On the Performance of Underwater Communication System in Noise with Gaussian Mixture Statistics

Sharbari Banerjee[1], Monika Agrawal[2]
Centre for Applied Research in Electronics
Indian Institute of Technology Delhi
New Delhi, India 110016
Email: crz098482@care.iitd.ac.in[1], maggarwal@care.iitd.ac.in[2]

***Abstract*—Underwater acoustic (UWA) communication in shallow water areas is often affected by the impulsive components of ambient noise. The noise statistics in such cases does not follow the traditional Gaussian behavior. On the other hand, based on the central limit theorem (CLT), most communication receiver designs assume the channel noise to be Gaussian. Performance of such systems may not be optimum in the presence of non-Gaussian noise in the UWA channel, and, so, the scenario demands an investigation. Several non-Gaussian noise models in various UWA applications have been proposed in previous literature. Among them the Gaussian Mixture (GM) distribution is popular due to its "universal approximation" properties. However, despite its popularity in modeling UWA noise, none has yet reported any analytical analysis of error probability assuming GM noise statistics in the UWA channel, to the best of our knowledge. In this paper, we present a study on the error performance of a UWA communication system considering channel noise to be a mixture of Gaussian components. We derive the analytical expressions for probability of error with BPSK, QPSK and M-PAM constellations. We also simulate a UWA system where the performance of a traditional Gaussian receiver is studied in the presence of GM noise. It has been observed that in the presence of GM noise, the system performance degrades significantly from the usual one expected in a Gaussian noise environment. Also the performance becomes a function of the mixing coefficients of the GM distribution and varies if the same component densities are mixed in different proportions.**

I. Introduction

Performance of a communication system depends on the statistical characteristics of channel noise which is mostly assumed to be Gaussian in nature. This assumption is based on the central limit theorem (CLT) [1]: the sum of K independent and identically distributed (i.i.d.) random variables with finite variance converges to a Gaussian distribution as $K \to \infty$. The underwater acoustic (UWA) channel, often characterized by its frequency-selective, time-variant, dispersive and complex multipath nature, also contains several sources of random noise that are categorized as: (a) intermittent noise and (b) prevailing noise. The major sources of UWA noise are: merchant ships, underwater explosives, biological sources, waves, thermal noise, surface agitation, turbulance etc. [2]-[9]. The Gaussian statistics is thus the best approximation to describe the cumulative effect of noise from all these sources. However, several literature report the frequency spectrum of ocean noise [7]-[10], and, tells that for a given frequency band of operation, not all the sources contribute to the total channel noise; rather one or only a few of the sources dominate the spectral level of the overall noise and thus the probability density function (pdf) [11]. For example, in case of a communication system designed to operate in the band of 1KHz to 10KHz, the dominant sources of UWA noise are sea-surface agitation and snapping shrimp. In such situations, where the number of random noise sources is limited (i.e. $K \nrightarrow \infty$), the CLT does not hold and the channel noise can not be approximated by the classical Gaussian distribution [10]. Hence the scenario needs to be investigated as the system designed assuming Gaussian noise statistics may perform sub-optimally or even worse in the presence of non-Gaussian noise[12], [13] .

Noise components affecting signal transmission via the UWA channel are characterized as impulsive sequences of short width, high energy spikes with probability higher than the expected Gaussian density function. The noise thus shows a heavier tail in its pdf. Several non-Gaussian noise models in various UWA applications have been proposed in literature [13]-[18]. Among them the Middleton model is probably the first proposed non-Gaussian interference model [14]. However, its limitation is that the density function is represented by an infinite series of weighted Gaussian PDF's. Another one is the Generalized Gaussian (GG) model [13],[15] that offers a flexible parametric form and can describe a wide range of super-Gaussian to sub-Gaussian densities. Earlier in [13], we have reported the analytical error analysis of UWA communication using the GG moise model. In [16], an impulsive noise channel is well modeled by the symmetric α stable (SαS) distribution instead of the traditional Gaussian model. However, one drawback of the SαS distribution is: its pdf does not have a close form expression except for $\alpha = 1, 2$, and, so the density function is generally approximated by the Cauchy-Gaussian mixture model [17], Gaussian Mixture [18] model etc.

The Gaussian Mixture (GM) model is widely used due to its "universal approximation" properties [18],[19]. It is a parametric probability density function defined as a weighted sum of Gaussian component densities [20],[21]. Based on the Wiener's approximation theorem, it has been shown in [21] that any non-Gaussian density function can be well-approximated by a finite sum of Gaussian PDF's. This makes the GM distribution popular in modeling UWA noise in shallow water communication systems where the number of

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noise sources is limited (i.e., $K \nrightarrow \infty$) for a particular band of operation. There exists a number of UWA systems/applications including cooperative OFDM system [22], 3-D localization of UWA sources [23], submarine tracking [24], ROV navigation [25] etc., where the channel noise has been modeled with the GM statistics. However, despite its popularity in modeling UWA noise, no analytical error analysis for UWA communication system considering the GM noise statistics has yet been reported elsewhere, to the best of our knowledge.

In this paper, we present the analytical expression of error probability considering the channel noise to to be a weighted mixture of Gaussian density functions. We also study the performance of a Gaussian receiver in the presence of GM noise. Different modulation schemes (BPSK, QPSK, M-PAM) have been considered to analyze the effect of noise statistics on system performance. It has been observed that in the presence of GM noise, the system performance differs significantly from the usual one expected in a Gaussian noise environment. Also the performance becomes a function of the mixing coefficients of the GM distribution. As one mixing coefficient approaches unity, one component dominates and the overall noise pdf approaches the standard single component Gaussian density function with same mean and variance as the dominant noise source. So the system error performance approaches the standard Gaussian error rate with same noise power as the dominant component.

The rest of this paper is organized as follows: section II presents a brief review of GM statistics; in section III, we elaborate the analytical derivations of probability of error for BPSK, QPSK, M-PAM constellations considering GM noise distribution; results from analytical expressions are shown in section IV whereas simulation results are described in section V; and, finally section VI presents the conclusions.

