Dynamic Analysis and Active Damping Control for Underwater Vehicle-Manipulator Systems

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ABSTRACT

Underwater vehicle-manipulator (UVM) systems have been suggested for inspection, maintenance, repair and construction of underwater structures. The addition of manipulators to the vehicle makes control of the system more difficult due to the interaction forces between vehicle and manipulator. So, the efficient compensation of those interactions should be considered for accurate control of both subsystems. For precision control of the end-effectors, at the first step, it is essential to analyze the kinematics and dynamics of the whole UVM systems. And then, the robust controller has to be introduced over the interaction disturbances. In this paper, firstly, the kinematic and dynamic analysis about the whole UVM system is executed. The whole dynamics is divided by meaningful terms. And, a robust coordinated motion control algorithm for autonomous UVM systems is considered. For the manipulation, the vehicle movement may be not desirable, because the small movement of vehicle affect significant changes of the end-effectors. And, a vehicle is relatively smaller bandwidth system than manipulators. For this reason, in this paper, active damping control with two-time scale approach for the manipulator is used with passive vehicle. Using this algorithm, the manipulator controller can deal with the vehicle state additionally.

KEY WORDS: Underwater vehicle-manipulator system; kinematic and dynamic modeling; two-time scale control; robust control; active damping control

INTRODUCTION

In recent years, the need for the exploration and preservation of the oceanic environments has gained significant momentum. It is recognized that there have been increased activity in the exploration of the ocean's resources. Also, for some special tasks, a robot manipulator is needed. According to this reason, autonomous underwater vehicles (AUVs) and remotely operated vehicles (ROVs) equipped with robotic manipulators have an important role in to play in a number of shallow- and deep-water missions for marine science, oil and gas, survey, exploration, exploitation, and military applications.

However, adding the manipulator, the UVM systems have a new and more complex problem to deal with. There are many researchers to solve problems in the UVM systems, for example, drag optimization with redundancy (Sarkar and Podder, 2001), external and unified force control (Antonelli, Chiaverini, and Sarkar, 2001, Cui and Sarkar, 2001), interaction from the manipulator compensation algorithm with force sensor (Ryu, Kwon, and Lee, 2001), tracking control with velocity estimation (Antonelli, Caccavale, Chiaverini, and Villani, 2000), and etc. The main control difficulties stem from the highly coupling model, the system nonlinearities, and the external unmodeled disturbances acting on the vehicle dynamics. Also, the system has different characteristics, that is, the vehicle has lower bandwidth system than the manipulator. Canudas *et al.* (Canudas, Diaz, and Perrier, 2000) dealt with the different system bandwidth, however, they only focused on vehicle motion, not the end-effector movement. And, the whole UVM system base is not fixed but floating, the disturbances could be amplified and the sensing of the states be more difficult. Moreover, the interaction between two subsystems makes control difficult. To resolve this problem, we may use the whole system dynamics at once by unified system approach, but it causes the complexity of modeling, and the system differences, as mentioned above, can be hardly included in the whole control system. Hence, decentralized controller is suitable for those kinds of composite dynamic systems. And also, interaction disturbances and system bandwidth have to be considered carefully. In addition, adequate kinematic and dynamic decoupled algorithm is needed.

For the manipulation, the vehicle control may be not desirable, because the small movement of vehicle affect significant changes of the end-effectors. And, an UVM system is a kinematically redundant system. That is, it has more degrees-of-freedom(DOF) in the joint-space that are required to execute a task specified in the Cartesian space. This is because the AUV itself has six DOF, and any additional DOF that comes from the manipulator will allow the combined system to exceed the maximum task-space dimension, which is six. Therefore, the position change of the end-effector due to the vehicle movement can be compensated by the manipulator system. Using above characteristic, the passive vehicle approach, which means we do not control directly the vehicle system with vehicle actuator, is useful, because a vehicle is relatively smaller bandwidth system than manipulators (Chung, Eom, Yi, Suh, Oh, Chung, and Kim, 2001). It is a kind of conventional under-actuated system controller. But, we do not need to fully concentrate on controlling the under-actuated part(vehicle). Even though it is possible, it is not desirable, because the vehicle generally has much larger mass and inertia system than the manipulator, which means the manipulator can easily fall into the actuator saturation.

