2.8 Case Study – Submarine Depth Control System

Automatic depth control of a submarine is an interesting and important problem. Figure 8.1 below illustrates a representative situation, where the actual depth of the submarine is denoted $c(t)$. This depth $c(t)$ is measured by a sensor and compared with the desired depth $r(t)$.

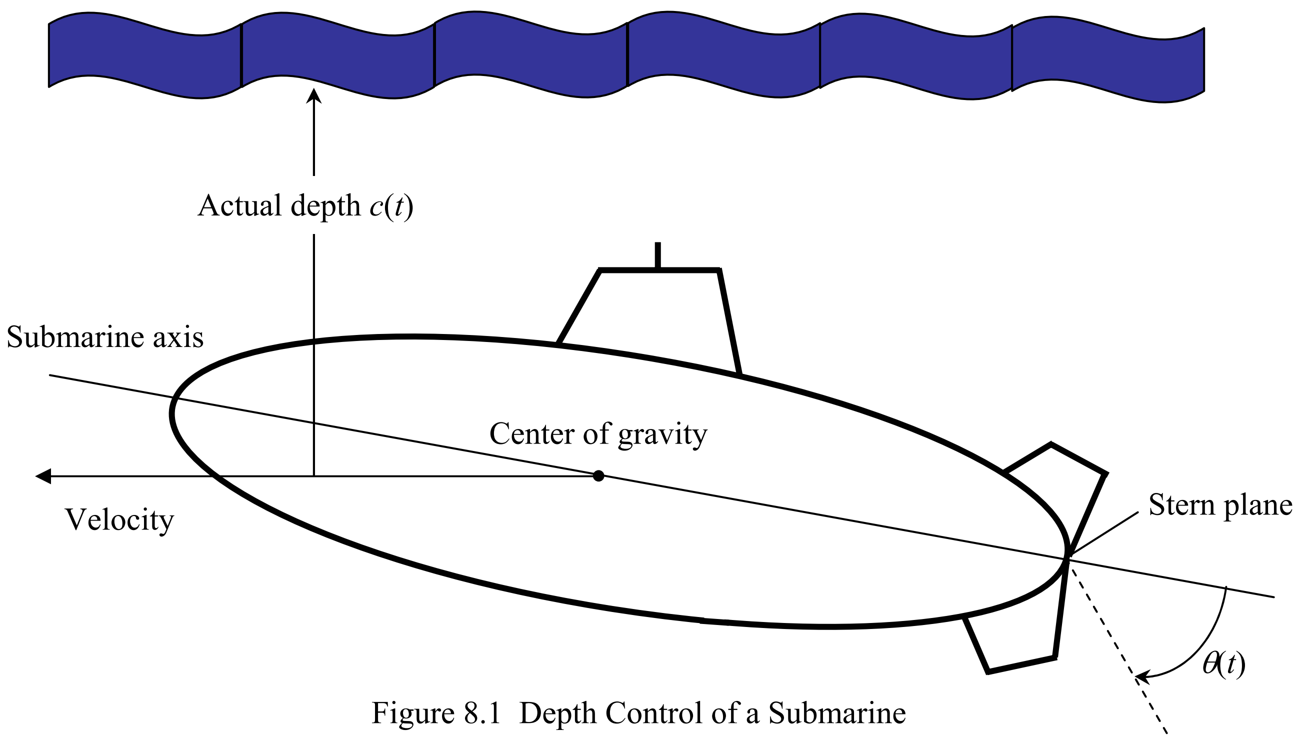

Figure 8.1 Depth Control of a Submarine

An equivalent block diagram of the depth control system is shown in Figure 8.2. The error signal $e(t)$ is the difference between the commanded depth $r(t)$ and the actual depth $c(t)$. It is fed back to the controller which sends a signal to the stern plane actuator motor that controls the stern plane actuator angle $\theta(t)$. The submarine depth responds to changes in the stern plane angle.

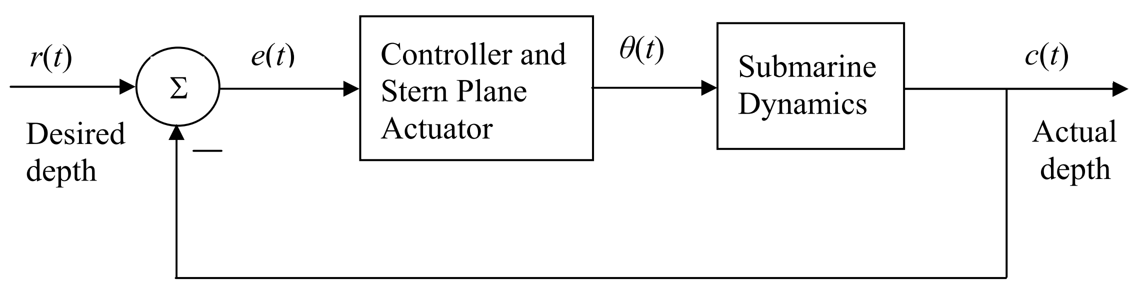

Figure 8.2 Equivalent Block Diagram of Submarine Depth Control System

The controller and stern plane actuator combination are modeled as a gain K, i.e.