II. A Brief Review of Gaussian Mixture Statistics

The Gaussian Mixture (GM) model is a parametric probability density function (pdf) which is defined by a weighted sum of K Gaussian component densities [20]. It is widely used as a parametric model for the probability distribution of continuous measurements of UWA noise [22]-[25]. For a stochastic noise process N with K-component GM statistics, the pdf can be given by,

$$f_{N-GM}(n) = \sum_{j=1}^{K} \epsilon_j f_j = \sum_{j=1}^{K} \epsilon_j \mathcal{N}(\mu_{nj}, \sigma_{nj}) = \sum_{j=1}^{K} \frac{\epsilon_j}{\sqrt{2\pi\sigma_{nj}^2}} exp\left\{ -\left(\frac{n-\mu_{nj}}{\sqrt{2}\sigma_{nj}} \right)^2 \right\} \quad (1)$$

where ϵ_j is the mixing coefficient to the j^{th} component constrained by $0 \leq \epsilon_j \leq 1, \forall j = 1, \cdots, K$, and $\sum_{j=1}^{K} \epsilon_j = 1$. $f_j = \mathcal{N}(\mu_{nj}, \sigma_{nj})$ is the j^{th} Gaussian density functions with mean μ_{nj} and variance σ_{nj}^2. the mean and variance of the resultant noise process can be given by,

$$\mu_{N-GM} = \sum_{j=1}^{K} \epsilon_j \mu_{nj}, \quad (2)$$

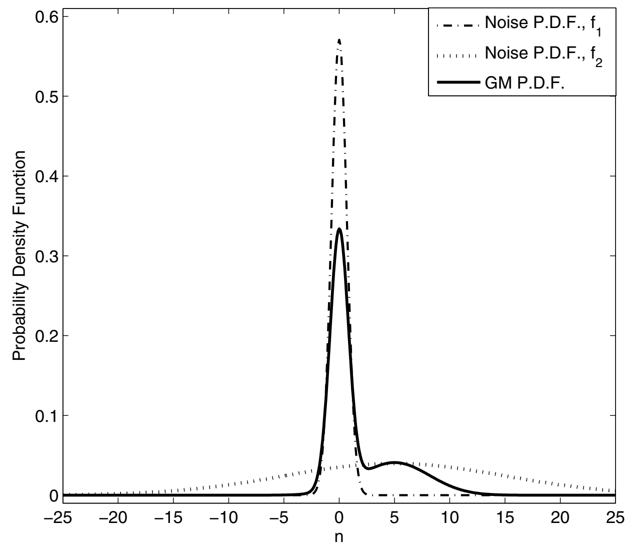


Fig. 1. Probability Density Function, $f_{N-GM}(n)$, from (1) for a two component Gaussian Mixture Stochastic Process.

$$\sigma_{N-GM}^2 = \sum_{j=1}^{K} \epsilon_j \sigma_{nj}^2. \quad (3)$$

Fig.1 shows the pdf of a two component GM distributed noise with $f_1 = \mathcal{N}(0, 0.836)$ and $f_2 = \mathcal{N}(5, 3.71)$. With the mixing coefficients being $\epsilon_1 = 0.7$ and $\epsilon_2 = 0.3$, the mean and variance of the resultant mixture pdf turns out to be $\mu_{N-GM} = 1.5$ and $\sigma_{N-GM}^2 = 3.5039$. Based on the Weiner's approximation theorem, it has been shown in [21] that the GM distribution can approximate any arbitrarily shaped non-Gaussian densities. This attribute of the GM distribution is named as the "Universal-approximation" property which makes it very popular in modeling UWA noise. The choice of density function configuration i.e., number of components, structure of covariance matrices (full or diagonal), parameter sharing among components etc., is often governed by the number of data samples available for estimating the mixing parameters and how the GM distribution is used in the target UWA application.

A detailed review of UWA noise statistics suggests that there exists a low-power nearly Gaussian background noise randomly spiked with high amplitude short duration pulses. The noise density in such cases becomes a mixture of two Gaussian densities weighted according to the impact of each component on the overall ambient noise. Such noise can be modeled by the empirical ϵ-mixture or ϵ-contaminated GM density function [26], [27] of the following form:

$$f_{N-GM}(n) = (1-\epsilon)\mathcal{N}_b(\mu_{nb}, \sigma_{nb}) + \epsilon\mathcal{N}_i(\mu_{ni}, \sigma_{ni}) \quad (4)$$

where $\epsilon \in [0, 1]$, controls the contribution from each component. Typical values of ϵ ranges from 0.01 to 0.3. $\mathcal{N}_b(\mu_{nb}, \sigma_{nb})$ defines noise pdf from of the nearly Gaussian component whereas $\mathcal{N}_i(\mu_{ni}, \sigma_{ni})$ describes impulsive noise pdf where $\sigma_{nb} \leq 1 \leq \sigma_{ni}$. The impulsive component variance $\sigma_{ni} =$

$n\sigma_{nb}$ where typically $n \in [1, 10000]$. The model is widely used in a variety of application due to its analytic simplicity and can give a fairly well approximation to many naturally occurring ocean noise components. The GM distribution not only provides a smooth fit to the overall noise distribution, but it also clearly describes the multi-modal nature of the mixture density.

III. Error Probability of a UWA System in the presence of GM Noise

Let us consider a UWA communication system that transmits symbol, s_i, i^{th} point of a particular constellation. In the presence of only noise, signal received at the receiver can be given by,

$$r = s_i + n \tag{5}$$

where the noise sample n belongs to a K-component GM noise process, N, which follows the same PDF as given in (1). The PDF of the received process can be given by,

$$f_r(x) = f_{N-GM}(x - s_i). \tag{6}$$

Therefore, the received signal belongs to a stochastic process that follows the same density functions as that of the noise process but with mean, $\mu_{rj} = \mu_{nj} + s_i$ and variances, $\sigma_{rj}^2 = \sigma_{nj}^2 \forall j = 1 \cdots K$ components of the mixture model.

Now we consider different constellations and present the average probability of error for each of them in the following sub-sections.

