position to estimate the target position. If these two criteria are met, the outer vehicles can *enlarge* the formation by placing themselves farther apart from the target and thus improving the estimation confidence by augmenting the metric M according to the derivations of the previous section. In practice, the largest radius of the formation must be limited because of the attenuation of the acoustic signal and, in some scenarios, because of the bounds of the operation area.

IV. RESULTS

In July 2013, field trials have been carried out in the Douro river. The operation is described and the collected data is analyzed next.

A. Assets

Four heterogeneous ASVs were used to estimate and optimally position themselves according to the TOA measurements and to the corresponding estimate. All of the vehicles were equipped with GPS devices and compasses or inertial measurement units (IMUs). The formers were providing precise time information to the acoustic receivers enclosed on board of each vehicle. The coordination algorithm has been ran on a computer on the margin interacting with the vehicles by means of radio communications. The overall operation ran autonomously.

An acoustic pinger emitting a signal of $35\,\mathrm{kHz}$ lasting $11\,\mathrm{ms}$ has been anchored in the river, in the water column, which has a maximum depth of about 10 meters at the deployment location. As the pinger was not rigidly attached to the bottom nor any structure, it is impossible to provide ground-truth

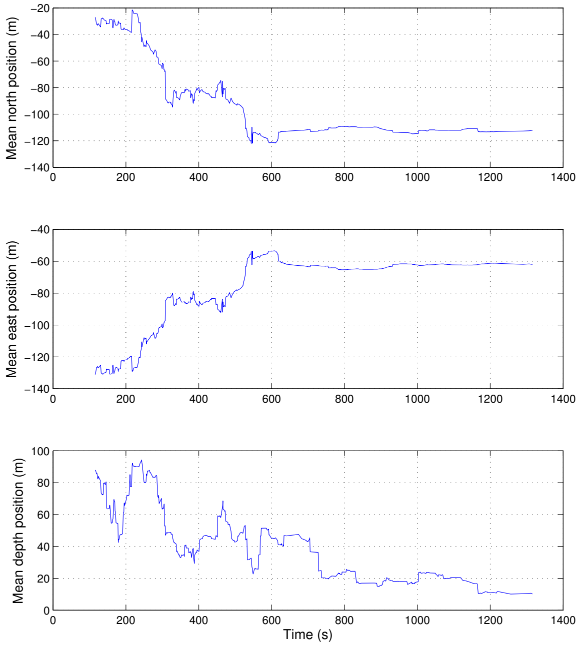

Fig. 2. Mean estimated position over a window of 60 sequential estimates

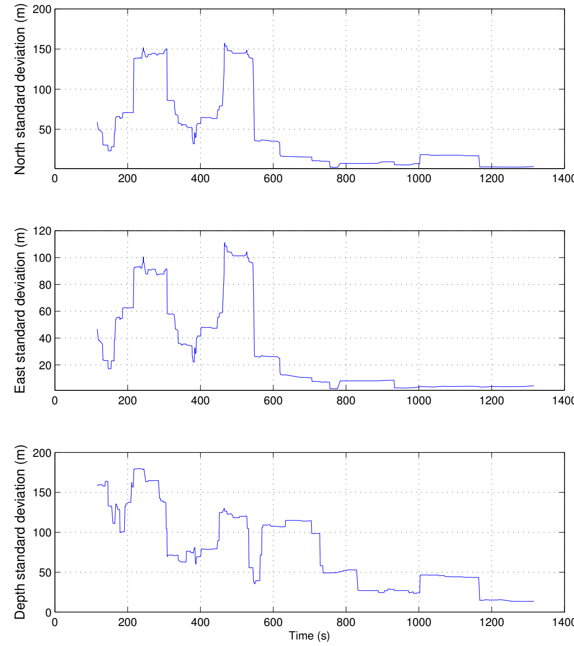

Fig. 3. Standard deviation of the estimate

information about its position. Nonetheless, it is expected that the vehicles can track the pinger and estimate its position. Moreover, the convergence of the formation to a given point would suggest that the algorithm can track and estimate the position of the target with success.

B. Operation

The vehicles were placed in formation at an arbitrary point in the operation area. This point was chosen to be $[x, y] = [-35, -115]$. According to the section III, the formation must first approach the target using the homing method. To do this, the radius of the formation was set to $\rho_f = 20$ meters. For practical reasons, the radius adopted for the estimation-based positioning behavior was set to $\rho_f = 50$ meters because of bounded operation area.

During all the operation, the estimation algorithm was continuously ran to decide when the formation should switch from homing behavior to estimation-based positioning. The results are presented and discussed hereafter. It must be highlighted that some incoherent GPS fixes occurred on two vehicles during the operation, conducting to poor positioning and to poor estimates during some instants. These malfunctions were observed in the field via an interface providing feedback on the positions of the vehicles in real time. As it is not trivial to filter them at this point, the results presented include these occurrences, which were robustly accommodated by the algorithm though, as the results hereafter demonstrate.

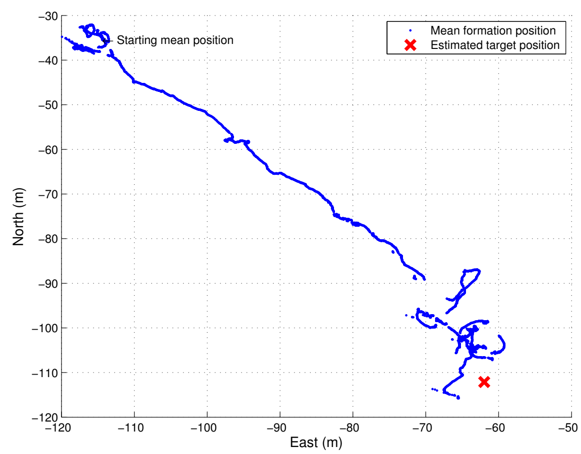

Fig. 4. Mean position of the formation and estimated position of the target

C. Performances

The estimated coordinates of the target over time are depicted in figure 2. The results shown in this figure have been computed using the last 60 estimates of the target to smooth the obtained results. Incoherent estimates caused by the divergence of the estimator have been filtered. All the estimates that were not contained in a cube centered at the origin with 2000 meters side length were removed to avoid results biased by incoherent estimates. Nevertheless, it should be noted that these were only two over 537 estimates. The standard deviations of these estimates are shown in figure 3.

It is important to note that the mean estimate converges to a constant value, suggesting a successful tracking and position estimation of the target. Moreover, the standard deviation of the estimate generally shows a decreasing trend although it was affected by the erroneous measurements of the positions of the vehicles.

Figure 4 depicts the mean position of the formation as well as the final estimated target position. For the sake of clarity, the trajectories of the vehicles have not been included. Note that there are several discontinuities in the mean position of the formation. These are caused by the erroneous GPS fixes, which also have impacted on the estimate during some intervals of time. From the figure, it is clear that the center of mass of the formation ends over a small region very close to the mean estimated position of the target.

V. Conclusions

This paper presented the results for optimal positioning of TOA sensors in the sense of minimization of the volume of the confidence region of an estimate. A two-step procedure was given to enable precise tracking and estimation of an acoustic target. The special case with four sensors was explored and subsequently implemented on a formation of four ASVs. The first results from field trials have provided very motivating results that still require tuning to accelerate the tracking task and to improve the estimate by means of more precise positioning of vehicles in the formation.

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