Stopping Distance for an Unpowered Drop-weight Descent

Rob McEwen
Revision 1
December 3, 2002

Introduction:

The general objective is to save power on deep descents by attaching a drop-weight to the AUV and allowing gravity to perform the descent. The weight is on a rope to give the vehicle room to coast to a stop after the weight hits the bottom. The WHOI vehicle works in this way.

The objective here is to compute the vertical distance that the vehicle coasts before drag and buoyancy bring it to a stop. The rope can then be made long enough to prevent the vehicle from hitting the bottom.

This analysis makes the basic simplifying assumption that the vehicle descends at a constant attitude and rate. Dynamic effects during the descent are neglected.

We consider here the two extreme cases; first where the vehicle remains exactly vertical, and second where the vehicle remains horizontal.

Other assumptions are:

1. The only forces acting on the vehicle and weight system are gravity, drag, and buoyancy.
2. All forces are in the vertical plane, implying a single degree of freedom.
3. The weight is made of steel (chosen for the sake of an example).
4. The weight and rope have no drag.
5. The weight and rope also remain exactly vertical, with constant tension on the rope.

Conclusions:

See Figure 1. The blue and green curves are stopping distances in meters. The red curve is the mass of the drop-weight as a function of steady-state speed, in kg.

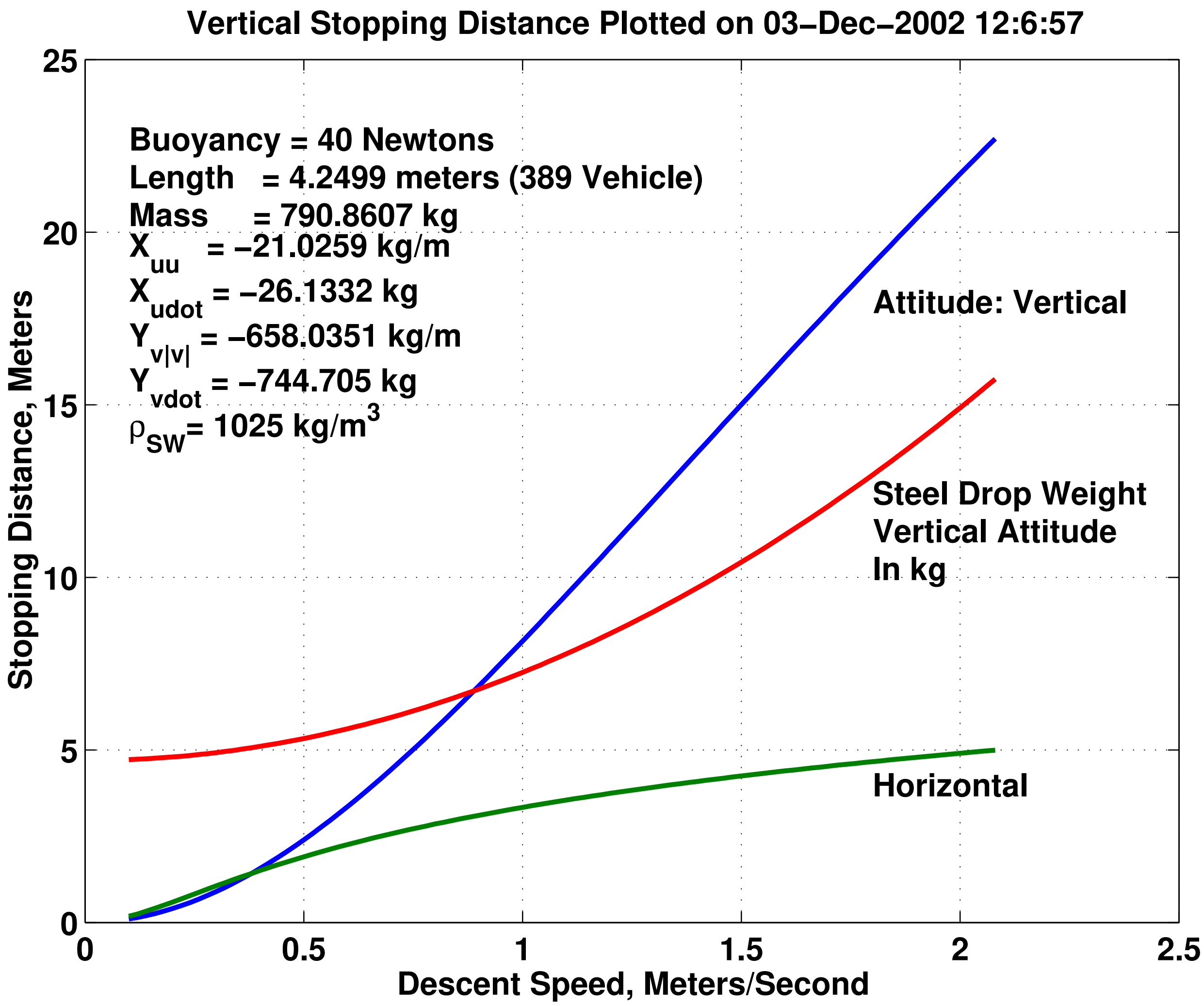

Figure 1: Vertical Stopping Distance Versus Descent Speed

Discussion:

Summing the forces in the vertical direction gives

$$m\dot{v} + Dv^2 + B = 0 \tag{1}$$