A. BPSK constellation

In this case, let us consider, s_1 and s_2 are the two possible symbols that are equally likely to be transmitted $\left(s_1 = \sqrt{\mathcal{E}_b}, s_2 = -\sqrt{\mathcal{E}_b}, \text{where } \mathcal{E}_b \text{ is the energy per bit}\right)$. The conditional PDF's can be given as,

$$P(r|s_1) = \sum_{j=1}^{K} \frac{\epsilon_j}{\sqrt{2\pi\sigma_{nj}^2}} exp\left\{-\left(\frac{r - \{\mu_{nj} + \sqrt{\mathcal{E}_b}\}}{\sqrt{2}\sigma_{nj}}\right)^2\right\}, \tag{7}$$

$$P(r|s_2) = \sum_{j=1}^{K} \frac{\epsilon_j}{\sqrt{2\pi\sigma_{nj}^2}} exp\left\{-\left(\frac{r - \{\mu_{nj} - \sqrt{\mathcal{E}_b}\}}{\sqrt{2}\sigma_{nj}}\right)^2\right\}. \tag{8}$$

The average probability of error for equi-probable transmission can be derived as follows:

$$P_{E-GM}^{BPSK} = \tfrac{1}{2} P(r < 0 | s_1 \text{ sent}) + \tfrac{1}{2} P(r > 0 | s_2 \text{ sent})$$

$$= \sum_{j=1}^{K} \frac{\epsilon_j}{2\sigma_{nj}\sqrt{2\pi}} \left\{ \int_{-\infty}^{0} e^{-\frac{(r+x)^2}{2\sigma_{nj}^2}} dr + \int_{0}^{\infty} e^{-\frac{(r+y)^2}{2\sigma_{nj}^2}} dr \right\} \tag{9}$$

where $x = -(\sqrt{\mathcal{E}_b + \mu_{nj}})$ and $y = -(\sqrt{\mathcal{E}_b - \mu_{nj}})$. Considering $\frac{r+x}{\sigma_{nj}} = u$, $\frac{r+y}{\sigma_{nj}} = v$, and, with appropriate change of variables, we obtain,

$$P_{E-GM}^{BPSK} = \sum_{j=1}^{K} \frac{\epsilon_j}{2\sigma_{nj}\sqrt{2\pi}} \left\{ \int_{\frac{x}{\sigma_{nj}}}^{\infty} e^{-\frac{u^2}{2}} du + \int_{\frac{y}{\sigma_{nj}}}^{\infty} e^{-\frac{v^2}{2}} dv \right\}$$

$$= \sum_{j=1}^{K} \frac{\epsilon_j}{2} \left\{ \mathcal{Q}\left(\frac{\sqrt{\mathcal{E}_b} + \mu_{nj}}{\sigma_{nj}}\right) + \mathcal{Q}\left(\frac{\sqrt{\mathcal{E}_b} - \mu_{nj}}{\sigma_{nj}}\right) \right\} \tag{10}$$

where $\mathcal{Q}(\cdot)$ is the standard $\mathcal{Q}$-function. For single component zero mean Gaussian process, equation (10) reduces to the usual expression for probability of error derived in [1] with $\sigma_{n1}^2 = \frac{N_0}{2}$ and $\epsilon_1 = 1$.

B. QPSK Constellation

The QPSK constellation can be thought of as two BPSK signals in phase quadrature. Since noise is statistically independent on the quadrature components, the probability of correct decision for a 2-bit symbol can be given by,

$$P_c = (1 - P_2)^2 \tag{11}$$

where P_2 is the symbol error probability for BPSK constellation. Here $P_2 = P_{E-GM}^{BPSK}$ and thus the symbol error probability for QPSK becomes,

$$P_{E-GM}^{QPSK} = 1 - P_c = 2P_{E-GM}^{BPSK} \left\{ 1 - \frac{1}{2} P_{E-GM}^{BPSK} \right\}. \tag{12}$$

Replacing P_{E-GM}^{BPSK} from (12), we get,

$$P_{E-GM}^{QPSK} = \sum_{j=1}^{K} \epsilon_j \left\{ \mathcal{Q}\left(\frac{\sqrt{\mathcal{E}_b} + \mu_{nj}}{\sigma_{nj}}\right) + \mathcal{Q}\left(\frac{\sqrt{\mathcal{E}_b} - \mu_{nj}}{\sigma_{nj}}\right) \right\}$$

$$\left[1 - \sum_{j=1}^{K} \frac{\epsilon_j}{4} \mathcal{Q}\left(\frac{\sqrt{\mathcal{E}_b} + \mu_{nj}}{\sigma_{nj}}\right) - \sum_{j=1}^{K} \frac{\epsilon_j}{4} \mathcal{Q}\left(\frac{\sqrt{\mathcal{E}_b} - \mu_{nj}}{\sigma_{nj}}\right) \right]. \tag{13}$$

In this case also, error probability given in equation (13) for a single component zero mean Gaussian process, reduces to the usual expression of QPSK error probability given in [1] considering $\sigma_{n1}^2 = \frac{N_0}{2}$ and $\epsilon_1 = 1$.

C. M-ary PAM

The signal points of M-ary PAM are defined as,

$$s_i = A_i \sqrt{\frac{\mathcal{E}_g}{2}} = (2i - 1 - M)\, d \sqrt{\frac{\mathcal{E}_g}{2}}, \quad \forall i = 1, ..., M \tag{14}$$

where $\mathcal{E}_g$ is energy of the basic pulse shape, and $d\sqrt{2\mathcal{E}_g}$ is the Euclidean distance between two adjacent signal points. Considering equi-probable signal points, the average energy is given by,