Therefore, in this paper, active damping control with two-time scale approach for the manipulator is used with passive vehicle

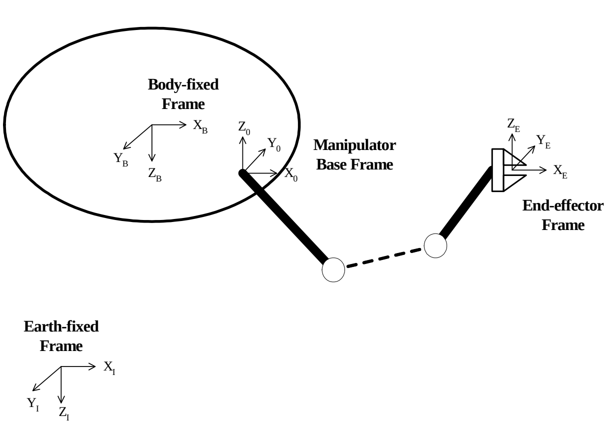

Fig. 1. Schematic diagram of UVM system

(Lew and Moon, 2001). The manipulator controller can deal with the vehicle state additionally by means of active damping controller for the vehicle. Consequently, using different time scale controller acting on the manipulator, we can control the whole system simultaneously. In other words, fast time scale controller is used for the manipulator and slow time scale controller for the vehicle according to the system bandwidth.

In this paper, firstly, the modeling of UVM systems will be described in the sense of kinematics and dynamics. And then, using the analysis of modeling, we propose two-time scale controller. It will be designed in the operational space, and can robustly control the end-effector under the interaction between the vehicle and the manipulator.

MODELING OF UVM SYSTEMS

The kinematic task is more tractable and systematic if it is described in the Denavit-Hatenberg(D-H) coordinate frames. Fig. 1 illustrates the coordinates of the UUV(Unmanned Underwater Vehicle) and n-link manipulator. The 0-th frame, $\boldsymbol{\Sigma}_0$, is the reference frame located at the manipulator base, and each i-th frame are located at the i-th link along the D-H coordinate rule. $\boldsymbol{\Sigma}_B$ and $\boldsymbol{\Sigma}_I$ denote the vehicle body-fixed and the earth-fixed reference frame, respectively. In the earth-fixed frame, X_I directs North and Z_I is chosen as the gravity direction.

Since the earth-fixed frame does not have motion, it can be regarded as the inertial frame. The position and orientation of the vehicle, $\boldsymbol{\eta}$, can be described relative to the inertial reference frame while the linear and angular velocities of the vehicle, $\boldsymbol{\nu}$, can be expressed in the body-fixed frame.

The state vectors for the vehicle are described as following:

$$\begin{aligned} &\boldsymbol{\eta} = [\boldsymbol{\eta}_1^T, \boldsymbol{\eta}_2^T]^T; \qquad \boldsymbol{\eta}_1 = [x, y, z]^T; \qquad \boldsymbol{\eta}_2 = [\phi, \theta, \psi]^T; \\ &\boldsymbol{\nu} = [\boldsymbol{\nu}_1^T, \boldsymbol{\nu}_2^T]^T; \qquad \boldsymbol{\nu}_1 = [v_x, v_y, v_z]^T; \qquad \boldsymbol{\nu}_2 = [\omega_x, \omega_y, \omega_y]^T; \end{aligned}$$

And, the inertial and body-fixed velocity relation can be represented with vehicle jacobian matrix (Fossen, 1994, Yuh, 1990).

$$\begin{bmatrix} \dot{\boldsymbol{\eta}}_1 \\ \dot{\boldsymbol{\eta}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}_{v1}(\boldsymbol{\eta}_2) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{J}_{v2}(\boldsymbol{\eta}_2) \end{bmatrix} \begin{bmatrix} \boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2 \end{bmatrix} \Longleftrightarrow \dot{\boldsymbol{\eta}} = \boldsymbol{J}_v(\boldsymbol{\eta})\boldsymbol{\nu}. \tag{1}$$

And, for the n-link manipulator, the joint position state vector is defined by $\boldsymbol{q}$ and, the position and orientation of the end-effector, $\boldsymbol{p}_e$, can be described relative to the inertial reference frame while the local position and orientation of the end-effector, $\boldsymbol{p}$, can be expressed in the vehicle body-fixed frame.