where D is a drag coefficient, B is buoyancy, and m is mass, including entrained water and added mass. The positive direction is downward.

Separating variables and integrating:

$$\begin{aligned}
\frac{dv}{dt} &= -\frac{Dv^2 + B}{m} && (2)\\
\int_{v(0)}^{v(t)} \frac{d\sigma}{D\sigma^2 + B} &= -\int_0^t \frac{dt}{m} && (3)\\
\frac{1}{D}\int_{v(0)}^{v(t)} \frac{d\sigma}{\sigma^2 + \frac{B}{D}} &= -\frac{1}{m}t && (4)\\
\sqrt{\frac{D}{B}} \arctan \sqrt{\frac{D}{B}}\sigma \Bigg|_{v(0)}^{v(t)} &= -\frac{D}{m}t && (5)\\
\arctan \sqrt{\frac{D}{B}}v &= \arctan \sqrt{\frac{D}{B}}v(0) - \frac{\sqrt{DB}}{m}t && (6)\\
v(t) &= \sqrt{\frac{B}{D}} \tan\left(\arctan \sqrt{\frac{D}{B}}v(0) - \frac{\sqrt{DB}}{m}t\right) && (7)
\end{aligned}$$

To find the stopping distance, we must first find the time it takes for the vehicle to come to a stop, starting from the moment the drop-weight touches the ground and its weight ceases to act on the vehicle. That is, find t_f such that $v(t_f) = 0$, with $v(0) = v_0 \neq 0$. From Eqn. 7 we see that $v(t_f) = 0$ if the argument of the tangent is zero, that is,

$$\arctan \sqrt{\frac{D}{B}}v_0 - \frac{\sqrt{DB}}{m}t_f = 0 \tag{8}$$

or

$$t_f = \frac{m}{\sqrt{DB}} \arctan \sqrt{\frac{D}{B}}v_0 \tag{9}$$

We must now integrate Eqn. 7 to get position. First define

$$\begin{aligned}
a &\triangleq \arctan \sqrt{\frac{D}{B}}v_0 && (10)\\
b &\triangleq \frac{\sqrt{DB}}{m} && (11)\\
c &\triangleq \sqrt{\frac{B}{D}} && (12)
\end{aligned}$$

Then,

$$
\begin{aligned}
\dot{x} &= c\tan(a-bt) && (13)\\
x(t)-x(0) &= c\int_0^t \tan(a-bt)dt && (14)
\end{aligned}
$$

Let

$$
\begin{aligned}
u &\stackrel{\triangle}{=} a-bt && (15)\\
du &= -b\,dt && (16)
\end{aligned}
$$

