ROBUST ATTITUDE STABILIZATION OF AN UNDERACTUATED AUV

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Keywords : Nonlinear control, Feedback stabilization, Underactuated systems.

Abstract

Robustness to actuator failures may be obtained by changing to a control law that controls the vehicle using only the remaining actuators, when an actuator failure is detected. Such a control law is in this paper proposed for attitude stabilization of an AUV with only two control torques. To this end, a stability result for a class of homogeneous time-varying systems is presented. The proposed control law is a continuous, periodic time-varying feedback law. It provides exponential stability that does not depend on exact knowledge of the model parameters, and the exponential stability of the controlled system is thus robust to uncertainties in the model parameters.

1 Introduction

Control of underactuated vehicles, i.e. vehicles with fewer independent control actuators than degrees of freedom to be controlled, is a field of increasing interest. It is a continuation of the research on nonholonomic systems. The nonholonomic systems have a constraint on the velocity, while the underactuation leads to a constraint on the acceleration. Underactuated vehicles have been studied by e.g. Byrnes and Isidori [1] who gave results on stabilizability of a class of underactuated vehicles. Leonard [3, 4] showed how open-loop small-amplitude periodic time-varying forcing can be used to control both underactuated spacecraft and underwater vehicles. Morin and Samson [6] developed a continuous time-varying feedback law that exponentially stabilizes an underactuated spacecraft, using the homogeneity properties of the system.

Control of autonomous underwater vehicles has many interesting applications. The AUVs may perform environmental surveying, inspect undersea cables and offshore oil installations, and find sunken ships, aircraft and other lost artifacts. As they are untethered, they may operate under ice, opening up vast, largely unexplored Arctic areas that are inaccessible to any other kind of research vessel, and operate at depths too deep for tethered vehicles. They are also of military interest. Critical to their success is their capability for accurate and reliable autonomous control. Robustness to actuator failures may be obtained by equipping the vehicle with redundant actuators. A less costly option would be obtained by, when detecting an actuator failure, changing to a control law that controls the vehicle using only the remaining actuators. This represents a software solution to fault handling in the event of an actuator failure. This is a cost-reducing alternative to fault handling by actuator duplication or triplication, i.e. by hardware redundancy. The software solution is furthermore weight economical compared to the hardware solution, and this can be an important issue in underwater applications.

In this paper control of the attitude of an AUV with only two control torques is considered. As open-loop control does not compensate for model errors and external disturbances, we seek a feedback law. An exponentially stabilizing feedback law is derived following a procedure similar to that of [6]. However, using the results of [6] directly, hydrodynamic damping, gravitation and buoyancy would have to be cancelled. Cancellation of restoring forces and torques is not desirable, and it would furthermore require exact knowledge of the model parameters. The cancellation of the gravitation and buoyancy torques is needed to obtain a homogeneous system. Using the fact that the homogeneity properties of a system are coordinate dependent, we propose a change of coordinates. In [6] the Rodrigues parameters are used to represent the attitude. For the AUV, the gravitation and buoyancy vector field then is non-homogeneous. Making a change of coordinates using the Euler parameters instead of the Rodrigues parameters, we obtain a homogeneous system without having to cancel the gravitation and buoyancy. Then, using the results of [6] directly except from this change of coordinates, the hydrodynamic damping and certain terms of the gravitation and buoyancy would still have to be cancelled. As a further development of [6] we present a result on stability of a class of homogeneous time-periodic systems. Using this new result, cancellation of hydrodynamic damping, gravitation and buoyancy is avoided. Furthermore, using this result the controller can be extended to exponentially stabilize systems that include thruster dynamics.

The resulting feedback law does not cancel any dynamics. The exponential stability result does not depend on exact knowledge of the model parameters, and consequently the exponential stability is robust to model parameter uncertainty.

The paper is organized as follows: In Section 2 the stability result for a class of homogeneous time-periodic systems is presented. In Section 3 the AUV attitude model is presented, and a continuous periodic time-varying feedback law is proposed. It is proved that the feedback law exponentially stabilizes the origin of the AUV. This is illustrated by simulations in Section 4.