The state vectors related to the manipulator are described as following:

$$\begin{aligned} &\boldsymbol{q} = [q_1, q_2, \cdots, q_n]^T; \\ &\boldsymbol{p} = [\boldsymbol{p}_1^T, \boldsymbol{p}_2^T]^T; \qquad \boldsymbol{p}_1 = [x_l, y_l, z_l]^T; \qquad \boldsymbol{p}_2 = [\phi_l, \theta_l, \psi_l]^T; \\ &\boldsymbol{p}_e = [\boldsymbol{p}_{e1}, \boldsymbol{p}_{e2}]^T; \qquad \boldsymbol{p}_{e1} = [x_e, y_e, z_e]^T; \qquad \boldsymbol{p}_{e2} = [\phi_e, \theta_e, \psi_e]^T; \end{aligned}$$

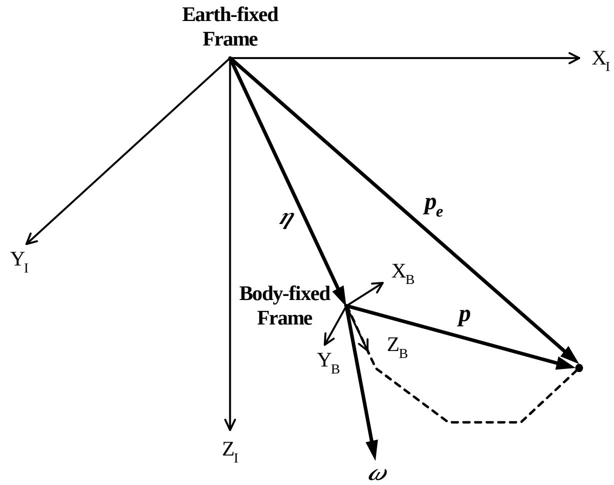

Fig. 2. Moving coordinate system representation of UVM system

The local velocity of end-effector relation with joint velocity is expressed by usual manipulator jacobian and rotation matrix.

$$\begin{bmatrix} \dot{\boldsymbol{p}}_1 \\ \dot{\boldsymbol{p}}_2 \end{bmatrix} = \begin{bmatrix} {}^B\boldsymbol{R}_0 \boldsymbol{J}_{m1}(\boldsymbol{q}) \\ {}^B\boldsymbol{R}_0 \boldsymbol{J}_{m2}(\boldsymbol{q}) \end{bmatrix} \dot{\boldsymbol{q}} = \begin{bmatrix} \boldsymbol{J}_{m1}'(\boldsymbol{q}) \\ \boldsymbol{J}_{m2}'(\boldsymbol{q}) \end{bmatrix} \dot{\boldsymbol{q}} \Longleftrightarrow \dot{\boldsymbol{p}} = \boldsymbol{J}_m'(\boldsymbol{q})\dot{\boldsymbol{q}}, \tag{2}$$

where the $\boldsymbol{J}_{m1}$ and $\boldsymbol{J}_{m2}$ represent the position and orientation jacobian matrix from manipulator base frame to end-effector, respectively. And ${}^B\boldsymbol{R}_0$ means the transformation matrix from the vehicle body fixed coordinate to the manipulator base coordinate. If the two coordinates are coincidence, ${}^B\boldsymbol{R}_0 = \boldsymbol{I}$, and the $\boldsymbol{J}_m'$ is the manipulator jacobian itself.

UVM Systems Kinematics

To find the overall velocity relation, we used the moving coordinate system relation. Fig. 1 can be represented as Fig. 2. Then the end-effector velocity relative to the inertial frame has below relations:

$$\begin{aligned} \dot{\boldsymbol{p}}_e = \begin{bmatrix} \boldsymbol{v}_e \\ \boldsymbol{\omega}_e \end{bmatrix} &= \begin{bmatrix} \dot{\boldsymbol{\eta}}_1 + (\dot{\boldsymbol{p}}_1)_I + \dot{\boldsymbol{\eta}}_2 \times (\boldsymbol{p}_1)_I \\ \dot{\boldsymbol{\eta}}_2 + (\dot{\boldsymbol{p}}_2)_I \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} \dot{\boldsymbol{\eta}}_1 + \dot{\boldsymbol{\eta}}_2 \times (\boldsymbol{p}_1)_I \\ \dot{\boldsymbol{\eta}}_2 \end{bmatrix}}_{\text{velocity induced by vehicle}} + \underbrace{\begin{bmatrix} (\dot{\boldsymbol{p}}_1)_I \\ (\dot{\boldsymbol{p}}_2)_I \end{bmatrix}}_{\text{velocity induced by manipulator}}, \end{aligned} \tag{3}$$