$$\theta = Ke \tag{8.1}$$

and the submarine dynamics are modeled in Eqs (8.2) and (8.3).

$$\frac{d^2v}{dt^2} + 0.1v = \frac{d^2\theta}{dt^2} + 2\frac{d\theta}{dt} + \theta \tag{8.2}$$

where $v = v(t)$ is the depth rate of the submarine. Integrating the depth rate yields the depth of the sub,

$$c = \int v dt \tag{8.3}$$

The error signal is output from the summer.

$$e = r - c \tag{8.4}$$

Eqs (8.1) thru (8.4) constitute the simplified mathematical model of the control system. Our goal is to choose the parameter K so that the sub responds to step changes in commanded depth in an acceptable manner.

A simulation diagram of the control system is a useful first step in helping us choose a set of state variables. Employing the technique discussed in Section 4 for drawing the simulation diagram of a system with input derivative terms present (see Sec 4, Prob 2), the diagram is shown in Figure 8.3.

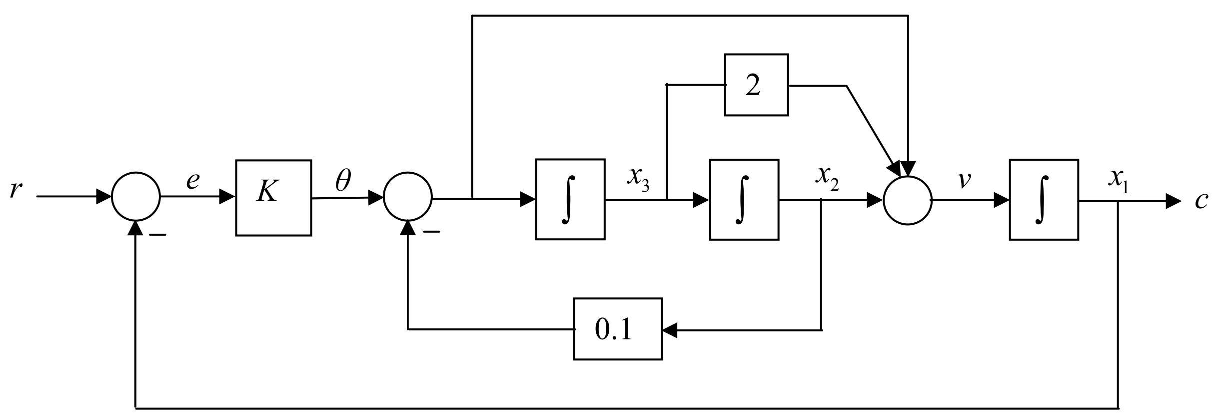

Figure 8.3 Simulation Diagram of Submarine Depth Control System

The stern plane angle θ is expressible in terms of the states x_1, x_2, x_3 and input r by

$$\theta = Ke = K(r - x_1) \tag{8.5}$$

From the simulation diagram and Eq (8.5), the state equations are

$$\dot{x}_3 = \theta - 0.1x_2 \tag{8.6}$$

$$= K(r - x_1) - 0.1x_2 \tag{8.7}$$

$$\Rightarrow \qquad \dot{x}_3 = -Kx_1 - 0.1x_2 + Kr \tag{8.8}$$

$$\dot{x}_2 = x_3 \tag{8.9}$$

$$\dot{x}_1 = x_2 + 2x_3 + \dot{x}_3 \tag{8.10}$$

Substituting $\dot{x}_3$ from Eq (8.8) into Eq (8.10) gives

$$\dot{x}_1 = -Kx_1 + 0.9x_2 + 2x_3 + Kr \tag{8.11}$$

Choosing the outputs as θ, v and c makes Eq (8.5) an output equation along with

$$v = \dot{x}_1 = -Kx_1 + 0.9x_2 + 2x_3 + Kr \tag{8.12}$$

$$c = x_1 \tag{8.13}$$

In summary, the state equations are

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -K & 0.9 & 2 \\ 0 & 0 & 1 \\ -K & -0.1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} K \\ 0 \\ K \end{bmatrix} \begin{bmatrix} r \end{bmatrix} \tag{8.14}$$

$$\begin{bmatrix} \theta \\ v \\ c \end{bmatrix} = \begin{bmatrix} -K & 0 & 0 \\ -K & 0.9 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} K \\ K \\ 0 \end{bmatrix} \begin{bmatrix} r \end{bmatrix} \tag{8.15}$$

We shall return to this state variable model of the submarine depth control system at a later point. After all, the main task is yet to be addressed, namely selecting the design parameter K to achieve a desirable dynamic response for the system.

Exercises

1. Suppose the model of the controller and stern plane actuator in Eq (8.1) is replaced by the following equation

$$\theta = K_C e + K_I \int e(t)dt$$

   A) Redraw the simulation diagram of the system.

   B) Determine the state variables and find the matrices A and B in the state equations.