2 Stability of a class of homogeneous time-periodic systems

For the design of stabilizing controllers for underactuated systems, one approach may be to first design a stabilizing control law for a reduced system where velocities are considered as inputs, and afterwards show stability of the original system. This approach was proposed by [5]. [6] presented a stability result for a class of asymptotically stable homogeneous time-periodic systems to which an integrator has been added at the input level. To obtain a system with a pure integrator at the input, it may however be necessary to cancel some of the dynamics. In this section we provide an extension of the result in [6], and conclude stability for a class of asymptotically stable homogeneous time-periodic system to which an integrator and dynamics with certain homogeneity properties has been added at the input level. By using this result, in Section 3 we can design an exponentially stabilizing control law that does not cancel any dynamics.

For the definitions of dilations, homogeneity, homogeneous norms and exponential stability with respect to a dilation, the reader is referred to [9].

Definition 1 *A* T-periodic *function is a time-periodic function with period T.*

In the following proposition we consider a system that is homogeneous of degree 0 with respect to a dilation $\delta(x,t)$. We denote this dilation by

$$\delta(x,t) = (\lambda^{r_1} x_1, \ldots, \lambda^{r_n} x_n, t) \tag{1}$$

We define the dilation $\delta_e(x,y,t)$ by

$$\delta_e(x,y,t) = (\lambda^{r_1} x_1, \ldots, \lambda^{r_n} x_n, \lambda^{q_1} y_1, \ldots, \lambda^{q_m} y_m, t) \tag{2}$$

where the q_i's are defined in the proposition.

Proposition 1 *Consider the system*

$$\dot{x} = f(x, v(x,t), t) \tag{3}$$

Let $f(x,y,t) : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}^n$ be a continuous T-periodic function and assume that the system (3) is homogeneous of degree 0 with respect to a dilation $\delta(x,t)$. Let $v(x,t) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^m$ be a vector function whose components $v_1(x,t), \ldots, v_m(x,t)$ are continuous T-periodic functions, differentiable with respect to t, of class C^1 on $(\mathbb{R}^n - \{0\}) \times \mathbb{R}$ and homogeneous respectively of degree $q_1, \ldots, q_m > 0$ with respect to the dilation $\delta(x,t)$. Let $g_i(x, y_1, \ldots, y_i, t) : \mathbb{R}^n \times \mathbb{R}^i \times \mathbb{R} \to \mathbb{R}$ be continuous, T-periodic functions that are homogeneous of degree q_i, for $i \in [1,m]$, with respect to the dilation $\delta_e(x,y,t)$.

Assume that the origin $x = 0$ is an asymptotically stable equilibrium point of the system (3). Then, for positive and sufficiently large values of $k_1, \ldots, k_m$, the origin $(x,y) = (0,0)$ is an asymptotically stable equilibrium point of the system

$$\begin{aligned}
\dot{x} &= f(x, y_1, \ldots, y_m, t) \\
\dot{y}_1 &= g_1(x, y_1, t) - k_1(y_1 - v_1(x,t)) \\
\dot{y}_2 &= g_2(x, y_1, y_2, t) - k_2(y_2 - v_2(x,t)) \\
&\vdots \\
\dot{y}_m &= g_m(x, y_1, \ldots, y_m, t) - k_m(y_m - v_m(x,t))
\end{aligned} \tag{4}$$

For the proof of Proposition 1 the reader is referred to [8].

3 Exponential attitude stabilization of the underactuated AUV

In this section a continuous periodic time-varying feedback law is proposed, and proved to exponentially stabilize the origin of the AUV using only the two available control torques.

We consider the attitude dynamics of an autonomous underwater vehicle described by:

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau \tag{5}$$

$$\dot{\eta} = J(\eta)\nu \tag{6}$$

It is here assumed that the linear velocities are zero. The matrix M is the inertia matrix, including added mass. $C(\nu)$ is the Coriolis and centripetal matrix, also including added mass. $D(\nu)$ is the damping matrix and $g(\eta)$ is the vector of gravitational and buoyant torques. Equation (6) represents the kinematics. The vector $\nu = [p,q,r]^T$ is the angular velocities in roll, pitch and yaw. The vector $\tau = [\tau_1, \tau_2, 0]^T$ is the vector of control torques. For parametrization of $SO(3)$, both the Rodrigues parameters and the Euler parameters give homogeneous kinematic equations. However, to also obtain a homogeneous gravitational and buoyancy vector field $g(\eta)$ we have to use the Euler parameters [8].