where $(\cdot)_I$ means the vector with respect to the inertial frame. And the cross product term in the linear velocity induced by vehicle is expressed as:

$$\dot{\boldsymbol{\eta}}_2 \times (\boldsymbol{p}_1)_I = -\boldsymbol{S}((\boldsymbol{p}_1)_I)\dot{\boldsymbol{\eta}}_2 = -\boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1)\dot{\boldsymbol{\eta}}_2, \tag{4}$$

where $\boldsymbol{S}(\cdot)$ is skew-symmetric matrix defined such that $\boldsymbol{a} \times \boldsymbol{b} \triangleq \boldsymbol{S}(\boldsymbol{a})\boldsymbol{b}$, that is:

$$\boldsymbol{S}(\boldsymbol{a}) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \tag{5}$$

Then, the velocity induced by vehicle can be rewritten as following:

$$\begin{aligned} &\begin{bmatrix} \dot{\boldsymbol{\eta}}_1 - \boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1)\dot{\boldsymbol{\eta}}_2 \\ \dot{\boldsymbol{\eta}}_2 \end{bmatrix}, \\ &= \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1) \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\eta}}_1 \\ \dot{\boldsymbol{\eta}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1) \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \boldsymbol{J}_v \begin{bmatrix} \boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2 \end{bmatrix}, \\ &= \boldsymbol{J}_v \begin{bmatrix} \boldsymbol{I} & -{\boldsymbol{J}_{v1}}^T \boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1)\boldsymbol{J}_{v2} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2 \end{bmatrix} \triangleq \boldsymbol{J}_v \boldsymbol{J}_1 \boldsymbol{\nu}. \end{aligned} \tag{6}$$

And, the velocity induced by manipulator is rewritable as same manner:

$$\begin{aligned}\begin{bmatrix}(\dot{\boldsymbol{p}}_1)_I\\(\dot{\boldsymbol{p}}_2)_I\end{bmatrix}&=\begin{bmatrix}\boldsymbol{J}_{v1}\dot{\boldsymbol{p}}_1\\\boldsymbol{J}_{v2}\dot{\boldsymbol{p}}_2\end{bmatrix}=\begin{bmatrix}\boldsymbol{J}_{v1}\boldsymbol{J}_{m1}{}'\\\boldsymbol{J}_{v2}\boldsymbol{J}_{m2}{}'\end{bmatrix}\dot{\boldsymbol{q}},\\&=\boldsymbol{J}_v\boldsymbol{J}_m{}'\dot{\boldsymbol{q}}.\end{aligned}\tag{7}$$

Finally, we can obtain the end-effector velocity as below:

$$\dot{\boldsymbol{p}}_e=\boldsymbol{J}_v(\boldsymbol{\eta})\begin{bmatrix}\boldsymbol{J}_1(\boldsymbol{\eta},\boldsymbol{q}) & \boldsymbol{J}_m{}'(\boldsymbol{q})\end{bmatrix}\begin{bmatrix}\boldsymbol{\nu}\\\dot{\boldsymbol{q}}\end{bmatrix}.\tag{8}$$

For the decoupled system control, we need both the vehicle and the end-effector velocity relations:

$$\begin{bmatrix}\dot{\boldsymbol{\eta}}\\\dot{\boldsymbol{p}}_e\end{bmatrix}=\boldsymbol{J}(\boldsymbol{\eta},\boldsymbol{q})\begin{bmatrix}\boldsymbol{\nu}\\\dot{\boldsymbol{q}}\end{bmatrix},\tag{9}$$

where

$$\begin{aligned}\boldsymbol{J}(\boldsymbol{\eta},\boldsymbol{q})&\triangleq\begin{bmatrix}\boldsymbol{J}_v(\boldsymbol{\eta}) & \boldsymbol{0}\\\boldsymbol{0} & \boldsymbol{J}_v(\boldsymbol{\eta})\end{bmatrix}\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\\\boldsymbol{J}_1(\boldsymbol{\eta},\boldsymbol{q}) & \boldsymbol{J}_m{}'(\boldsymbol{q})\end{bmatrix},\\\boldsymbol{J}_1(\boldsymbol{\eta},\boldsymbol{q})&\triangleq\begin{bmatrix}\boldsymbol{I} & -\boldsymbol{J}_{v1}{}^T\boldsymbol{S}(\boldsymbol{J}_{v1}\boldsymbol{p}_1)\boldsymbol{J}_{v2}\\\boldsymbol{0} & \boldsymbol{I}\end{bmatrix}.\end{aligned}$$

