Dynamic Analysis and Two-Time Scale Control for Underwater Vehicle-Manipulator Systems

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***Abstract*— Underwater vehicle-manipulator (UVM) system becomes popular for various applications although it has complexity in control and analysis when compared to the vehicle itself. To resolve the complexity, firstly, thorough kinematic and dynamic analysis on the whole UVM system is performed. Then, the whole dynamics is divided into several meaningful terms. A robust coordinated motion control algorithm for autonomous UVM systems is considered next based on the analysis of divided dynamics. Since the vehicle has relatively smaller bandwidth than manipulators, active damping control with two-time scale approach for the manipulator is proposed. Using this algorithm, the manipulator controller can cover the vehicle state. The performance of the proposed algorithm is verified by computer simulations.**

I. Introduction

In recent years, the need for exploration and preservation of oceanic environments has gained significant momentum, and there has been increased activity using a robot manipulators for some special tasks. However, adding a manipulator to the vehicle produces new and more complex problems. There are many researchers to solve problems in the UVM systems. The main control difficulties stem from the dynamics, for example, highly coupling model, system nonlinearities, and external unmodeled disturbances acting on the vehicle dynamics. Another control difficulties are that the system has different characteristics, that is, the vehicle has lower bandwidth than the manipulator. These difficulties can be theoretically tackled if we have thorough analysis of dynamic model of the complex UVM system.

The interaction between two subsystems makes control difficult. To resolve this problem, we may use the whole system dynamics at once by unified system approach, but it results in complexity of modeling. And also, the different characteristics, as mentioned above, can be hardly included in the whole control system. Hence, decentralized controller was proposed for those kinds of composite dynamic systems. However, in this case, interaction disturbances have to be considered carefully based on system characteristics. Previous researchers only concentrated on the dynamic equation which is mainly taking care of vehicle motion [1] by lack of considering end-effector motion. And, they derived recursive formulation [2], which is not suitable to obtain the decoupled system modeling and control. So, we analyze in detail kinematics and dynamics of whole system using already derived equations of each subsystems, which help to easily construct motions of equation for decoupled controller.

Additionally, UVM system is a kinematically redundant system, that is, it has more degrees-of-freedom(DOF) in the joint-space that are required to execute a task specified in the Cartesian space. This is because the AUV/ROV itself has six DOF, and any additional DOF that comes from the manipulator will make the combined system redundant. Therefore, the position change of the end-effector due to the vehicle movement can be compensated by the manipulator. Using the redundancy, the passive vehicle approach, which means we do not control directly the vehicle system with vehicle actuator, is useful, because a vehicle is a relatively smaller bandwidth system than a manipulator [3].

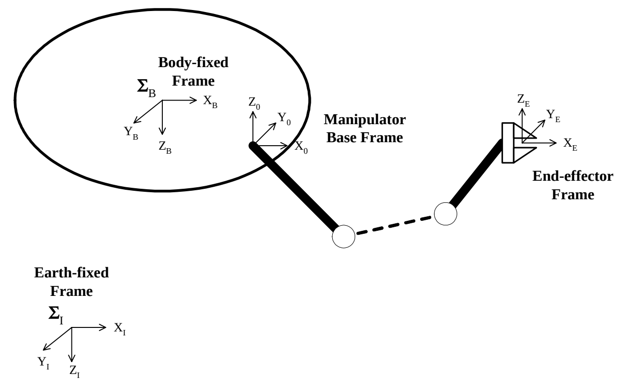


Fig. 1. Schematic diagram of UVM system

Therefore, in this paper, active damping control with two-time scale approach for the manipulator is proposed with passive vehicle. The manipulator controller can deal with the vehicle state additionally by means of active damping controller for the vehicle. Consequently, using different time scale controller acting on the manipulator, we can control the whole system simultaneously. In other words, fast time scale controller is used for the manipulator and slow time scale controller for the vehicle.

The modeling of UVM systems will be described in the sense of kinematics and dynamics in section II. We propose two-time scale controller in section III in the operational space for robust control of the end-effector under the interaction between the vehicle and the manipulator. Finally, the performance of proposed system will be shown by simulation and conclusion follows.

II. Modeling of UVM Systems

Fig. 1 illustrates the coordinates of the UUV(Unmanned Underwater Vehicle) and n-link manipulator. The 0-th frame, $\mathbf{\Sigma}_0$, is the reference frame located at the manipulator base, and each i-th frame are located at the i-th link along the D-H coordinate rule. $\mathbf{\Sigma}_B$ and $\mathbf{\Sigma}_I$ denote the vehicle body-fixed and the earth-fixed reference frame, respectively. In the earth-fixed frame, X_I directs North and Z_I is chosen as the gravity direction. The earth-fixed frame can be regarded as the inertial frame.

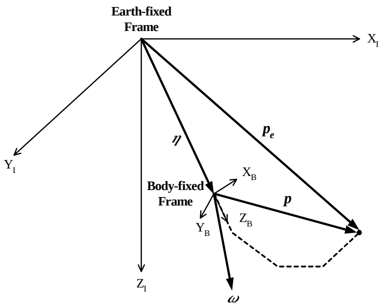


Fig. 2. Moving coordinate system representation of UVM system

The position and orientation of the vehicle, $\boldsymbol{\eta}$, are described relative to the inertial reference frame, while the linear and angular velocities of the vehicle, $\boldsymbol{\nu}$, can be expressed in the body-fixed frame.

The state vectors for the vehicle are described as following:

$$\boldsymbol{\eta} = [\boldsymbol{\eta}_1^T, \boldsymbol{\eta}_2^T]^T; \quad \boldsymbol{\eta}_1 = [x, y, z]^T; \quad \boldsymbol{\eta}_2 = [\phi, \theta, \psi]^T;$$
$$\boldsymbol{\nu} = [\boldsymbol{\nu}_1^T, \boldsymbol{\nu}_2^T]^T; \quad \boldsymbol{\nu}_1 = [v_x, v_y, v_z]^T; \quad \boldsymbol{\nu}_2 = [\omega_x, \omega_y, \omega_y]^T;$$

The inertial and body-fixed velocity relation can be represented with vehicle Jacobian matrix [4,5].

$$\begin{bmatrix}\dot{\boldsymbol{\eta}}_1 \\ \dot{\boldsymbol{\eta}}_2\end{bmatrix} = \begin{bmatrix}\boldsymbol{J}_{v1}(\boldsymbol{\eta}_2) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{J}_{v2}(\boldsymbol{\eta}_2)\end{bmatrix} \begin{bmatrix}\boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2\end{bmatrix} \Longleftrightarrow \dot{\boldsymbol{\eta}} = \boldsymbol{J}_v(\boldsymbol{\eta})\boldsymbol{\nu}. \quad (1)$$

For n-link manipulator, the joint position state vector is defined by $\boldsymbol{q}$, and the position and orientation of the end-effector, $\boldsymbol{p}_e$, can be described relative to the inertial reference frame, while the local position and orientation of the end-effector, $\boldsymbol{p}$, can be expressed in the vehicle body-fixed frame.