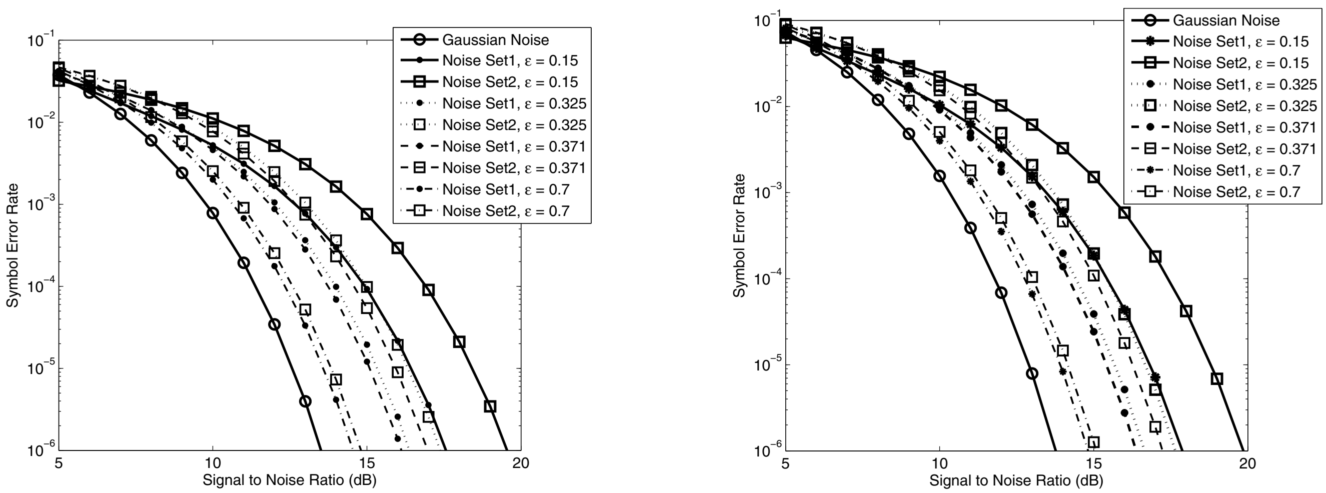
$$\mathcal{E}_{av} = \frac{1}{M} \sum_{i=1}^{M} \mathcal{E}_i = \frac{\left(M^2 - 1\right) d^2 \mathcal{E}_g}{6} \tag{15}$$

From the expression of received signal in (5), an error occurs only when the magnitude of noise sample n exceeds half of the distance between signal points. Since all signal points are equally probable, the average probability of symbol error can be given by,

$$P_{E-GM}^{M-PAM} = \frac{M-1}{M} P\left(|n| > d\sqrt{\frac{\mathcal{E}_g}{2}}\right)$$

TABLE I
PARAMETERS OF GM DISTRIBUTION FOR DIFFERENT RECORDINGS OF UWA NOISE

Noise Data	GM Model Parameters						
	ϵ_1 $\epsilon_2 = (1-\epsilon_1)$	μ_1	μ_2	μ_{N-GM}	σ_1	σ_2	σ_{N-GM}
Set 1	0.371	0	0	0	0.888	1.93	1.3704
Set 2	0.325	0	0	0	0.836	3.17	1.9333



(a) BPSK (b) QPSK

Fig. 2. Probability of error evaluated directly from the analytical expression given in (10) and (13) for BPSK and QPSK constellation respectively.

$$= \frac{2(M-1)}{M}\int_{\sqrt{\frac{d^2\mathcal{E}_g}{2}}}^{\infty} f_{N-GM}(n)\,dn$$

$$= \frac{2(M-1)}{M}\sum_{j=1}^{K}\frac{\epsilon_j}{\sqrt{2\pi\sigma_{nj}^2}}\int_{\sqrt{\frac{d^2\mathcal{E}_g}{2}}}^{\infty} exp\left\{-\frac{(n-\mu_{nj})^2}{2\sigma_{nj}^2}\right\}dn \tag{16}$$

since $\sqrt{\frac{d^2\mathcal{E}_g}{2}}$ is always > 0. Considering $\frac{n-\mu_{nj}}{\sigma_{nj}} = u$, and, with appropriate change of variables, (16) can be rewritten as,

$$\frac{2(M-1)}{M}\sum_{j=1}^{K}\epsilon_j\frac{1}{\sqrt{2\pi}}\int_{Z}^{\infty} e^{-\frac{u^2}{2}}$$

$$= \frac{2(M-1)}{M}\cdot\mathcal{Q}\left(\sqrt{\frac{d^2\mathcal{E}_g}{2\sigma_{nj}^2}}-\frac{\mu_{nj}}{\sigma_{nj}}\right). \tag{17}$$

where $Z = \sqrt{\frac{d^2\mathcal{E}_g}{2\sigma_{nj}^2}} - \frac{\mu_{nj}}{\sigma_{nj}}$. From (15), $d^2\mathcal{E}_g = \frac{6\mathcal{E}_{av}}{M^2-1}$. Again $\mathcal{E}_{av} = \mathcal{E}_b log_2 M$. Thus putting them in (17), the average probability of error for M-PAM constellation becomes,

$$P_{E-GM}^{M-PAM} = \frac{2(M-1)}{M}\cdot\mathcal{Q}\left(\sqrt{\frac{3log_2M}{M^2-1}\cdot\frac{\mathcal{E}_b}{\sigma_{nj}^2}}-\frac{\mu_{nj}}{\sigma_{nj}}\right) \tag{18}$$

where, $\mathcal{E}_b$ is the average bit energy. For a single component zero mean Gaussian process, the probability of error given in equation (19) , reduces to the usual expression of M-ary PAM error probability given in [1] considering $\sigma_{n1}^2 = \frac{N_0}{2}$ and $\epsilon_1 = 1$.

IV. ANALYTICAL RESULTS

In this section, we present the error performance [symbol error rate (SER) vs. signal-to-noise ratio (SNR)] for different constellations using a 2-component GM noise statistics. Table 1 presents parameters of GM density function for two different sets of noise recordings. Since the individual components (f_1 and f_2) are zero mean ($\mu_{nj} = 0, j = 1,2$) the resultant mixture process is also zero mean ($\mu_{N-GM} = 0$) for both sets of noise recording. Variance of the resultant mixture process is calculated as given in (3) and presented in table I. These parameters are used in the analytical expressions given in section III in order to evaluate the error performance of UWA systems with BPSK, QPSK and PAM constellation. The average SNR has been defined as,

$$SNR = 10log\left\{\frac{\mathcal{E}_b}{(1-\epsilon)\sigma_{n1}^2+\epsilon\sigma_{n2}^2}\right\} \tag{19}$$

where the symbols have their usual meaning as defined in the previous sections. We have GM parameters from different noise observations as presented in table I. These parameters are used to evaluate the analytical expressions.