And, the jacobian inverse has the form

$$\begin{aligned}&\boldsymbol{J}^{\#}(\boldsymbol{\eta},\boldsymbol{q})=\\&\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\\-\boldsymbol{J}_m{}'^{\#}(\boldsymbol{q})\boldsymbol{J}_1(\boldsymbol{\eta},\boldsymbol{q}) & \boldsymbol{J}_m{}'^{\#}(\boldsymbol{q})\end{bmatrix}\begin{bmatrix}\boldsymbol{J}_v{}^{-1}(\boldsymbol{\eta}) & \boldsymbol{0}\\\boldsymbol{0} & \boldsymbol{J}_v{}^{-1}(\boldsymbol{\eta})\end{bmatrix},\end{aligned}\tag{10}$$

where $\boldsymbol{J}_m{}'^{\#}(\boldsymbol{q})$ is Moore-Penrose pseudo-inverse of $\boldsymbol{J}_m{}'$. The form of $\boldsymbol{J}_m{}'^{\#}(\boldsymbol{q})$ is dependant on whether the manipulator is redundant or non-redundant.

UVM Systems Dynamics

To control dynamic system, firstly, we have to construct and analysis for the equations of motion. But, the UVM system is generally highly coupled and complicated system, the dynamic formulation is not a easy work. There are many ways to express a dynamic system, for example, the Newton-Euler method, the Lagrange method, and the Kane's method (Tarn, Shoults, and Yang, 1996). Naturally, they have all some merits and demerits. The Newton-Euler method is a recursive formulation and it is adequate to high-order system, but it is not convenient to design a controller like model-based controller. Whereas, using the Lagrangian method, the controller design is easy and it is energy based approach, so it is easy to update additional forces on the system such as drag forces and added mass(inertia) forces. But, for high-order systems like UVM system, it is very hard work to derive the equations of motion as the analytic form. And, for the moving base system which is not fixed in an inertial frame, it is not convenient to derive the dynamic formulation using the Lagrangian in terms of the velocities expressed in a body-fixed frame. In UVM systems, the sensor information and actuator forces are exerted on body-fixed frame, so it is more natural to write up the dynamics using body-fixed velocities.

To supplement the above drawbacks, we used quasi-Lagrange approach (Meirovitch, 1990) and partitioned dynamic equations. The quasi-Lagrange method can give the equations of motion in terms of the body-fixed velocities. And, for the decentralized control of the vehicle and manipulator, the partitioned submatrices of dynamic terms are absolutely needed to design the controller. Using this partitioned dynamics, we can easily deal with the coupling effect between two subsystems. Moreover, it is possible to use the already driven dynamic equations for each subsystems.

The hydrodynamic forces and moments of vehicle and manipulator have been used as the previous works by others and omitted here (Schjølberg and Fossen, 1994, Lévesque and Richard, 1994).

Quasi-Lagrange System

For the underwater vehicle, Fossen derived the dynamics using quasi-velocities (Fossen, 1994), and for the hybrid system which is composed of both quasi-velocities and generalized-velocities such as UVM system, Meirovitch and Sarkar has developed using same approach (Meirovitch and Stemple, 1995, Sarkar and Podder, 2001).

The Lagrangian in general form is defined as

$$L=L[\boldsymbol{\eta},\boldsymbol{q},\dot{\boldsymbol{\eta}},\dot{\boldsymbol{q}}].\tag{11}$$

And, with the extended Hamiltonian principle, Eq. (11) gives below differential equations.

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial[\dot{\boldsymbol{\eta}}^T,\dot{\boldsymbol{q}}^T]^T}\right)-\frac{\partial L}{\partial[\boldsymbol{\eta}^T,\boldsymbol{q}^T]^T}=\boldsymbol{Q}.\tag{12}$$

Nextly, the quasi-Lagrangian is defined as form

$$\overline{L}=\overline{L}[\boldsymbol{\delta},\boldsymbol{\mu}],\tag{13}$$

where $\boldsymbol{\delta}=[\boldsymbol{\eta}^T,\boldsymbol{q}^T]^T$ and $\boldsymbol{\mu}=[\boldsymbol{\nu}^T,\dot{\boldsymbol{q}}^T]^T$. And the overline symbol used to distinguish from original Lagrangian.