The state vectors of the manipulator are described as following:

$$\boldsymbol{q} = [q_1, q_2, \cdots, q_n]^T;$$
$$\boldsymbol{p} = [\boldsymbol{p}_1^T, \boldsymbol{p}_2^T]^T; \quad \boldsymbol{p}_1 = [x_l, y_l, z_l]^T; \quad \boldsymbol{p}_2 = [\phi_l, \theta_l, \psi_l]^T;$$
$$\boldsymbol{p}_e = [\boldsymbol{p}_{e1}, \boldsymbol{p}_{e2}]^T; \quad \boldsymbol{p}_{e1} = [x_e, y_e, z_e]^T; \quad \boldsymbol{p}_{e2} = [\phi_e, \theta_e, \psi_e]^T;$$

The local velocity of end-effector relation with joint velocity is expressed by usual manipulator Jacobian and rotation matrix.

$$\begin{bmatrix}\dot{\boldsymbol{p}}_1 \\ \dot{\boldsymbol{p}}_2\end{bmatrix} = \begin{bmatrix}{}^B\boldsymbol{R}_0 \boldsymbol{J}_{m1}(\boldsymbol{q}) \\ {}^B\boldsymbol{R}_0 \boldsymbol{J}_{m2}(\boldsymbol{q})\end{bmatrix} \dot{\boldsymbol{q}} = \begin{bmatrix}\boldsymbol{J}_{m1}'(\boldsymbol{q}) \\ \boldsymbol{J}_{m2}'(\boldsymbol{q})\end{bmatrix} \dot{\boldsymbol{q}} \Longleftrightarrow \dot{\boldsymbol{p}} = \boldsymbol{J}_m'(\boldsymbol{q})\dot{\boldsymbol{q}}, \quad (2)$$

where the $\boldsymbol{J}_{m1}$ and $\boldsymbol{J}_{m2}$ represent the position and orientation Jacobian matrix from manipulator base frame to end-effector, respectively. ${}^B\boldsymbol{R}_0$ means the transformation matrix from the vehicle body fixed coordinate to the manipulator base coordinate. If the two coordinates coincides, ${}^B\boldsymbol{R}_0 = \boldsymbol{I}$, and $\boldsymbol{J}_m'$ is the manipulator Jacobian itself.

A. UVM Systems Kinematics

To find the overall velocity relation, we used the moving coordinate system relation as shown in Fig. 2. Then the end-effector velocity relative to the inertial frame has below relations:

$$\begin{aligned}\dot{\boldsymbol{p}}_e = \begin{bmatrix}\boldsymbol{v}_e \\ \boldsymbol{\omega}_e\end{bmatrix} &= \begin{bmatrix}\dot{\boldsymbol{\eta}}_1 + (\dot{\boldsymbol{p}}_1)_I + \dot{\boldsymbol{\eta}}_2 \times (\boldsymbol{p}_1)_I \\ \dot{\boldsymbol{\eta}}_2 + (\dot{\boldsymbol{p}}_2)_I\end{bmatrix} \\ &= \underbrace{\begin{bmatrix}\dot{\boldsymbol{\eta}}_1 + \dot{\boldsymbol{\eta}}_2 \times (\boldsymbol{p}_1)_I \\ \dot{\boldsymbol{\eta}}_2\end{bmatrix}}_{\text{velocity induced by vehicle}} + \underbrace{\begin{bmatrix}(\dot{\boldsymbol{p}}_1)_I \\ (\dot{\boldsymbol{p}}_2)_I\end{bmatrix}}_{\text{velocity induced by manipulator}},\end{aligned} \quad (3)$$

where $(\cdot)_I$ means the vector with respect to the inertial frame. The cross product term in the linear velocity induced by vehicle is expressed as:

$$\dot{\boldsymbol{\eta}}_2 \times (\boldsymbol{p}_1)_I = -\boldsymbol{S}((\boldsymbol{p}_1)_I)\dot{\boldsymbol{\eta}}_2 = -\boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1)\dot{\boldsymbol{\eta}}_2, \quad (4)$$

where $\boldsymbol{S}(\cdot)$ is skew-symmetric matrix defined such that $\boldsymbol{a} \times \boldsymbol{b} \triangleq \boldsymbol{S}(\boldsymbol{a})\boldsymbol{b}$. Then, the velocity induced by vehicle can be rewritten as following:

$$\begin{aligned}&\begin{bmatrix}\dot{\boldsymbol{\eta}}_1 - \boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1)\dot{\boldsymbol{\eta}}_2 \\ \dot{\boldsymbol{\eta}}_2\end{bmatrix} \\ &= \begin{bmatrix}\boldsymbol{I} & -\boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1) \\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix} \begin{bmatrix}\dot{\boldsymbol{\eta}}_1 \\ \dot{\boldsymbol{\eta}}_2\end{bmatrix} = \begin{bmatrix}\boldsymbol{I} & -\boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1) \\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix} \boldsymbol{J}_v \begin{bmatrix}\boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2\end{bmatrix} \\ &= \boldsymbol{J}_v \begin{bmatrix}\boldsymbol{I} & -\boldsymbol{J}_{v1}{}^T \boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1)\boldsymbol{J}_{v2} \\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix} \begin{bmatrix}\boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2\end{bmatrix} \triangleq \boldsymbol{J}_v \boldsymbol{J}_1 \boldsymbol{\nu}.\end{aligned} \quad (5)$$

The velocity induced by manipulator can be also rewritten with the same manner:

$$\begin{bmatrix}(\dot{\boldsymbol{p}}_1)_I \\ (\dot{\boldsymbol{p}}_2)_I\end{bmatrix} = \begin{bmatrix}\boldsymbol{J}_{v1}\dot{\boldsymbol{p}}_1 \\ \boldsymbol{J}_{v2}\dot{\boldsymbol{p}}_2\end{bmatrix} = \begin{bmatrix}\boldsymbol{J}_{v1}\boldsymbol{J}_{m1}' \\ \boldsymbol{J}_{v2}\boldsymbol{J}_{m2}'\end{bmatrix} \dot{\boldsymbol{q}} = \boldsymbol{J}_v \boldsymbol{J}_m' \dot{\boldsymbol{q}}. \quad (6)$$

Finally, we can obtain the end-effector velocity as below:

$$\dot{\boldsymbol{p}}_e = \boldsymbol{J}_v(\boldsymbol{\eta}) \begin{bmatrix}\boldsymbol{J}_1(\boldsymbol{\eta}, \boldsymbol{q}) & \boldsymbol{J}_m'(\boldsymbol{q})\end{bmatrix} \begin{bmatrix}\boldsymbol{\nu} \\ \dot{\boldsymbol{q}}\end{bmatrix}. \quad (7)$$