To a rotation ϕ about an axis of rotation defined by the unit vector λ the following four parameters are associated:

$$\epsilon = [\epsilon_1, \epsilon_2, \epsilon_3]^T = \lambda \sin(\frac{\phi}{2}) \qquad \epsilon_4 = \cos(\frac{\phi}{2}) \tag{7}$$

We assume in the following that $|\phi| < \pi$, i.e. $\epsilon_4 > 0$. Let $\bar{\epsilon}_4 = \epsilon_4 - 1$. We define $\eta = [\epsilon_1, \epsilon_2, \epsilon_3, \bar{\epsilon}_4]^T$. The kinematic equation (6) may then be written

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \\ \dot{\epsilon}_3 \\ \dot{\bar{\epsilon}}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \bar{\epsilon}_4 + 1 & -\epsilon_3 & \epsilon_2 \\ \epsilon_3 & \bar{\epsilon}_4 + 1 & -\epsilon_1 \\ -\epsilon_2 & \epsilon_1 & \bar{\epsilon}_4 + 1 \\ -\epsilon_1 & -\epsilon_2 & -\epsilon_3 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{8}$$

The matrix M is

$$M = \mathrm{diag}\{m_{11}, m_{22}, m_{33}\} \tag{9}$$

The damping matrix is

$$D(\nu) = \mathrm{diag}\{d_{11} + \sum_{i \geq 2} d_{p^i} |p|^{i-1}, d_{22} + \sum_{i \geq 2} d_{q^i} |q|^{i-1}, \\ d_{33} + \sum_{i \geq 2} d_{r^i} |r|^{i-1}\} \tag{10}$$

Both M and D are positive definite matrices. The matrix $C(\nu)$ is given by

$$C(\nu) = \begin{bmatrix} 0 & m_{33} r & -m_{22} q \\ -m_{33} r & 0 & m_{11} p \\ m_{22} q & -m_{11} p & 0 \end{bmatrix} \tag{11}$$

The AUV is assumed to be neutrally buoyant. The distance between the centre of gravity and the centre of buoyancy is defined by the vector $\overline{BG}$. It is assumed that $\overline{BG} = [0, 0, \overline{BG}_z]^T$ decomposed in the body-fixed frame, and $\overline{BG}_z > 0$. Using theory from [2] the gravitational and buoyancy vector field is found to be

$$g(\eta) = \begin{bmatrix} 2\overline{BG}_z W((\bar{\epsilon}_4 + 1)\epsilon_1 + \epsilon_2 \epsilon_3) \\ 2\overline{BG}_z W((\bar{\epsilon}_4 + 1)\epsilon_2 - \epsilon_1 \epsilon_3) \\ 0 \end{bmatrix} \tag{12}$$

We consider the case where, due to an actuator failure $\tau_3 = 0$. The underactuated AUV then belongs to the class of vehicles which it in [7] is shown that cannot be asymptotically stabilized by either continuous state feedback (e.g. linear feedback) or by discontinuous state feedback. Instead we propose a continuous periodic time-varying feedback law:

Proposition 2 *Consider the functions*

$$\begin{aligned} v_1(\epsilon, r, t) &= -k_1 \epsilon_1 - \rho(\epsilon, r) \sin(t/\varepsilon) \\ v_2(\epsilon, r, t) &= -k_2 \epsilon_2 + \frac{2}{\rho(\epsilon, r)} (k \epsilon_3 + d r) \sin(t/\varepsilon) \end{aligned} \tag{13}$$

where $d(m_{11} - m_{22}) > 0$, $k(m_{11} - m_{22}) > 0$, $k_1, k_2 > 0$ and $\rho(\epsilon, r)$ is any homogeneous norm associated with the dilation $\delta_\lambda^\alpha(\epsilon, r, t) = (\lambda\epsilon_1, \lambda\epsilon_2, \lambda^2\epsilon_3, \lambda^2 r, t)$. Given the following continuous periodic time-varying feedback law

$$\begin{aligned} \tau_1(\eta, \nu, t) &= -k_3 m_{11}(p - v_1(\epsilon, r, t)) \\ \tau_2(\eta, \nu, t) &= -k_4 m_{22}(q - v_2(\epsilon, r, t)) \end{aligned} \tag{14}$$

Then there exists an $\varepsilon_0 > 0$ such that for any $\varepsilon \in (0, \varepsilon_0)$ and for positive and sufficiently large parameters k_3 and k_4, the feedback law (14) locally exponentially stabilizes the origin of the system (5)–(8), with respect to the dilation $\delta_\lambda^\beta(\eta, \nu, t) = (\lambda\epsilon_1, \lambda\epsilon_2, \lambda^2\epsilon_3, \lambda^2\bar{\epsilon}_4, \lambda p, \lambda q, \lambda^2 r, t)$.

