AUV Buoyancy Versus Seawater Density

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Introduction:

The objective is to find the buoyancy of the Spokes-cruise AUV as a function of seawater density. That is, find $b(\rho)$.

Conclusions:

Figure 1 shows the AUV buoyancy as a function of seawater density. It also shows a more conservative plot which is the buoyancy of the AUV when the entrained water remains constant at 1027 kg/m^3. This situation could occur if the AUV moved through water where the salinity changed relatively quickly from saline to less saline, and then stopped, not giving the entrained water a chance to flush out and reach the ambient salinity.

The vehicle was ballasted in the test tank to 40N, ± 5N, at a seawater density of 1021.8 kg/m^3 (subsequent measurement). If we take 10N (twice the measurment uncertainty) as the minimum allowable buoyancy, then the AUV drops below the minimum buoyancy at 1013.7 kg/m^3, presuming the entrained water has the ambient salinity. If the entrained water remains constant at 1027 kg/m^3, the minimum allowable water density is 1020.5 kg/m^3.

Discussion:

Let the total vehicle mass be given by

$$m = m_D + m_{EW} \tag{1}$$

where m_D is the dry mass, and m_{EW} is the mass of the entrained water. Given that the buoyant force on a submerged object is equal to the weight of the displaced water (Archimedes second prinicple), the weight of the vehicle in water is

$$W = m_D g - \rho V_D g, \tag{2}$$

where V_D is the corresponding volume of displaced water, and ρ is the density of seawater. Here we will define "buoyancy", b, as

$$b = -W, \tag{3}$$

and so

$$b = (\rho V_D - m_D)g. \tag{4}$$

To find $b(g)$, we need now to solve for V_D. Then,

$$V = V_D + V_{EW}, \tag{5}$$

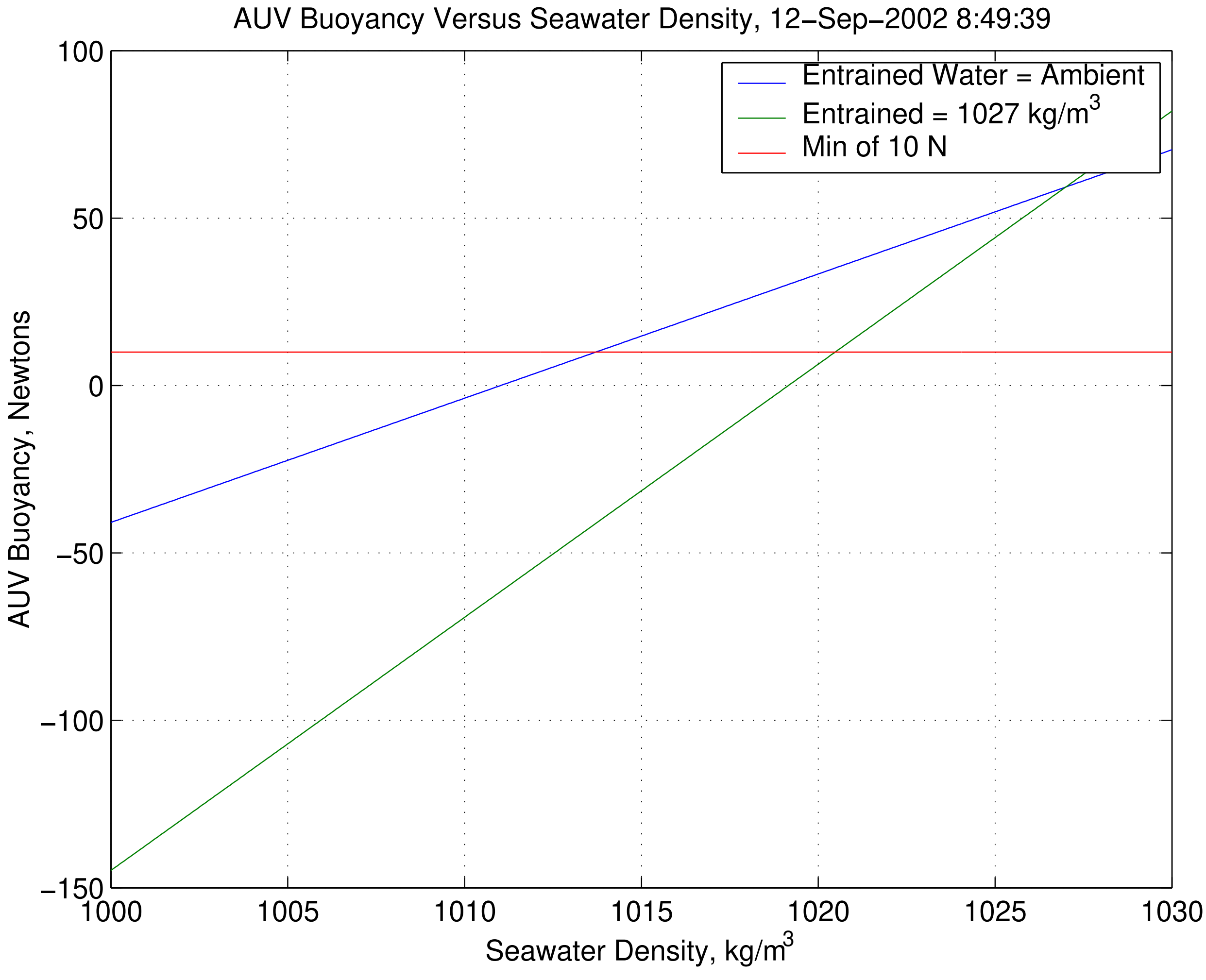

Figure 1: Buoyancy Versus Seawater Density

where V is the total volume of the hull and propulsor.

From Eqn. 1, the volume of the entrained water is

$$V_{EW} = \frac{m_{EW}}{\rho} = \frac{m - m_D}{\rho}. \tag{6}$$

The vehicle was ballasted to have a buoyancy of $b_o = 40\text{N}$ in the test tank. The density of the tank water, ρ_o, was later measured to be 1021.8 kg/m^2. From Eqn. 4, the corresponding total mass of the vehicle, m_o, is

$$m_o = V\rho_o - \frac{b_o}{g}. \tag{7}$$

Substituting this into Eqn. 6 gives

$$V_{EW} = V - \frac{b_o + m_D g}{\rho_o g}. \tag{8}$$

From Eqn. 5

$$V_D = \frac{b_o + m_D g}{\rho_o g}, \tag{9}$$

and substituting this into Eqn. 4 gives

$$b(\rho) = \rho \frac{b_o + m_D g}{\rho_o} - m_D g. \tag{10}$$

Figure 1 is a plot of this equation.

A related issue is the buoyancy of the vehicle when it moves into fresher water, and the entrained water is still more saline and has not yet flushed out of the hull. For this case, let's call the buoyancy b_S.

Eqn. 4 is also true for the vehicle as a whole, that is,

$$b = (\rho V - m)g. \tag{11}$$

Substituting Eqns. 1 and 6,

$$b_S(\rho) = (\rho V - m_D - \rho_S V_{EW})g, \tag{12}$$

where ρ_S is the density of the more saline water. Eqn. 12 is the second (steeper slope) line in Figure 1.