For the decentralized system control, we need both vehicle and end-effector velocity relations:

$$\begin{bmatrix}\dot{\boldsymbol{\eta}} \\ \dot{\boldsymbol{p}}_e\end{bmatrix} = \boldsymbol{J}(\boldsymbol{\eta}, \boldsymbol{q}) \begin{bmatrix}\boldsymbol{\nu} \\ \dot{\boldsymbol{q}}\end{bmatrix}, \quad (8)$$

where

$$\boldsymbol{J}(\boldsymbol{\eta}, \boldsymbol{q}) \triangleq \begin{bmatrix}\boldsymbol{J}_v(\boldsymbol{\eta}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{J}_v(\boldsymbol{\eta})\end{bmatrix} \begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{J}_1(\boldsymbol{\eta}, \boldsymbol{q}) & \boldsymbol{J}_m'(\boldsymbol{q})\end{bmatrix},$$
$$\boldsymbol{J}_1(\boldsymbol{\eta}, \boldsymbol{q}) \triangleq \begin{bmatrix}\boldsymbol{I} & -\boldsymbol{J}_{v1}{}^T \boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1)\boldsymbol{J}_{v2} \\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}.$$

And, the Jacobian inverse has the form

$$\begin{aligned}&\boldsymbol{J}^{\#}(\boldsymbol{\eta}, \boldsymbol{q}) = \\ &\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{J}_m'^{\#}(\boldsymbol{q})\boldsymbol{J}_1(\boldsymbol{\eta}, \boldsymbol{q}) & \boldsymbol{J}_m'^{\#}(\boldsymbol{q})\end{bmatrix} \begin{bmatrix}\boldsymbol{J}_v^{-1}(\boldsymbol{\eta}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{J}_v^{-1}(\boldsymbol{\eta})\end{bmatrix},\end{aligned} \quad (9)$$

where $\boldsymbol{J}_m'^{\#}(\boldsymbol{q})$ is Moore-Penrose pseudo-inverse of $\boldsymbol{J}_m'$. The form of $\boldsymbol{J}_m'^{\#}(\boldsymbol{q})$ is dependant on whether the manipulator is redundant or non-redundant.

B. UVM Systems Dynamics

To control dynamic system properly, we need to understand its dynamics clearly. The UVM system is generally a highly coupled and complicated system, the dynamic formulation itself is not an easy work. For the moving base system which is not fixed in an inertial frame, it is not convenient to derive the dynamic formulation using the Lagrangian in terms of the velocities expressed in a body-fixed frame. In UVM systems, the sensor information and actuator forces are exerted on body-fixed frame, so it is more natural to write up the dynamics using body-fixed velocities. To supplement this, we used quasi-Lagrange approach [6] and partitioned dynamic equations. The quasi-Lagrange method can give the equations of motion in terms of the body-fixed velocities. And, for the decentralized control of the vehicle and manipulator, the partitioned submatrices of dynamic terms are absolutely needed to design the controller. Using this partitioned dynamics, we can easily deal with the coupling effect between two subsystems. Moreover, it is possible to use the already derived dynamic equations for each subsystems.

The hydrodynamic forces and moments of vehicle and manipulator have been used from previous works [7] and omitted here.

1) Quasi-Lagrange System: For the underwater vehicle, Fossen [4] derived the dynamics using quasi-velocities. Meirovitch and Sarkar [1,8] also derived the dynamics for the hybrid system which is composed of both quasi-velocities and generalized-velocities.

The quasi-Lagrangian is defined as

$$\overline{L} = \overline{L}[\boldsymbol{\delta}, \boldsymbol{\mu}], \tag{10}$$

where $\boldsymbol{\delta} = [\boldsymbol{\eta}^T, \boldsymbol{q}^T]^T$ and $\boldsymbol{\mu} = [\boldsymbol{\nu}^T, \dot{\boldsymbol{q}}^T]^T$. Then, we obtain differential equation as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \overline{L}}{\partial \boldsymbol{\mu}}\right) + \boldsymbol{A}\frac{\partial \overline{L}}{\partial \boldsymbol{\mu}} - \boldsymbol{B}^T\frac{\partial \overline{L}}{\partial \boldsymbol{\delta}} = \boldsymbol{\tau}, \tag{11}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{S}(\boldsymbol{\nu}_2) & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{S}(\boldsymbol{\nu}_1) & \boldsymbol{S}(\boldsymbol{\nu}_2) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{J}_{v1}(\boldsymbol{\eta}_2) & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{J}_{v2}(\boldsymbol{\eta}_2) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix},$$

and $\boldsymbol{\tau} = \boldsymbol{B}^T\boldsymbol{Q}$.

Next, we wish to develop an expression for the partitioned dynamics. In Eq. (11), $\boldsymbol{A}\frac{\partial \overline{L}}{\partial \boldsymbol{\mu}}$ has another expression as:

$$\boldsymbol{A}\frac{\partial \overline{L}}{\partial \boldsymbol{\mu}} = \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_1}) & \boldsymbol{0} \\ -\boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_1}) & -\boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_2}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\boldsymbol{\mu} \triangleq \boldsymbol{A}'\boldsymbol{\mu}. \tag{12}$$

For convenience, rigid body kinetic energy is firstly considered. Then, $\overline{L} = \overline{T}$ and $\overline{T}$ is given as

$$\overline{T} = \frac{1}{2}\boldsymbol{\mu}^T\boldsymbol{M}\boldsymbol{\mu}, \tag{13}$$

where $\boldsymbol{M}$ is the inertia matrix of UVM system:

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} & \boldsymbol{M}_{13} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} & \boldsymbol{M}_{23} \\ \boldsymbol{M}_{31} & \boldsymbol{M}_{32} & \boldsymbol{M}_{33} \end{bmatrix}, \tag{14}$$

and then

$$\begin{aligned} \overline{T} = \frac{1}{2}(\quad & \boldsymbol{\nu}_1^T\boldsymbol{M}_{11}\boldsymbol{\nu}_1 + \boldsymbol{\nu}_1^T\boldsymbol{M}_{12}\boldsymbol{\nu}_2 + \boldsymbol{\nu}_1^T\boldsymbol{M}_{13}\dot{\boldsymbol{q}} \\ & + \boldsymbol{\nu}_2^T\boldsymbol{M}_{21}\boldsymbol{\nu}_1 + \boldsymbol{\nu}_2^T\boldsymbol{M}_{22}\boldsymbol{\nu}_2 + \boldsymbol{\nu}_2^T\boldsymbol{M}_{23}\dot{\boldsymbol{q}} \\ & + \dot{\boldsymbol{q}}^T\boldsymbol{M}_{31}\boldsymbol{\nu}_1 + \dot{\boldsymbol{q}}^T\boldsymbol{M}_{32}\boldsymbol{\nu}_2 + \dot{\boldsymbol{q}}^T\boldsymbol{M}_{33}\dot{\boldsymbol{q}} \quad). \end{aligned} \tag{15}$$

Eq. (14) is expressed as the partitioned submatrices which are related to $\boldsymbol{\nu}_1$, $\boldsymbol{\nu}_2$, and $\dot{\boldsymbol{q}}$, respectively. The inertia matrix has symmetric property due to the action and the reaction principle, and positive definite($\boldsymbol{M} > 0$). $\boldsymbol{M}_{33}$ represent the manipulator inertia matrix itself($\boldsymbol{M}_m$), because the lower part of the inertia matrix does not affect the upper part of the inertia matrix. And, Eq. (11) can be rewritten as

$$\boldsymbol{M}\dot{\boldsymbol{\mu}} + \boldsymbol{C}(\boldsymbol{q}, \boldsymbol{\mu})\boldsymbol{\mu} = \boldsymbol{\tau}. \tag{16}$$

Now, we have to find the Coriolis and Centripetal Matrix.