Then, we obtain another differential equations as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\overline{L}}{\partial\boldsymbol{\mu}}\right)+\boldsymbol{A}\frac{\partial\overline{L}}{\partial\boldsymbol{\mu}}-\boldsymbol{B}^T\frac{\partial\overline{L}}{\partial\boldsymbol{\delta}}=\boldsymbol{\tau},\tag{14}$$

where

$$\boldsymbol{A}=\begin{bmatrix}\boldsymbol{S}(\boldsymbol{\nu}_2) & \boldsymbol{0} & \boldsymbol{0}\\\boldsymbol{S}(\boldsymbol{\nu}_1) & \boldsymbol{S}(\boldsymbol{\nu}_2) & \boldsymbol{0}\\\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix},\quad\boldsymbol{B}=\begin{bmatrix}\boldsymbol{J}_{v1}(\boldsymbol{\eta}_2) & \boldsymbol{0} & \boldsymbol{0}\\\boldsymbol{0} & \boldsymbol{J}_{v2}(\boldsymbol{\eta}_2) & \boldsymbol{0}\\\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix},$$

and $\boldsymbol{\tau}=\boldsymbol{B}^T\boldsymbol{Q}$. The detail procedure is well explained in reference (Meirovitch and Stemple, 1995).

Next, we wish to develop an expression for the partitioned dynamics. In Eq. (14), $\boldsymbol{A}\frac{\partial\overline{L}}{\partial\boldsymbol{\mu}}$ has another expression as below:

$$\boldsymbol{A}\frac{\partial\overline{L}}{\partial\boldsymbol{\mu}}=\begin{bmatrix}\boldsymbol{0} & -\boldsymbol{S}(\frac{\partial\overline{L}}{\partial\boldsymbol{\nu}_1}) & \boldsymbol{0}\\-\boldsymbol{S}(\frac{\partial\overline{L}}{\partial\boldsymbol{\nu}_1}) & -\boldsymbol{S}(\frac{\partial\overline{L}}{\partial\boldsymbol{\nu}_2}) & \boldsymbol{0}\\\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}\boldsymbol{\mu}\triangleq\boldsymbol{A}'\boldsymbol{\mu}.\tag{15}$$

For the convenience, firstly rigid body kinetic energy is only considered. Then, the $\overline{L}=\overline{T}$ and $\overline{T}$ is given as

$$\overline{T}=\frac{1}{2}\boldsymbol{\mu}^T\boldsymbol{M}\boldsymbol{\mu},\tag{16}$$

where $\boldsymbol{M}$ is the inertia matrix of UVM system:

$$\boldsymbol{M}=\begin{bmatrix}\boldsymbol{M}_{11} & \boldsymbol{M}_{12} & \boldsymbol{M}_{13}\\\boldsymbol{M}_{21} & \boldsymbol{M}_{22} & \boldsymbol{M}_{23}\\\boldsymbol{M}_{31} & \boldsymbol{M}_{32} & \boldsymbol{M}_{33}\end{bmatrix},\tag{17}$$

and then

$$\begin{aligned}\overline{T}=\frac{1}{2}(\ &\boldsymbol{\nu}_1^T\boldsymbol{M}_{11}\boldsymbol{\nu}_1+\boldsymbol{\nu}_1^T\boldsymbol{M}_{12}\boldsymbol{\nu}_2+\boldsymbol{\nu}_1^T\boldsymbol{M}_{13}\dot{\boldsymbol{q}}\\&+\boldsymbol{\nu}_2^T\boldsymbol{M}_{21}\boldsymbol{\nu}_1+\boldsymbol{\nu}_2^T\boldsymbol{M}_{22}\boldsymbol{\nu}_2+\boldsymbol{\nu}_2^T\boldsymbol{M}_{23}\dot{\boldsymbol{q}}\\&+\dot{\boldsymbol{q}}^T\boldsymbol{M}_{31}\boldsymbol{\nu}_1+\dot{\boldsymbol{q}}^T\boldsymbol{M}_{32}\boldsymbol{\nu}_2+\dot{\boldsymbol{q}}^T\boldsymbol{M}_{33}\dot{\boldsymbol{q}}\quad).\end{aligned}\tag{18}$$