Then, letting $x(0)=0$, Eqn. 14 becomes

$$
\begin{aligned}
x(t) &= -\frac{c}{b}\int_a^{a-bt} \tan u\,du && (17)\\
&= \frac{c}{b}\ln\left(|\cos u|\right)\Big|_a^{a-bt} && (18)\\
&= \frac{c}{b}\ln\left(|\cos a-bt|\right)-\ln\left(|\cos a|\right) && (19)
\end{aligned}
$$

Substituting Eqns. 10- 12 into the above yields

$$
x(t)=\frac{m}{D}\left\{\ln|\cos(\arctan\sqrt{\frac{D}{B}}v_0-\frac{\sqrt{DB}}{m}t)|-\ln|\cos(\arctan\sqrt{\frac{D}{B}}v_0)|\right\} \tag{20}
$$

We want to find $x(t_f)$. Comparing Eqn. 8 with the above, we can see that a substitution of $t=t_f$ will make the first term on the right hand side drop out, leaving

$$
x(t_f)=-\frac{m}{D}\ln|\cos(\arctan\sqrt{\frac{D}{B}}v_0)| \tag{21}
$$

Assuming that $v_0>0$, and applying a trigonometric identity gives the final form

$$
x(t_f)=-\frac{m}{D}\ln\left(\frac{1}{\sqrt{\frac{D}{B}v_0^2+1}}\right) \tag{22}
$$

Figure 1 contains plots of this function for two sets of values.

We find numerical values for D and m from the set of stability derivatives given in the Appendix. For the vertical case,

$$
\begin{aligned}
m &= m_V - X_{\dot{u}} && (23)\\
D &= -X_{uu} && (24)
\end{aligned}
$$

where m_V is the static mass of the vehicle including the entrained water. For the horizontal case,

$$\begin{aligned} m &= m_V - Y_{\dot{v}} && (25) \\ D &= -Y_{v|v|} && (26) \end{aligned}$$

The static force necessary to achieve a steady descent rate of v_0 is found by summing all forces, and setting the acceleration to zero:

$$\begin{aligned} -Dv_0^2 - B + W &= 0 && (27) \\ W &= Dv_0^2 + B && (28) \end{aligned}$$

This can be converted to the mass required in seawater for a particular material, such as steel:

$$m_{DW} = \frac{W}{(1 - \frac{\rho_{SW}}{\rho})g} \quad (29)$$

where m_{DW} is the mass of the drop-weight, ρ and ρ_{SW} are the densities of steel (7840 kg/m^3) and seawater (1025 kg/m^3), respectively.