***Theorem** 1 : Coriolis and Centripetal Matrix from Inertia Matrix* If the inertia matrix is given as Eq. (14), then we can always parameterize the Coriolis and centripetal matrix such that $\dot{\boldsymbol{M}} - (\boldsymbol{C} + \boldsymbol{C}^T) = \boldsymbol{0}$ by defining:

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} & \boldsymbol{C}_{13} \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} & \boldsymbol{C}_{23} \\ \boldsymbol{C}_{31} & \boldsymbol{C}_{32} & \boldsymbol{C}_{33} \end{bmatrix} = \left[\begin{array}{c|c|c} \boldsymbol{\Gamma}_1 & \boldsymbol{\Gamma}_2 & \boldsymbol{\Gamma}_3 \end{array} \right], \tag{17}$$

where

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} \boldsymbol{0} \\ -\boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_1}) \\ \dot{\boldsymbol{M}}_{31} - \left\lfloor \frac{\partial \boldsymbol{M}_{31}}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} \right\rfloor \end{bmatrix},$$

$$\boldsymbol{\Gamma}_2 = \begin{bmatrix} \dot{\boldsymbol{M}}_{12} - \boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_1}) \\ \frac{1}{2}\dot{\boldsymbol{M}}_{22} - \boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_2}) \\ \dot{\boldsymbol{M}}_{32} - \left\lfloor \frac{\partial \boldsymbol{M}_{32}}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} \right\rfloor - \left\lfloor \frac{\partial \boldsymbol{M}_{12}}{\partial \boldsymbol{q}}\boldsymbol{\nu}_1 \right\rfloor - \frac{1}{2}\left\lfloor \frac{\partial \boldsymbol{M}_{22}}{\partial \boldsymbol{q}}\boldsymbol{\nu}_2 \right\rfloor \end{bmatrix},$$

$$\boldsymbol{\Gamma}_3 = \begin{bmatrix} \left\lfloor \frac{\partial \boldsymbol{M}_{13}}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} \right\rfloor \\ \left\lfloor \frac{\partial \boldsymbol{M}_{23}}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} \right\rfloor + \left\lfloor \frac{\partial \boldsymbol{M}_{21}}{\partial \boldsymbol{q}}\boldsymbol{\nu}_1 \right\rfloor + \frac{1}{2}\left\lfloor \frac{\partial \boldsymbol{M}_{22}}{\partial \boldsymbol{q}}\boldsymbol{\nu}_2 \right\rfloor \\ \boldsymbol{C}_m(\boldsymbol{q}, \dot{\boldsymbol{q}}) \end{bmatrix}.$$

Here, $\boldsymbol{C}_m(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is Coriolis and Centripetal matrix for the manipulator, which already satisfies $\dot{\boldsymbol{M}}_m - (\boldsymbol{C}_m + \boldsymbol{C}_m^T) = \boldsymbol{0}$ and $\boldsymbol{C}_m\dot{\boldsymbol{q}} = \frac{1}{2}\dot{\boldsymbol{M}}_m\dot{\boldsymbol{q}}$. And, the symbol, $\left\lfloor \frac{\partial \boldsymbol{A}}{\partial \boldsymbol{q}}\boldsymbol{a} \right\rfloor$, means

$$\left\lfloor \frac{\partial \boldsymbol{A}}{\partial \boldsymbol{q}}\boldsymbol{a} \right\rfloor_{ij} = \Sigma_k^n \frac{\partial A_{kj}}{\partial q_i}a_k, \qquad n \text{ is dimension of vector } \boldsymbol{a}.$$

□

The proof of Theorem 1 is omitted here.

The added mass(inertia) and added Coriolis and Centripetal forces can be dealt similarly. The gravitational forces and buoyancy are also added by energy relations as $\overline{L} = \overline{T} + \overline{V}$, and, the drag forces can be considered with iterative Newton-Euler method. The details are omitted here.

2) Partitioned Dynamics: Now, the whole UVM system equations of motion will be derived, and then divided into several meaningful terms.

Similar to several other researches, it is clear that the whole system equations of motion can be written as the following equations:

$$\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{\mu}} + \boldsymbol{C}(\boldsymbol{q}, \boldsymbol{\mu})\boldsymbol{\mu} + \boldsymbol{D}(\boldsymbol{q}, \boldsymbol{\mu})\boldsymbol{\mu} + \boldsymbol{g}(\boldsymbol{\eta}, \boldsymbol{q}) = \boldsymbol{\tau}, \tag{18}$$

where

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{M}_{11}(\boldsymbol{q}) & \boldsymbol{M}_{12}(\boldsymbol{q}) \\ \boldsymbol{M}_{21}(\boldsymbol{q}) & \boldsymbol{M}_{22}(\boldsymbol{q}) \end{bmatrix}, \tag{19}$$

$$\boldsymbol{C}(\boldsymbol{q}, \boldsymbol{\mu}) = \begin{bmatrix} \boldsymbol{C}_{11}(\boldsymbol{q}, \boldsymbol{\mu}) & \boldsymbol{C}_{12}(\boldsymbol{q}, \boldsymbol{\mu}) \\ \boldsymbol{C}_{21}(\boldsymbol{q}, \boldsymbol{\mu}) & \boldsymbol{C}_{22}(\boldsymbol{q}, \boldsymbol{\mu}) \end{bmatrix}, \tag{20}$$

$$\boldsymbol{D}(\boldsymbol{q}, \boldsymbol{\mu}) = \begin{bmatrix} \boldsymbol{D}_{11}(\boldsymbol{q}, \boldsymbol{\mu}) & \boldsymbol{D}_{12}(\boldsymbol{q}, \boldsymbol{\mu}) \\ \boldsymbol{D}_{21}(\boldsymbol{q}, \boldsymbol{\mu}) & \boldsymbol{D}_{22}(\boldsymbol{q}, \boldsymbol{\mu}) \end{bmatrix}, \tag{21}$$

$$\boldsymbol{g}(\boldsymbol{q}, \boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{g}_1(\boldsymbol{\eta}, \boldsymbol{q}) \\ \boldsymbol{g}_2(\boldsymbol{\eta}, \boldsymbol{q}) \end{bmatrix}, \tag{22}$$

$$\boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{\tau}_v \\ \boldsymbol{\tau}_m \end{bmatrix}. \tag{23}$$

Notice that the subscripts of matrices are different from those in previous section.

For the advanced and systematic control, the equations have to be divided by reasonable way. The vehicle and manipulator dynamics and interaction dynamics will be considered.