Eq. (17) is expressed as the partitioned submatrices which are related to $\boldsymbol{\nu}_1$, $\boldsymbol{\nu}_2$, and $\dot{\boldsymbol{q}}$, respectively. The inertia matrix has symmetric property due to the action and the reaction principle, and positive definite($\boldsymbol{M}>$

0). $\boldsymbol{M}_{33}$ represent the manipulator inertia matrix itself($\boldsymbol{M}_m$), because the lower part of the inertia matrix does not affect the upper part of the inertia matrix. And, Eq. (14) can be rewritable as

$$\boldsymbol{M}\dot{\boldsymbol{\mu}} + \boldsymbol{C}(\boldsymbol{q},\boldsymbol{\mu})\boldsymbol{\mu} = \boldsymbol{\tau}. \tag{19}$$

Now, we have to find the Coriolis and Centripetal Matrix which is possible to be enormous parameterizations.

Theorem 1 (Coriolis and Centripetal Matrix from Inertia Matrix) *If the inertia matrix is given as Eq. (17), then we can always parameterize the Coriolis and centripetal matrix such that* $\dot{\boldsymbol{M}} - (\boldsymbol{C} + \boldsymbol{C}^T) = \boldsymbol{0}$ *by defining:*

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} & \boldsymbol{C}_{13} \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} & \boldsymbol{C}_{23} \\ \boldsymbol{C}_{31} & \boldsymbol{C}_{32} & \boldsymbol{C}_{33} \end{bmatrix} = \left[\ \boldsymbol{\Gamma}_1 \ | \ \boldsymbol{\Gamma}_2 \ | \ \boldsymbol{\Gamma}_3 \ \right], \tag{20}$$

where

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} \boldsymbol{0} \\ -\boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_1}) \\ \dot{\boldsymbol{M}}_{31} - \left\lfloor \frac{\partial \boldsymbol{M}_{31}}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} \right\rfloor \end{bmatrix}, \tag{21a}$$

$$\boldsymbol{\Gamma}_2 = \begin{bmatrix} \dot{\boldsymbol{M}}_{12} - \boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_1}) \\ \frac{1}{2}\dot{\boldsymbol{M}}_{22} - \boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_2}) \\ \dot{\boldsymbol{M}}_{32} - \left\lfloor \frac{\partial \boldsymbol{M}_{32}}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} \right\rfloor - \left\lfloor \frac{\partial \boldsymbol{M}_{12}}{\partial \boldsymbol{q}} \boldsymbol{\nu}_1 \right\rfloor - \frac{1}{2} \left\lfloor \frac{\partial \boldsymbol{M}_{22}}{\partial \boldsymbol{q}} \boldsymbol{\nu}_2 \right\rfloor \end{bmatrix} \tag{21b}$$

$$\boldsymbol{\Gamma}_3 = \begin{bmatrix} \left\lfloor \frac{\partial \boldsymbol{M}_{13}}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} \right\rfloor \\ \left\lfloor \frac{\partial \boldsymbol{M}_{23}}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} \right\rfloor + \left\lfloor \frac{\partial \boldsymbol{M}_{21}}{\partial \boldsymbol{q}} \boldsymbol{\nu}_1 \right\rfloor + \frac{1}{2} \left\lfloor \frac{\partial \boldsymbol{M}_{22}}{\partial \boldsymbol{q}} \boldsymbol{\nu}_2 \right\rfloor \\ \boldsymbol{C}_m(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix}. \tag{21c}$$

And, $\boldsymbol{C}_m(\boldsymbol{q},\dot{\boldsymbol{q}})$ *is Coriolis and Centripetal matrix for the manipulator, which already satisfies* $\dot{\boldsymbol{M}}_m - (\boldsymbol{C}_m + \boldsymbol{C}_m^T) = \boldsymbol{0}$ *and* $\boldsymbol{C}_m\dot{\boldsymbol{q}} = \frac{1}{2}\dot{\boldsymbol{M}}_m\dot{\boldsymbol{q}}$. *And, the symbol,* $\left\lfloor \frac{\partial \boldsymbol{A}}{\partial \boldsymbol{q}} \boldsymbol{a} \right\rfloor$, *means*

$$\left\lfloor \frac{\partial \boldsymbol{A}}{\partial \boldsymbol{q}} \boldsymbol{a} \right\rfloor_{ij} = \Sigma_k^n \frac{\partial A_{kj}}{\partial q_i} a_k, \qquad n \text{ is dimension of vector } \boldsymbol{a}.$$