Appendix

```
DORADO389 PARAMETERS
-- Mass Properties --
790.860714      : (mass) Mass, kg.
856.942962      : (Izz) Inertia about (principal) z axis, kg-m^2.
26.048046       : (Ixx) Inertia about (principal) x axis, kg-m^2.
4.249875        : (len) Length, Meters.
13.943159       : (len) Length, feet.
0.533400        : (dia) Diameter, Meters.
21.000000       : (dia) Diameter, Inches.
-2.124937       : (xR) Distance from CG to CP of ring, Meters.
-2.201137       : (xP) Distance from CG to pivot point, Meters.
0.118832        : (xG) x CG offset, Meters.
0.000000        : (yG) y CG offset, Meters.
0.006800        : (zG) z CG offset, Meters.
0.118832        : (xB) x CB offset, Meters.
0.000000        : (yB) y CB offset, Meters.
0.000000        : (zB) z CB offset, Meters.
-0.076200       : (xPcp) Dist from pivot to ring CP, Meters.
-3.000000       : (xPcp) Dist from pivot to ring CP, In.
0.063500        : (aR)  Dist from c.l. to rudder actuator base, m
0.063500        : (rR2) Dist from c.l. to rudder actuator pivot, m
0.025400        : (rR1) Body x Dist from Tailc. Pivot to Rudder Act. Pvt, m
-- Vehicle Control System --
0.200000        : (Ts) Control System Sampling Interval, Sec.
3.000000        : (vel) Foward speed, knots.
52.000000       : (Tp) Thrust, Newtons.
0.400000        : (Kph) Heading position gain, unitless.
2.000000        : (Krh) Heading rate gain, seconds.
0.000000        : (Kvh) Sway rate (v) gain, rad/(meters/sec).
0.020000        : (Kih) Original Heading integral gain, 1/sec.
0.000000        : (Kwp) Sway (Waypoint) gain, rad/meter.
0.000000        : (Kiwp)Sway (Waypoint) integral gain, rad/sec/m.
0.750000        : (Kpp) Pitch position gain, unitless.
1.100000        : (Krp) Pitch rate gain, seconds.
0.025000        : (Kip) Pitch integral gain, 1/sec.
0.017000        : (Kpd) Depth position gain, rad/meter.
0.000800        : (Kid) Depth integral gain, rad/meter/sec.
-- Tailcone --
```

```
15.000000      : (maxDeltaR) Max. Rudder Deflection, Deg.
2400.000000    : (maxActDisp) Max Act. Displacement, counts.
1200.000000    : (maxActRate) Max Actuator Displacement Rate, Counts/Sec
240.000000     : (maxActDispTs) Max Act. Displacment in one Ts, Counts/sample
1.150000       : (ampsAct) Actuator current, full on, amps.
-- Control Surface Hydrodynamic Properties --
0.129677       : (TotalArea) Area of ring and struts, M^2.
371.252039     : (YdeltaRFixed) Sway force due to fixed rudder.
370.523439     : (YdeltaR) Sway force coeff. due to rudder deflection.
   - Fins or Struts -
0.010161       : (StrutArea) Control surface Area, Meters^2.
15.750000      : (StrutArea) Control surface Area, Inches^2.
0.077803       : (LinCtrlArea) Equivalent control surface area, Meters^2.
120.595454     : (LinCtrlArea) Equivalent control surface area, Inches^2.
6.280000       : (dCl) Coeff. of lift slope.
0.810000       : (Cdc) Crossflow drag coefficient
0.012000       : (Cd0) Constant component of drag coeff.
3.111111       : (Ar) Ring/strut aspect ratio.
0.900000       : (Ef) Oswald efficiency factor.
   - Ring -
0.048387       : (RingArea) Control surface Area, Meters^2.
75.000000      : (RingArea) Control surface Area, Inches^2.
4.800000       : (dClR) Coeff. of lift slope.
0.810000       : (CdcR) Crossflow drag coefficient
0.010000       : (Cd0R) Constant component of drag coeff.
4.260000       : (ArR) Ring/strut aspect ratio.
0.900000       : (EfR) Oswald efficiency factor.
-- Fuselage Hydrodynamic Properties --
-26.133175     : (Xudot) Deriv of X wrto u-dot, kg.
744.704979     : (Xrv) Deriv of x wrto rv, kg.
79.522820      : (Xrr) Deriv of x wrto rr, N-sec^2.
-21.025893     : (Xuu) Deriv of x wrto uu, kg/m.
-56.853571     : (Xvv) Deriv of x wrto vv, kg/m.
-355.887321    : (Nr) Deriv of N wrto r, kg-m.
-625.157469    : (Nv) Deriv of N wrto v (Munk Moment), kg.
52.952896      : (Yr) Deriv of Y wrto r, kg.
-117.680396    : (Yv) Deriv of Y wrto v, kg/m.
-744.704979    : (Yvdot) Deriv of Y wrto v-dot (Added Mass),kg.
-722.037739    : (Nrdot) Deriv of N wrto r-dot (Added Inr),kg-m^2.
-79.522820     : (Yrdot) Deriv of Y wrto r-dot.
```

```
-79.522820     : (Nvdot) Deriv of N wrto v-dot.
0.000000       : (Yvabsr) Deriv of Y wrto r|v|.
-658.035108    : (Yvabsv) Deriv of Y wrto v|v|.
343.836608     : (Nvabsv) Deriv of N wrto v|v|.
-875.305915    : (Nrabsr) Deriv of N wrto r|r|.
```