The vehicle and manipulator dynamics with added dynamics to the vehicle are shown as following:

$$\begin{bmatrix} \boldsymbol{M}_v & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \dot{\boldsymbol{\mu}} + \begin{bmatrix} \boldsymbol{C}_v(\boldsymbol{\nu}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{D}_v(\boldsymbol{\nu}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{g}_v(\boldsymbol{\eta}) \\ \boldsymbol{0} \end{bmatrix} = \boldsymbol{\tau}_1, \tag{24}$$

$$\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_m(\boldsymbol{q}) \end{bmatrix} \dot{\boldsymbol{\mu}} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C}_m(\boldsymbol{q}, \dot{\boldsymbol{q}}) \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D}_m(\boldsymbol{q}, \dot{\boldsymbol{q}}) \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{g}_m(\boldsymbol{\eta}, \boldsymbol{q}) \end{bmatrix} = \boldsymbol{\tau}_2. \tag{25}$$

The effect of added dynamics to the vehicle by manipulator has to be considered. This dynamic effect can be introduced as following:

$$\begin{bmatrix} \boldsymbol{M}_s(\boldsymbol{q}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \dot{\boldsymbol{\mu}} + \begin{bmatrix} \boldsymbol{C}_s(\boldsymbol{q}, \boldsymbol{\nu}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{D}_s(\boldsymbol{q}, \boldsymbol{\nu}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{g}_s(\boldsymbol{\eta}, \boldsymbol{q}) \\ \boldsymbol{0} \end{bmatrix} = \boldsymbol{\tau}_3. \tag{26}$$

In addition, the interaction dynamics between two subsystems are expressed in the form:

$$\begin{bmatrix} \boldsymbol{0} & \boldsymbol{M}_{12}(\boldsymbol{q}) \\ \boldsymbol{M}_{12}{}^T(\boldsymbol{q}) & \boldsymbol{0} \end{bmatrix} \dot{\boldsymbol{\mu}} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{C}_{12}(\boldsymbol{q}, \boldsymbol{\mu}) \\ \boldsymbol{C}_{21}(\boldsymbol{q}, \boldsymbol{\mu}) & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{D}_{12}(\boldsymbol{q}, \boldsymbol{\mu}) \\ \boldsymbol{D}_{21}(\boldsymbol{q}, \boldsymbol{\mu}) & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} = \boldsymbol{\tau}_4, \tag{27}$$

where the inertia matrix is symmetric due to the characteristics of Lagrangian system, and it can be easily proved. Finally, Eq. (18) can be expressed as follows

$$\boldsymbol{\tau} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + \boldsymbol{\tau}_3 + \boldsymbol{\tau}_4. \tag{28}$$

Eqs. (19) – (22) can be rewritten by meaningful terms using the above expression as:

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{M}_v + \boldsymbol{M}_s(\boldsymbol{q}) & \boldsymbol{M}_{12}(\boldsymbol{q}) \\ \boldsymbol{M}_{12}{}^T(\boldsymbol{q}) & \boldsymbol{M}_m(\boldsymbol{q}) \end{bmatrix}, \tag{29}$$

$$\boldsymbol{C}(\boldsymbol{q}, \boldsymbol{\mu}) = \begin{bmatrix} \boldsymbol{C}_v(\boldsymbol{\nu}) + \boldsymbol{C}_s(\boldsymbol{q}, \boldsymbol{\nu}) & \boldsymbol{C}_{12}(\boldsymbol{q}, \boldsymbol{\mu}) \\ \boldsymbol{C}_{21}(\boldsymbol{q}, \boldsymbol{\mu}) & \boldsymbol{C}_m(\boldsymbol{q}, \dot{\boldsymbol{q}}) \end{bmatrix}, \tag{30}$$

$$\boldsymbol{D}(\boldsymbol{q}, \boldsymbol{\mu}) = \begin{bmatrix} \boldsymbol{D}_v(\boldsymbol{\nu}) + \boldsymbol{D}_s(\boldsymbol{q}, \boldsymbol{\nu}) & \boldsymbol{D}_{12}(\boldsymbol{q}, \boldsymbol{\mu}) \\ \boldsymbol{D}_{21}(\boldsymbol{q}, \boldsymbol{\mu}) & \boldsymbol{D}_m(\boldsymbol{q}, \dot{\boldsymbol{q}}) \end{bmatrix}, \tag{31}$$

$$\boldsymbol{g}(\boldsymbol{q}, \boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{g}_v(\boldsymbol{\eta}) + \boldsymbol{g}_s(\boldsymbol{\eta}, \boldsymbol{q}) \\ \boldsymbol{g}_m(\boldsymbol{\eta}, \boldsymbol{q}) \end{bmatrix}. \tag{32}$$

Since we already considered $\boldsymbol{M}(\boldsymbol{q})$, $\boldsymbol{C}(\boldsymbol{q}, \boldsymbol{\mu})$ and $\boldsymbol{g}(\boldsymbol{q}, \boldsymbol{\eta})$ in the previous subsections, only unknown terms is the drag force part.

When the vehicle has an initial velocity and acceleration, $\boldsymbol{\nu}$ and $\dot{\boldsymbol{\nu}}$, then:

$$\begin{aligned} &\boldsymbol{M}_m(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}_m(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{D}_m(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}_m(\boldsymbol{\eta}, \boldsymbol{q}) + \\ &\quad \boldsymbol{M}_{12}{}^T(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{21}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}_{21}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau}_m. \end{aligned} \tag{33}$$

Eq. (33) gives $\boldsymbol{D}_{21}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu})$.

For the vehicle, the motion of manipulator will give interaction effects. These forces and moments, $\boldsymbol{\sigma}_m = [\boldsymbol{\sigma}_f{}^T, \boldsymbol{\sigma}_n{}^T]^T$, which are derived from the last iteration of the manipulator of the Newton-Euler method, can be calculated as:

$$\begin{aligned} \boldsymbol{\sigma}_f &= {}^B\boldsymbol{R}_0{}^0\boldsymbol{f}_0, \\ \boldsymbol{\sigma}_n &= {}^B\boldsymbol{R}_0{}^0\boldsymbol{n}_0 + {}^B\boldsymbol{p}_0 \times {}^B\boldsymbol{R}_0{}^0\boldsymbol{f}_0. \end{aligned} \tag{34}$$

It can be categorized as follows:

$$\begin{aligned} &\boldsymbol{M}_s(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{C}_s(\boldsymbol{q}, \boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}_s(\boldsymbol{q}, \boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}_s(\boldsymbol{\eta}, \boldsymbol{q}) + \\ &\quad \boldsymbol{M}_{12}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}_{12}(\boldsymbol{q}, \boldsymbol{\mu})\dot{\boldsymbol{q}} + \boldsymbol{D}_{12}(\boldsymbol{q}, \boldsymbol{\mu})\dot{\boldsymbol{q}} = \begin{bmatrix} \boldsymbol{\sigma}_f \\ \boldsymbol{\sigma}_n \end{bmatrix}. \end{aligned} \tag{35}$$

From Eq. (35), we can fill up the last unknown terms, $\boldsymbol{D}_s(\boldsymbol{q}, \boldsymbol{\nu})$ and $\boldsymbol{D}_{12}(\boldsymbol{q}, \boldsymbol{\mu})$.