Fig. 2(a) shows the BER vs. SNR graphs obtained directly from the analytical expression for BPSK constellation given in (10) with parameter values given in table I. For an SER

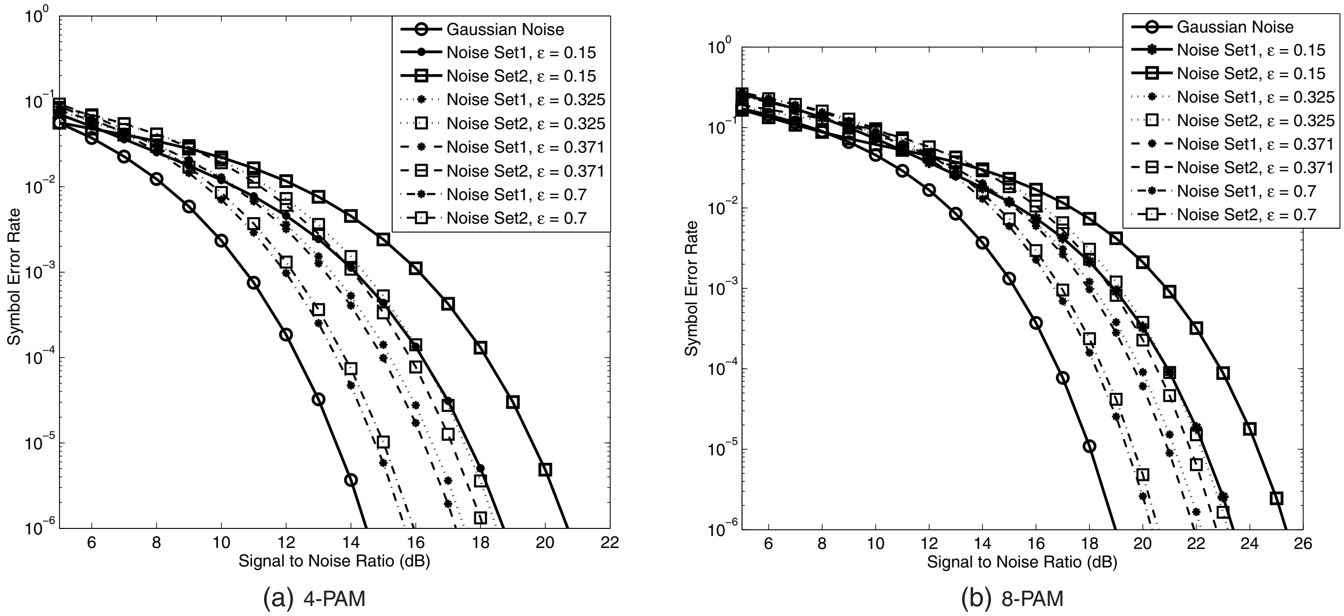


Fig. 3. Probability of error evaluated directly from the analytical expression given in (18) for PAM constellation

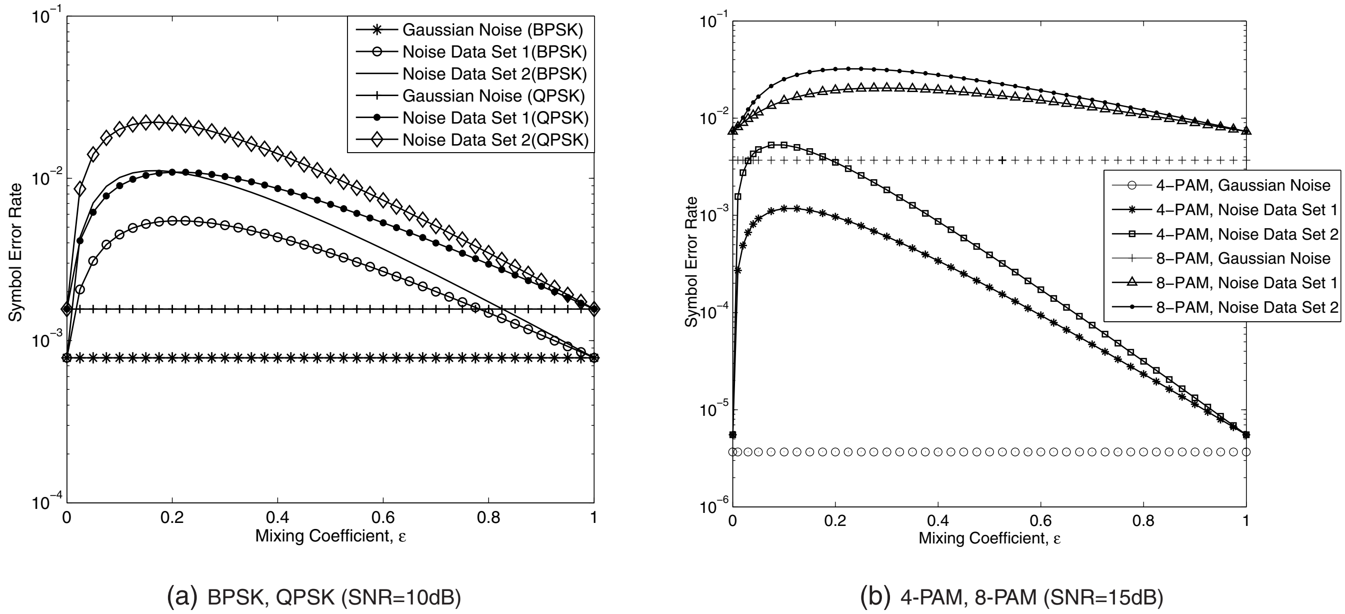
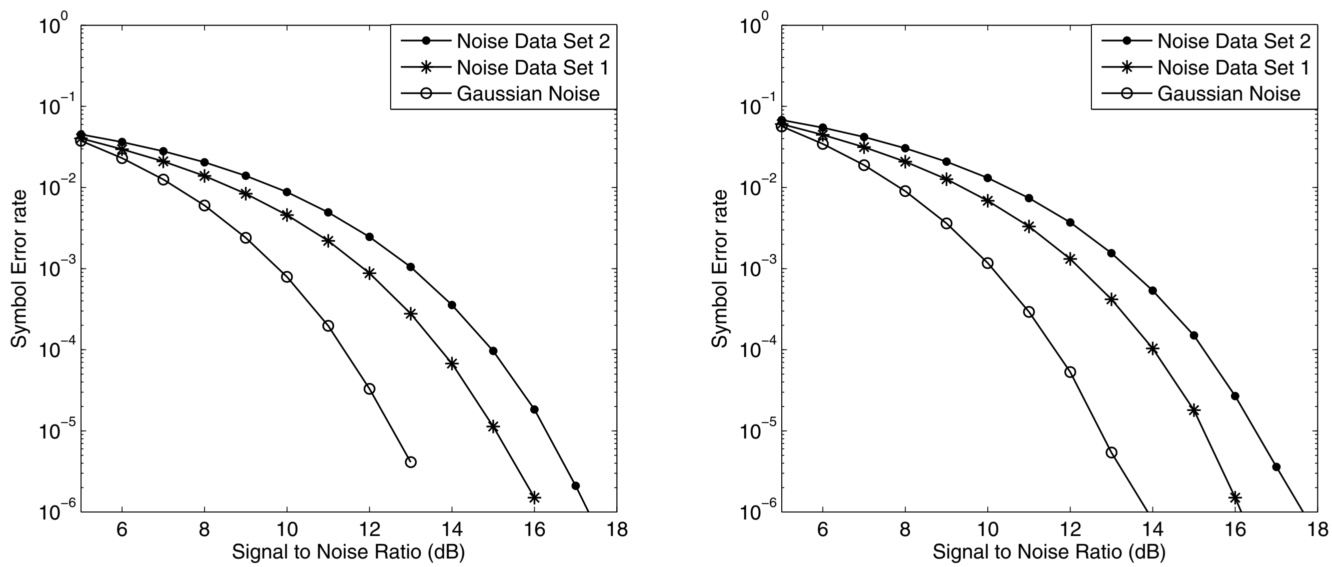