Proof. The system (5)–(8) with control (14) can be written

$$\begin{bmatrix} \dot{\eta} \\ \dot{\nu} \end{bmatrix} = l(\eta, \nu, t) + h(\eta, \nu, t) \tag{15}$$

where

$$l(\eta, \nu, t) = \begin{bmatrix} \frac{1}{2} p \\ \frac{1}{2} q \\ \frac{1}{2} r + \frac{1}{2}(\epsilon_1 q - \epsilon_2 p) \\ -\frac{1}{2}(\epsilon_1 p + \epsilon_2 q) \\ \frac{1}{m_{11}} \tau_1 - \frac{d_{11}}{m_{11}} p - \frac{2\overline{BG}_z W}{m_{11}} \epsilon_1 \\ \frac{1}{m_{22}} \tau_2 - \frac{d_{22}}{m_{22}} q - \frac{2\overline{BG}_z W}{m_{22}} \epsilon_2 \\ \frac{m_{11} - m_{22}}{m_{33}} pq - \frac{d_{33}}{m_{33}} r \end{bmatrix} \tag{16}$$

and $h(\eta, \nu, t)$ consists of the remaining terms. The functions τ_1 and τ_2 are homogeneous of degree 1 with respect to δ_λ^β and continuous for $(\eta, \nu) \neq (0, 0)$. Thus they are also continuous at $(\eta, \nu) = (0, 0)$. The vector function l is thus continuous. It is furthermore T-periodic and $l(0, 0, t) = 0$. The vector function h is continuous and is a sum of homogeneous vector fields of degree strictly positive with respect to δ_λ^β. The system

$$\begin{bmatrix} \dot{\eta} \\ \dot{\nu} \end{bmatrix} = l(\eta, \nu, t) \tag{17}$$

is homogeneous of degree 0 with respect to δ_λ^β. Thus by [6, Prop. 2] the solution $(\eta, \nu) = (0, 0)$ of the system (15) is locally exponentially stable with respect to δ_λ^β if the equilibrium $(\eta, \nu) = (0, 0)$ is a locally asymptotically stable equilibrium of the system (17).

The variable $\bar{\epsilon}_4$ is uniquely defined by ϵ since the Euler parameters satisfy the equation

$${\epsilon_1}^2 + {\epsilon_2}^2 + {\epsilon_3}^2 + {\epsilon_4}^2 = 1 \tag{18}$$

and we have assumed that $\epsilon_4 > 0$. We reduce the system (17) by removing the equation for $\dot{\bar{\epsilon}}_4$. If the reduced system is locally asymptotically stable, so is the system (17).

We further reduce the system by defining $v_1 \stackrel{\text{def}}{=} p$ and $v_2 \stackrel{\text{def}}{=} q$ as control variables:

$$\begin{bmatrix} \dot{\epsilon} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}v_1 \\ \frac{1}{2}v_2 \\ \frac{1}{2}r + \frac{1}{2}(v_2\epsilon_1 - v_1\epsilon_2) \\ \frac{m_{11}-m_{22}}{m_{33}}v_1 v_2 - \frac{d_{33}}{m_{33}}r \end{bmatrix} \tag{19}$$

The controls v_1 and v_2 are chosen to be the functions $v_1(\epsilon, r, t)$ and $v_2(\epsilon, r, t)$ given in Equation (13). The "averaged system" of the closed loop system is then

$$\begin{bmatrix} \dot{\epsilon} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}k_1\epsilon_1 \\ -\frac{1}{2}k_2\epsilon_2 \\ \frac{1}{2}r + \frac{1}{2}(k_1 - k_2)\epsilon_1\epsilon_2 \\ \frac{m_{11}-m_{22}}{m_{33}}(k_1k_2\epsilon_1\epsilon_2 - k\epsilon_3 - dr) - \frac{d_{33}}{m_{33}}r \end{bmatrix} \tag{20}$$