Finally, the equations of motion for total system in the earth-fixed reference frame can be written in the form:

$$\boldsymbol{M}_I(\boldsymbol{q}, \boldsymbol{\eta})\ddot{\boldsymbol{\xi}} + \boldsymbol{c}_I(\boldsymbol{q}, \boldsymbol{\eta}, \boldsymbol{\mu}) + \boldsymbol{d}_I(\boldsymbol{q}, \boldsymbol{\eta}, \boldsymbol{\mu}) + \boldsymbol{g}_I(\boldsymbol{q}, \boldsymbol{\eta}) = \boldsymbol{\tau}_I, \tag{36}$$

where $\boldsymbol{\xi} = [\boldsymbol{\eta}^T, \boldsymbol{p}_e^T]^T$.

III. Two-time Scale Control

In the underwater manipulation, the vehicle control is not effective, because high frequency interaction disturbance by manipulator can not be completely compensated by small bandwidth vehicle actuator. It may also cause large energy consumption. Therefore, it is desirable that the vehicle remains a passive system or just use a simple P-type controller. In this case, the residual error of the end-effector induced by vehicle motion should be compensated using redundancy.

We propose here an efficient control scheme for UVM system considering the system characteristics, which is active damping control with two-time scale approach in operational space. The basic idea is to have passive vehicle or simple use of P-type controller for vehicle and two-time scale controller for the manipulator to control the total system. Detail derivation is shown in the following section.

A. Partially Linearized Form

To control the whole UVM system as a decentralized system, a partially linearized form is useful. The overall system can be represented as

$$\begin{bmatrix} \boldsymbol{M}_{11}(\boldsymbol{q}) & \boldsymbol{M}_{12}(\boldsymbol{q}) \\ \boldsymbol{M}_{12}{}^T(\boldsymbol{q}) & \boldsymbol{M}_{22}(\boldsymbol{q}) \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\nu}} \\ \ddot{\boldsymbol{q}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{h}_1(\boldsymbol{\eta}, \boldsymbol{q}, \boldsymbol{\mu}) \\ \boldsymbol{h}_2(\boldsymbol{\eta}, \boldsymbol{q}, \boldsymbol{\mu}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\tau}_v \\ \boldsymbol{\tau}_m \end{bmatrix}, \tag{37}$$

where the $\boldsymbol{h}_1(\boldsymbol{\eta}, \boldsymbol{q}, \mu)$ and $\boldsymbol{h}_2(\boldsymbol{\eta}, \boldsymbol{q}, \boldsymbol{\mu})$ are nonlinear functions which contain the Coriolis and Centripetal, drag forces and total buoyancy forces.

If the P-type vehicle controller in the operational space is given as

$$\boldsymbol{\tau}_v = -\boldsymbol{J}_v{}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_v\boldsymbol{\eta}, \tag{38}$$

where $\boldsymbol{K}_v$ is the proportional gain matrix. For the passive vehicle system, we just set the proportional gain matrix as $\boldsymbol{0}$.

Then, we can invoke $\dot{\boldsymbol{\nu}}$ as:

$$\dot{\boldsymbol{\nu}} = -\boldsymbol{M}_{11}{}^{-1}(\boldsymbol{q})\left(\boldsymbol{M}_{12}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}_1(\boldsymbol{\eta}, \boldsymbol{q}, \boldsymbol{\mu}) + \boldsymbol{J}_v{}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_v\boldsymbol{\eta}\right). \tag{39}$$

Substituting Eq. (39) to Eq. (37), then

$$\boldsymbol{\tau}_m = \boldsymbol{M}^*\ddot{\boldsymbol{q}} + \boldsymbol{h}^*, \tag{40}$$

where

$$\begin{aligned} \boldsymbol{M}^* &= \boldsymbol{M}_{22}(\boldsymbol{q}) - \boldsymbol{M}_{12}{}^T(\boldsymbol{q})\boldsymbol{M}_{11}{}^{-1}(\boldsymbol{q})\boldsymbol{M}_{12}(\boldsymbol{q}), \\ \boldsymbol{h}^* &= \boldsymbol{h}_2(\boldsymbol{\eta}, \boldsymbol{q}, \boldsymbol{\mu}) \\ &\quad - \boldsymbol{M}_{12}{}^T(\boldsymbol{q})\boldsymbol{M}_{11}{}^{-1}(\boldsymbol{q})\{\boldsymbol{h}_1(\boldsymbol{\eta}, \boldsymbol{q}, \boldsymbol{\mu}) + \boldsymbol{J}_v{}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_v\boldsymbol{\eta}\}. \end{aligned} \tag{41}$$

Then, for the manipulator controller, a feedback linearizing control law is obtained as follows:

$$\boldsymbol{\tau}_m = \boldsymbol{M}^*\boldsymbol{u} + \boldsymbol{h}^*, \tag{42}$$

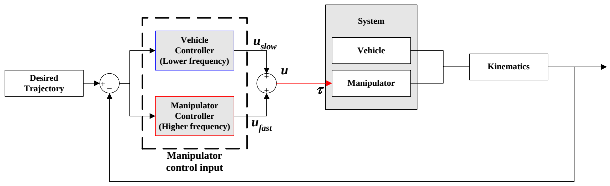


Fig. 3. Block diagram of the control system

where $\boldsymbol{u} \in \Re^m$ is an additional control input to be specified later. The substitution of Eq. (42) into Eq. (37) yields a partially linearized system as follows:

$$\boldsymbol{M}_{11}\dot{\boldsymbol{\nu}} + \boldsymbol{h}_1 + \boldsymbol{J}_v{}^{-1}\boldsymbol{K}_v\boldsymbol{\eta} \;=\; -\boldsymbol{M}_{12}\boldsymbol{u}, \tag{43}$$

$$\ddot{\boldsymbol{q}} \;=\; \boldsymbol{u}. \tag{44}$$

To implement this controller on a passive system, we also need the controllability definition [9–11]. These will be discussed in the following sections.

B. Controllability Condition

To ensure the manipulator control input to be able to control the vehicle, we have to check the controllability condition for the vehicle control input, $-\boldsymbol{M}_{12}(\boldsymbol{q})\boldsymbol{u}$. To deliever effective acceleration to the passive vehicle from the active manipulator, a *strong inertial coupling* between the joints is required [9].

$$rank(\boldsymbol{M}_{12}(\boldsymbol{q})) = l \qquad \text{for all} \quad \boldsymbol{q} \in \Re^n, \tag{45}$$

where l is the dimension of vehicle state, $\boldsymbol{\eta} \in \Re^l$. Note that strong inertial coupling requires $n \geq l$, i.e., that the number of manipulator degrees of freedom is at least as large as the number of vehicle degrees of freedom.

Based on the above analysis, it is now possible to design a two-time scale controller.