Fig. 4. SER vs. mixing coefficient from the analytical expressions for PSK and PAM constellations

of 10^{-5}, using noise parameters from data set 1 and 2, the system requires almost 15.1dB and 16.4dB SNR respectively. The same performance is achieved using the standard Gaussian noise parameters, $\mathcal{N}(0,1)$, when SNR is only 9.6dB. Thus, the presence of non-Gaussian noise degrades the system performance by around 5.5dB to 6.8dB depending on noise parameters.

Similarly, for QPSK constellation, the analytical results from equation (13) are shown in Fig. 2(b) for noise parameters from both data sets. It can be observed that, with the usually assumed Gaussian noise parameters [$\mathcal{N}(0,1)$], 10dB SNR is required to achieve an SER of 10^{-5}. On the other hand, the system requires 15.5dB and 16.7 dB SNR if the noise parameters belong to data set 1 and 2 respectively. Thus SNR loss in the presence of GM noise, in this case, turns out to be 5.5dB to 6.7dB. Also comparing the BPSK and QPSK results, we see that there exists a performance loss of ≈ 0.4dB as we move to the higher order constellation which fully complies with the previous theoretical results given in [1].

Next we present results obtained from (17) for M-PAM constellation. The results for $M = 2$ are verified and perfectly matches with the BPSK case. As shown in Fig. 3(a) and 3(b)



(a) BPSK

(b) QPSK

Fig. 5. Probability of error obtained by simulating a BPSK and a QPSK UWA system in the presence of 2-component GM noise

an SER of 10^{-5} is achieved at 13.6dB and 18dB SNR for the standard Gaussian noise case, $\mathcal{N}(0,1)$ if $M = 4$ and $M = 8$ respectively. When the GM noise is parameterized in accordance with data set 1, the system provides same error rate with 16.2dB and 21 dB of SNR for 4-PAM and 8-PAM constellations respectively. The system performance thus degrades by almost 3dB in both the cases of 4 and 8-PAM constellations. Similarly when the noise parameters belong to the 2nd data set, performance loss is around 4dB compared to the Gaussian noise case.