The linearization of (20) about $(\epsilon, r) = (0, 0)$ is asymptotically stable as $k_1, k_2 > 0$, $d(m_{11} - m_{22}) > 0$ and $k(m_{11} - m_{22}) > 0$. The system (20) is thus locally asymptotically stable. The system (19) is continuous, T-periodic and homogeneous of degree 0 with respect to the dilation $\delta_\lambda^\alpha(\epsilon, r, t) = (\lambda\epsilon_1, \lambda\epsilon_2, \lambda^2\epsilon_3, \lambda^2 r, t)$. Thus by [5, Th. 4.1] there exists an $\varepsilon_0 > 0$ such that for any $\varepsilon \in (0, \varepsilon_0)$ the origin of (19) is locally asymptotically stable.

The functions $v_1(\epsilon, r, t)$ and $v_2(\epsilon, r, t)$ are continuous, T-periodic, differentiable with respect to t, of class C^1 on $(\mathbb{R}^3 \times \mathbb{R} - \{0, 0\}) \times \mathbb{R}$ and homogeneous of degree 1 with respect to the dilation $\delta_\lambda^\alpha(\epsilon, r, t)$. The equations for $\dot{p}$ and $\dot{q}$ in (17) are

$$\dot{p} = -\frac{d_{11}}{m_{11}}p - \frac{2\overline{BG}_z W}{m_{11}}\epsilon_1 - k_3(p - v_1(\epsilon, r, t)) \tag{21}$$

$$\dot{q} = -\frac{d_{22}}{m_{22}}q - \frac{2\overline{BG}_z W}{m_{22}}\epsilon_2 - k_4(q - v_2(\epsilon, r, t)) \tag{22}$$

The first two terms of these equations are continuous and homogeneous of degree 1 with respect to δ_λ^β. Thus, by Proposition 1, as the origin of (19) is locally asymptotically stable, for positive and large enough values of k_3 and k_4 the origin $(\eta, \nu) = (0, 0)$ of the system (17) is locally asymptotically stable. Thus, by [6, Prop. 2] $(\eta, \nu) = (0, 0)$ is a locally exponentially stable equilibrium of (15), with respect to the dilation δ_λ^β. □

Remark 1 Using the new result of Section 2 it can be shown that the feedback law (14) can be extended to exponentially stabilize the attitude of an AUV with actuator dynamics described by

$$\dot{\tau}_i = -\frac{1}{T_i}(\tau_i - \tau_i^{com}) \tag{23}$$

without having to cancel any dynamics. Here τ_i^{com} is the commanded thrust and T_i is the time constant for the roll control torque ($i = 1$) and pitch control torque ($i = 2$). This is done by including τ_1 and τ_2 in the state vector.

4 Simulations

The action of the control law (14) has been simulated for an AUV of mass $m = 185$ kg, given by the following parameters:

$$M = \operatorname{diag}\{40, 80, 80\} \tag{24}$$

$$D(\nu) = \operatorname{diag}\{30 + 50|p|,\ 50 + 100|q|,\ 50 + 100|r|\} \tag{25}$$

The matrix $C(\nu)$ is given by (11) and (24). The AUV is neutrally buoyant, and the centre of buoyancy coincides with the centre of gravity.

The control parameters were chosen to be

$$\begin{array}{cccc} k_1 = 1 & k_2 = 1 & k_3 = 2 & k_4 = 2 \\ k = -0.3 & d = -0.3 & \varepsilon = 2 & \end{array} \tag{26}$$

Note that since $m_{11} < m_{22}$ the parameters k and d are chosen negative to satisfy the conditions $d(m_{11} - m_{22}) > 0$ and $k(m_{11} - m_{22}) > 0$.