C. Two-time Scale Control

A composite (fast and slow time scale) controller is designed based on the partially decoupled model Eqs. (43) and (44). Recall that the control objective is to determine the control input $\boldsymbol{u}$ that stabilizes both Eqn. (43) and (44). Define the control input, $\boldsymbol{u}$, to have two parts as shown in Fig. 3:

$$\boldsymbol{u} = \boldsymbol{u}_{slow} + \boldsymbol{u}_{fast}. \tag{46}$$

Using two-time scale theory, $\boldsymbol{u}_{fast}$ and $\boldsymbol{u}_{slow}$ are chosen in two different time scales. It is intended that $\boldsymbol{u}_{slow}$ affects mainly on Eq. (43), slow vehicle motion and $\boldsymbol{u}_{fast}$ mainly on Eq. (44), fast decoupled manipulator dynamics. For $\boldsymbol{u}_{fast}$, a typical linear tracking controller is proposed with high gains such that the control bandwidth is much higher than the natural frequency of the vehicle motion.

From Eqn. (8) and (9), we can obtain the manipulator joint acceleration as following :

$$\begin{aligned}\ddot{\boldsymbol{q}} = \boldsymbol{J}_m{}'^{\#}[\boldsymbol{J}_v{}^{-1}\{\ddot{\boldsymbol{p}}_e - (\dot{\boldsymbol{J}}_v\boldsymbol{J}_1 + \boldsymbol{J}_v\dot{\boldsymbol{J}}_1)\boldsymbol{\nu} \\ - (\dot{\boldsymbol{J}}_v\boldsymbol{J}_m{}' + \boldsymbol{J}_v\dot{\boldsymbol{J}}_m{}')\dot{\boldsymbol{q}}\} - \boldsymbol{J}_1\dot{\boldsymbol{\nu}}].\end{aligned} \tag{47}$$

Then, $\boldsymbol{u}_{fast}$ can be chosen as

$$\begin{aligned}\boldsymbol{u}_{fast} = \quad & \boldsymbol{J}_m{}'^{\#}[\boldsymbol{J}_v{}^{-1}\{(\ddot{\boldsymbol{p}}_e)_{ref} - (\dot{\boldsymbol{J}}_v\boldsymbol{J}_1 + \boldsymbol{J}_v\dot{\boldsymbol{J}}_1)\boldsymbol{\nu} \\ & -(\dot{\boldsymbol{J}}_v\boldsymbol{J}_m{}' + \boldsymbol{J}_v\dot{\boldsymbol{J}}_m{}')\dot{\boldsymbol{q}}\} - \boldsymbol{J}_1\dot{\boldsymbol{\nu}}],\end{aligned} \tag{48}$$

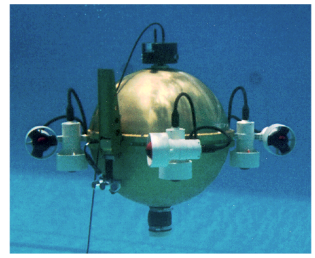
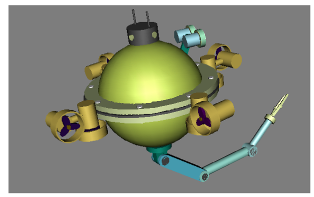
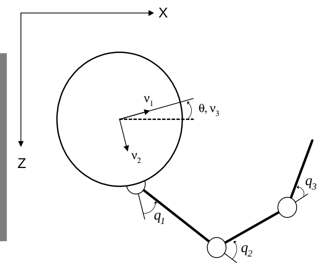


Fig. 4. ODIN in UH



(a) Concept of ODIN and manipulator



(b) Schematic diagram of the planar ODIN and manipulator system

Fig. 5. Example

where

$$(\ddot{\boldsymbol{p}}_e)_{ref} = (\ddot{\boldsymbol{p}}_e)_d + \boldsymbol{K}_D((\dot{\boldsymbol{p}}_e)_d - \dot{\boldsymbol{p}}_e) + \boldsymbol{K}_P((\boldsymbol{p}_e)_d - \boldsymbol{p}_e). \tag{49}$$

$\boldsymbol{K}_P$ and $\boldsymbol{K}_D$ are proportional and derivative gain matrix, respectively. Then, the closed loop error dynamics for the manipulator in the operational space becomes

$$\boldsymbol{J}_m{}'^{\#}(\boldsymbol{q})\boldsymbol{J}_v{}^{-1}(\boldsymbol{\eta})\{\ddot{\boldsymbol{e}}_p + \boldsymbol{K}_D\dot{\boldsymbol{e}}_p + \boldsymbol{K}_P\boldsymbol{e}_p\} = -\boldsymbol{u}_{slow}, \tag{50}$$

where $\boldsymbol{e}_p$ is defined by the position error, $(\boldsymbol{p}_e)_d - \boldsymbol{p}_e$. If $\boldsymbol{u}_{slow}$ changes slowly compared to the fast error dynamics, it can be treated as more like a constant. Then the error dynamics of Eq. (50) converge to be very small with positive definite matrices $\boldsymbol{K}_P$ and $\boldsymbol{K}_D$.

Now, the slow control input, $\boldsymbol{u}_{slow}$, is defined for the vehicle in Eq. (43) as

$$\boldsymbol{u}_{slow} = \quad \boldsymbol{M}_{12}{}^{\#}\{\boldsymbol{M}_{11}\boldsymbol{J}_v{}^{-1}((\ddot{\boldsymbol{\eta}})_{ref} + \dot{\boldsymbol{J}}_v\boldsymbol{\nu}) - \boldsymbol{h}_1\}, \tag{51}$$

where

$$(\ddot{\boldsymbol{\eta}})_{ref} = \boldsymbol{K}_d\dot{\boldsymbol{\eta}}. \tag{52}$$

Only derivative feedback of the vehicle with gain $\boldsymbol{K}_d$ is added to hold the vehicle velocity not increasing.

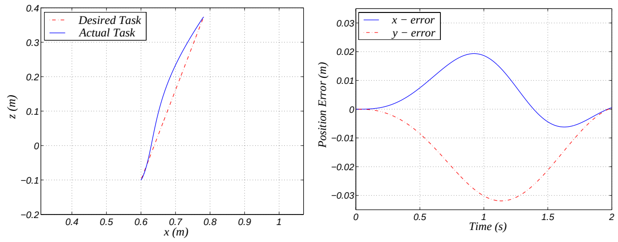
Then, after applying composite controller and let $\boldsymbol{J}_v{}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_v$ as $\boldsymbol{M}_{11}(\boldsymbol{q})\boldsymbol{J}_v{}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_p$, Eq. (43) becomes

$$\begin{aligned}\boldsymbol{M}_{11}(\boldsymbol{q})\boldsymbol{J}_v{}^{-1}(\boldsymbol{\eta})\{\ddot{\boldsymbol{e}}_\eta + \boldsymbol{K}_d\dot{\boldsymbol{e}}_\eta + \boldsymbol{K}_p\boldsymbol{e}_\eta\} &= -\boldsymbol{M}_{12}(\boldsymbol{q})\boldsymbol{u}_{fast}, \\ &\approx -\boldsymbol{M}_{12}(\boldsymbol{q})\ddot{\boldsymbol{q}}_{ref}.\end{aligned} \tag{53}$$

It can be easily shown that the second line of Eq. (53) is bounded since $\ddot{\boldsymbol{q}}_{ref}$ is bounded. Therefore, as long as $\boldsymbol{K}_d$ and $\boldsymbol{K}_p$ are chosen as a positive definite matrix, Eq. (53) remains stable.