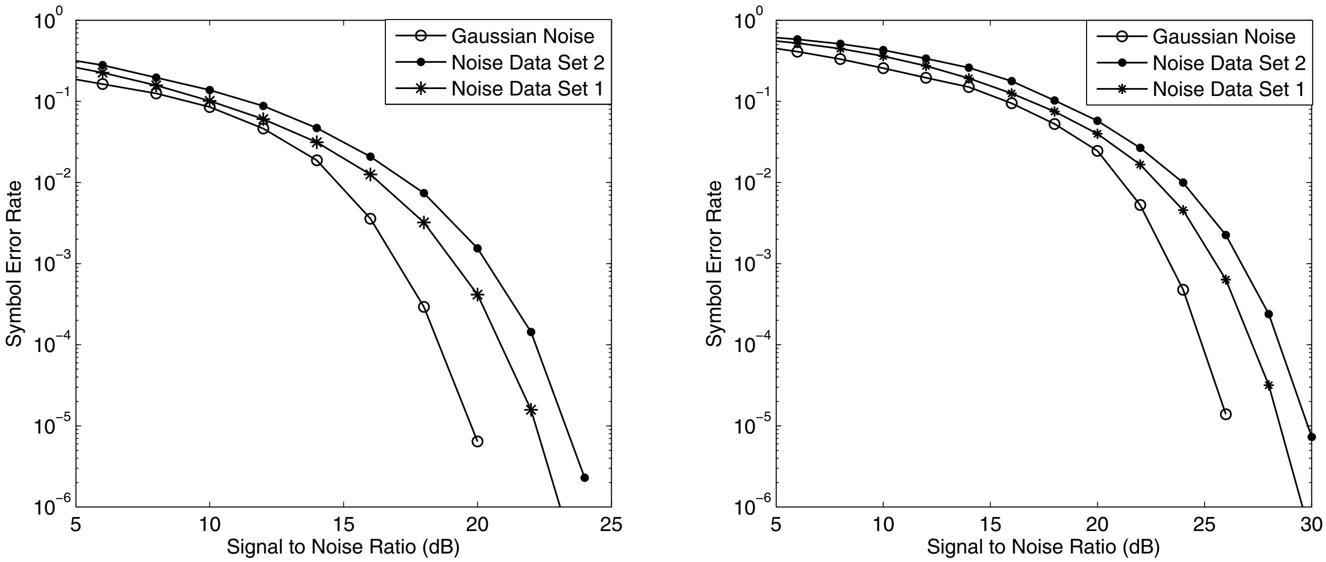
An interesting observation from all these graphs can be drawn: performance of the GM model varies with mixing coefficient. Let us consider the noise data set 1; the same component densities, if weighted with different mixing coefficients, results in different rate. A BPSK system achieves an SER of 10^{-5} at 16.5dB, 15.3dB, 15dB, 13.5dB as the mixing coefficient takes values 0.15, 0.325, 0.371, 0.7 respectively. So we study the error performance w.r.t. mixing coefficient. As shown in Fig. 4(a), for a fixed SNR of 10dB, if ϵ goes from 0 to 0.025, SER increases from 7.83×10^{-4} to 2.06×10^{-3} for the BPSK system in the presence of noise from data set 1. The SER keeps increasing till ϵ reaches 0.2. Then with further increase in its value, SER starts decreasing. While ϵ attains its maximum value of 1, SER is almost equal to the standard Gaussian noise case. Similar results follow in case of other noise data sets and constellations (Fig. 4(b)). This implies that for $\epsilon = 0$, only the first component prevails in the cumulative noise pdf which, thus, turns out to be the standard Gaussian density function, As ϵ increases, the other component starts contributing to the noise pdf giving it a non-Gaussian shape. Thus the error rates are increased. However as $\epsilon \to 1$, the second component becomes the dominant contributor to the overall noise pdf which again tends to be Gaussian. The results reflect this fact for all constellations.

V. Simulation Results

In this section, we consider a UWA communication system designed with the usual assumption that channel contains additive white Gaussian noise (AWGN), and, investigate how it performs when the channel noise is not Gaussian but a mixture of two weighted Gaussian densities. Several observations of UWA noise and their modelling with the GM statistics gave us the model parameters in table I. We used those observation data sets as the channel noise and simulate UWA communication systems with different modulation schemes. We have used a standard correlation receiver and simulate the UWA system model at different SNR. The SNR here is defined as follows:

$$SNR = 10log\left(\frac{P_r - P_n}{P_n}\right) \tag{20}$$

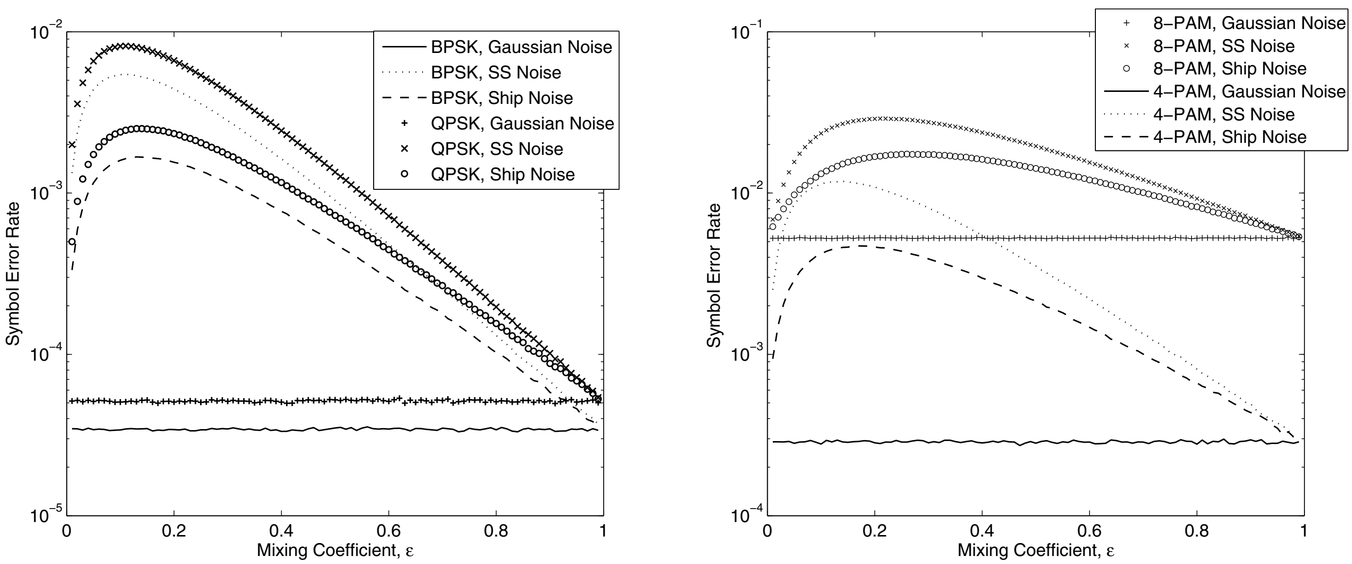
where P_r is the power of received signal in noise, and P_n is the power of channel noise.The error rates are averaged over a total of 2000 data frames each with 50000 data bits and are presented in Fig. 5 and 6. A similar pattern as that of the analytical results is found here as well, i.e., a performance loss of almost 2.5dB and 4dB exists in case of noise data set 1 and 2 respectively compared to the standard Gaussian noise $\mathcal{N}(0,1)$ at SER=10^{-5} for BPSK system. This can be verified from Fig. 5(a). Similarly for same SER in QPSK constellation, the error performance deteriorates by 2.5dB and 3.5dB in case of noise data set 1 and 2 respectively compared to Gaussian noise statistics, $\mathcal{N}(0,1)$ (shown in Fig.5(b)). In Fig. 6(a) and 6(b), for both 4-PAM and 8-PAM UWA system, the performance loss turns to be 3dB and 4dB respectively. Here also we