The homogeneous norm used in the feedback law was

$$\rho(\eta, r) = \sqrt{\epsilon_1^2 + \epsilon_2^2 + |\epsilon_3| + |r|} \tag{27}$$

and the initial values of the attitude dynamics were

$$[\epsilon_1(0), \epsilon_2(0), \epsilon_3(0), \bar{\epsilon}_4(0), p(0), q(0), r(0)]^T = [0.3, 0.3, 0.3, -0.15, 0, 0, 0]^T \tag{28}$$

The time evolution of the Euler parameters and the angular velocities are shown in Figure 1 and 2 respectively. It is seen that all the state variables converged to zero. In Section 3 it was shown that the origin of the system (5)–(8) with control (14) is exponentially stable with respect to the dilation δ_λ^β, for small enough values of ε and large enough values of k_3 and k_4. The function

$$\rho_\lambda^\beta(\eta, \nu) = \sqrt{\epsilon_1^2 + \epsilon_2^2 + |\epsilon_3| + |\bar{\epsilon}_4| + p^2 + q^2 + |r|} \tag{29}$$

is a homogeneous norm associated with δ_λ^β. In Figure 3 it is seen that the natural logarithm of ρ_λ^β was upper bounded by a decreasing straight line. This illustrates the result of Section 3. The simulations furthermore illustrate that ε does not have to be very small and k_3 and k_4 need not be very large to have exponential stability of the closed loop system.

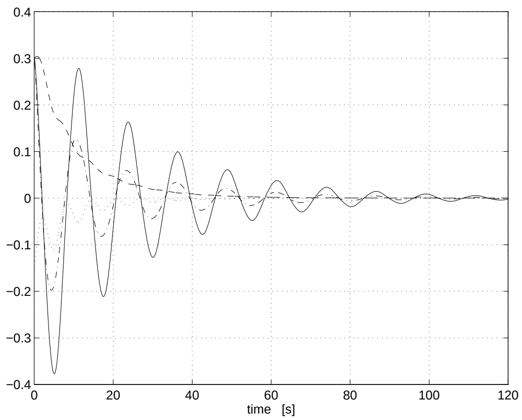

Figure 1: The time evolution of the Euler parameters $\epsilon_1(-)$, $\epsilon_2(-\cdot)$, $\epsilon_3(--)$ and $\epsilon_4(\cdots)$.

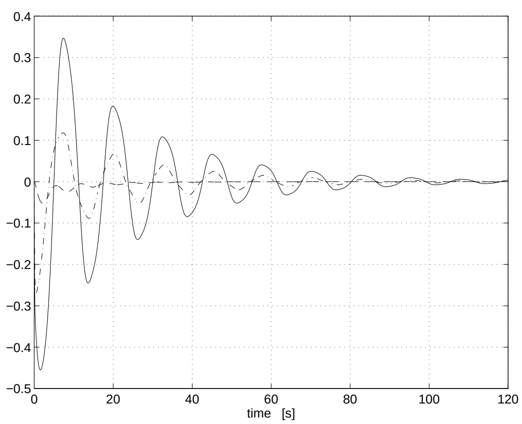

Figure 2: The time evolution of the angular velocities $p(-)$, $q(-\cdot)$ and $r(--)$ $[\frac{1}{s}]$.

5 Conclusions

Robustness to actuator failures may be obtained by, when detecting an actuator failure, changing to a control law that controls the vehicle using only the remaining actuators. Such a control law has been proposed for attitude stabilization of an AUV with only two control torques. It is shown that it exponentially stabilizes the origin of the AUV using only the two available control torques. The control is a continuous, periodic time-varying feedback law. It does not cancel any dynamics. The exponential stability does not depend on exact knowledge of the model parameters, and the exponential stability of the controlled system is thus robust to uncertainties in the model parameters.

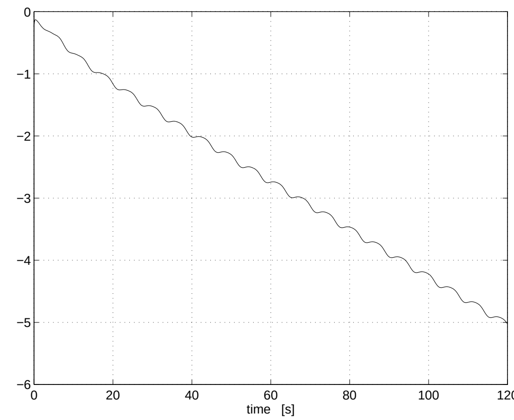

Figure 3: The natural logarithm of the homogeneous norm ρ^{β}_{λ}.

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