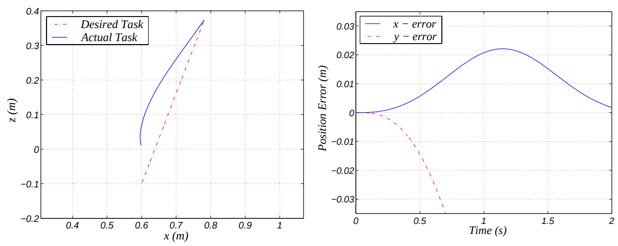
IV. Simulation

Figure 5(a) shows the ODIN in the University of Hawaii. Its symmetric structure enables us to easily get the dynamic



(a) Task in the x-z plane (b) Task errors

Fig. 6. Simulation results 1 with $\boldsymbol{u} = \boldsymbol{u}_{fast}$ ($\boldsymbol{K}_P = \text{diag}\{200, 300\}$ and $\boldsymbol{K}_D = \text{diag}\{10, 20\}$)



(a) Task in the x-z plane (b) Task errors

Fig. 7. Simulation results 2 with $\boldsymbol{u} = \boldsymbol{u}_{fast}$ ($\boldsymbol{K}_P = \text{diag}\{400, 500\}$ and $\boldsymbol{K}_D = \text{diag}\{40, 50\}$)

parameters with little modeling uncertainties. Now, we consider the manipulator as shown in Fig. 5(a) attached to ODIN.

To show the performance of the proposed controller, firstly, we consider only simple 2-D case. Then, there exist only surge, heave and pitch motions in vehicle movement as Fig. 5(b). The attached 3-DOF manipulator gives the strong inertial coupling condition, if $\boldsymbol{M}_{12}$ does not lose a rank. In the proposed system, $\boldsymbol{M}_{12}$ always has full rank except the case in which the manipulator meets the kinematic singularity. Hence, it is important to control precisely not to fall into kinematic singularities.

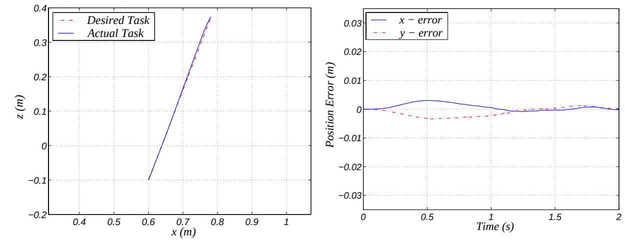
We performed simulation in two ways. At the first case, we only used $\boldsymbol{u} = \boldsymbol{u}_{fast}$, and at the second case, $\boldsymbol{u} = \boldsymbol{u}_{fast} + \boldsymbol{u}_{slow}$. Here, the fast and slow time scale are used as 1kHz and 10Hz, respectively.

Figure 6 and 7 show the simulation results with single time scale, $\boldsymbol{u}_{fast}$ only. Figure 7 shows unsatisfactory result using higher gains than those of Fig. 6. From these results, it is recognized that we can not increase the controller gain to reduce the errors due to the interaction effect. The system will go unstable if we keep increasing the gains. These results were caused by the remaining vehicle dynamics as described in Eq. (43), which is an interaction effect.

However, in Fig. 8 ($\boldsymbol{u} = \boldsymbol{u}_{fast} + \boldsymbol{u}_{slow}$), the system error is decreased up to 10 times than Fig 6. According to these results, we can say that the proposed controller is very effective under interaction disturbances.

V. Conclusions

In this paper, the kinematic and dynamic modeling for UVM systems are analyzed thoroughly. It enables us to design decentralized controller under the interaction between two subsystems. Nextly, two-time scale controller is proposed for the passive vehicle system based on the previous dynamic analysis. Using different time scale controller, fast time scale controller is used for the manipulator and slow time scale controller for the vehicle. Simulation results show the excellent performance over simple time scale controller.



(a) Task in the x-z plane (b) Task errors

Fig. 8. Simulation results with $\boldsymbol{u} = \boldsymbol{u}_{fast} + \boldsymbol{u}_{slow}$ ($\boldsymbol{K}_P = \text{diag}\{400, 500\}$, $\boldsymbol{K}_D = \text{diag}\{40, 50\}$ and $\boldsymbol{K}_d = \text{diag}\{20, 20, 40\}$)

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VII. References

[1] N. Sarkar and T. K. Podder, "Coordinated motion planning and control of autonomous underwater vehicle-manipulator systems subject to drag optimization," *IEEE J. of Oceanic Engineering*, vol. 26, pp. 228–239, Apr. 2001.

[2] S. McMillan, D. E. Orin, and R. B. McGhee, "Efficient dynamic simulation of an underwater vehicle with a robotic manipulator," *IEEE Trans. on Systems, Man, and Cybernetics*, vol. 25, pp. 1194–1204, Aug. 1995.

[3] C. Canudas, E. O. Diaz, and M. Perrier, "Nonlinear control of an underwater vehicle/manipulator with composite dynamics," *IEEE Trans. on Control Systems Technology*, vol. 8, pp. 948–960, Nov. 2000.

[4] T. I. Fossen, *Guidance and Control of Ocean Vehicles*. John Wiley & Sons Ltd., 1994.

[5] J. Yuh, "Modeling and control of underwater robotic vehicles," *IEEE Trans. on Systems, Man, and Cybernetics*, vol. 20, no. 6, pp. 1475–1483, 1990.

[6] L. Meirovitch, *Dynamics and Control of Structures*. John Wiley & Sons, 1990.

[7] I. Schjølberg and T. I. Fossen, "Modelling and control of underwater vehicle-manipulator systems," in *Proc. 3rd Conf. Marine Craft Maneuvering and Control*, (Southampton, U.K.), pp. 45–57, 1994.

[8] L. Meirovitch and T. Stemple, "Hybrid equation of motion for flexible multibody systems using quasicoordinates," *Journal of Guidance, Control, and Dynamics*, vol. 18, pp. 678–688, July 1995.

[9] M. W. Spong, "Partial feedback linearization of underactuated mechanical systems," in *Proc., IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pp. 314–321, 1994.

[10] A. D. Mahindrakar and R. N. Banavar, "Controllability properties of a planar 3R underactuated manipulator," in *Proc. of the 2002 IEEE International Conference on Control Applications*, pp. 489–494, 2002.

[11] H. Arai and S. Tachi, "Position control of a manipulator with passive joints using dynamic coupling," *IEEE Trans. on Robotics and Automation*, vol. 7, pp. 528–534, Aug. 1991.