(a) 4-PAM

(b) 8-PAM

Fig. 6. Probability of error obtained by simulating $M-$PAM UWA system in the presence of 2-component GM noise



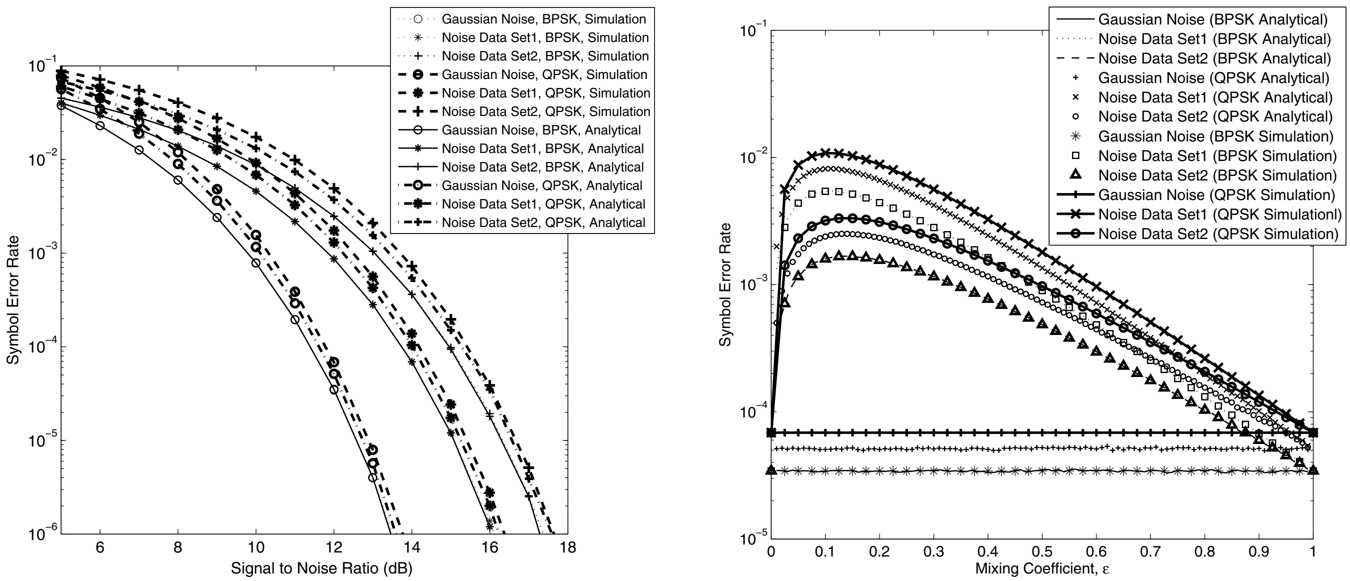
(a) BPSK, QPSK (SNR=12dB)

(b) 4-PAM (SNR=20dB), 8-PAM (SNR=22dB)

Fig. 7. SER vs. mixing coefficient from the analytical expressions for PSK and PAM constellations

study the error performance vs. mixing coefficient for all UWA systems with PSK and PAM constellations. The results given in Fig.7(a) and 7(b), are at SNR of 12 dB (for PSK), 18dB (for 4-PAM) and 22dB (for 8PAM). However all the results show us the same fact: As one mixing coefficient approaches unity, one component dominates and the overall noise pdf approaches the standard single component Gaussian density function with same mean and variance as the dominant noise source. So the system error performance approaches the standard Gaussian error rate with same noise power as the dominant component.

Now, a comparison of results from simulations with that obtained from analytical expressions is presented in Fig, 8. It can be seen from Fig. 8(a) that there exists an SNR difference of around 0.05dB (SER$=10^{-5}$) between the analytical and simulation results for QPSK system. Also, there is almost no SNR difference in case of BPSK constellation. However, for both 4 and 8 - PAM systems, the simulation results degrade by around 5dB compared to the analytical results. Any difference in the simulation and analytical results can be justified due to the fact that analytical results comes from



(a) Symbol Error Rate vs. Signal to Noise Ratio

(b) Symbol Error Rate vs. Mixing Coefficient

Fig. 8. Comparison between analytical and simulation results for BPSK and QPSK Constellations

the assumption of UWA noise being a GM stochastic process and the error probability has been found to be a function of its parameter metric; whereas the simulations are performed with an AWGN receiver in the presence of non-Gaussian noise that is approximated by a 2-component weighted Gaussian mixture. But more importantly, the simulation results comply with the analytical results, i.e., error performance of the system designed with conventional Gaussian noise statistics degrades in the presence of a 2-component GM noise process. As the noise parameter metric deviates from the traditional Gaussian statistics, the system performance gets worse. Thus UWA systems designed with conventional Gaussian noise assumptions seems to provide sub-optimal performance when channel noise follows non-Gaussian density function. Hence, to achieve optimum error performance, UWA system model with GM noise statistics appears to be more useful.

VI. Conclusion

Noise in an underwater acoustic (UWA) channel does not necessarily follow the classical Gaussian statistics and hugely affects signal transmission/recovery. In this paper, we present an analysis of error probability for a UWA communication system where the channel noise, unlike the usual Gaussian model, statistically follows a Gaussian Mixture (GM) distribution. The analytical expression of error probability for BPSK, QPSK and MPAM constellations are derived, each of which turns out to be a function of the noise parameter metric and so varies with mixing coefficient of the distribution. Also, we simulate a UWA system model with the traditional Gaussian noise assumption and see how it performs in the presence of non-Gaussian noise that are typically found in shallow water coastal areas. We observe that the performance of traditional Gaussian receiver degrades in the presence of a 2-component GM noise process depending on mixing coefficient and noise variance. Thus using the classical Gaussian assumptions to model UWA noise result in suboptimal performance of the system. Rather utilizing the "universal approximation" properties of the GM statistics, optimum error performance can be achieved for a UWA communication system operating in the presence of non-Gaussian noise.

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