Proof: *Inserting Eqs. (16) and (19) into Eq. (14), we obtain* $\boldsymbol{C}\boldsymbol{\mu}$ *as form of*

$$\begin{aligned} \boldsymbol{C}\boldsymbol{\mu} &= \dot{\boldsymbol{M}}\boldsymbol{\mu} + \boldsymbol{A}'\boldsymbol{\mu} - \boldsymbol{B}^T \frac{\partial \overline{L}}{\partial \delta} \\ &= \begin{bmatrix} \dot{\boldsymbol{M}}_{11}\boldsymbol{\nu}_1 + \dot{\boldsymbol{M}}_{12}\boldsymbol{\nu}_2 + \dot{\boldsymbol{M}}_{13}\dot{\boldsymbol{q}} \\ \dot{\boldsymbol{M}}_{21}\boldsymbol{\nu}_1 + \dot{\boldsymbol{M}}_{22}\boldsymbol{\nu}_2 + \dot{\boldsymbol{M}}_{23}\dot{\boldsymbol{q}} \\ \dot{\boldsymbol{M}}_{31}\boldsymbol{\nu}_1 + \dot{\boldsymbol{M}}_{32}\boldsymbol{\nu}_2 + \dot{\boldsymbol{M}}_{33}\dot{\boldsymbol{q}} \end{bmatrix} \\ &\quad + \begin{bmatrix} -\boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_1})\boldsymbol{\nu}_2 \\ -\boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_2})\boldsymbol{\nu}_1 - \boldsymbol{S}(\frac{\partial \overline{L}}{\partial \boldsymbol{\nu}_2})\boldsymbol{\nu}_2 \\ \boldsymbol{0} \end{bmatrix} - \begin{bmatrix} \boldsymbol{J}_{v1}^T \frac{\partial \overline{L}}{\partial \boldsymbol{\eta}_1} \\ \boldsymbol{J}_{v2}^T \frac{\partial \overline{L}}{\partial \boldsymbol{\eta}_2} \\ \frac{\partial \overline{L}}{\partial \boldsymbol{q}} \end{bmatrix}, \end{aligned} \tag{22}$$

where $\overline{L}$ *is a function of* $\boldsymbol{q}$ *only, so the last term of Eq. (22) is*

$$\begin{bmatrix} \boldsymbol{J}_{v1}^T \frac{\partial \overline{L}}{\partial \boldsymbol{\eta}_1} \\ \boldsymbol{J}_{v2}^T \frac{\partial \overline{L}}{\partial \boldsymbol{\eta}_2} \\ \frac{\partial \overline{L}}{\partial \boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \Delta \end{bmatrix}, \tag{23}$$

where

$$\begin{aligned} \Delta \triangleq & \left\lfloor \frac{\partial \boldsymbol{M}_{31}}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} \right\rfloor \boldsymbol{\nu}_1 + \left\lfloor \frac{\partial \boldsymbol{M}_{12}}{\partial \boldsymbol{q}} \boldsymbol{\nu}_1 \right\rfloor \boldsymbol{\nu}_2 + \left\lfloor \frac{\partial \boldsymbol{M}_{32}}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} \right\rfloor \boldsymbol{\nu}_2 \\ & + \frac{1}{2} \left\lfloor \frac{\partial \boldsymbol{M}_{22}}{\partial \boldsymbol{q}} \boldsymbol{\nu}_2 \right\rfloor \boldsymbol{\nu}_2 + \frac{1}{2}\dot{\boldsymbol{M}}_m\dot{\boldsymbol{q}}. \end{aligned} \tag{24}$$

And, we can easily show that Eq. (20) satisfies Eq. (22) by algebraic computation. □

We can deal the added mass(inertia) and added Coriolis and Centripetal forces as same way. The gravitational forces and buoyancy are added by kinematic relations as $\overline{L} = \overline{T} + \overline{V}$. And, the drag forces can be considered with iterative Newton-Euler method. The details are omitted here.

Partitioned Dynamics

Now, the whole UVM system equations of motion will be derived, and it will be also divided by several meaningful terms.

On the analogy of several other research, it is clear that the whole system equations of motion can be written as the following equations:

$$\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{\mu}} + \boldsymbol{C}(\boldsymbol{q},\boldsymbol{\mu})\boldsymbol{\mu} + \boldsymbol{D}(\boldsymbol{q},\boldsymbol{\mu})\boldsymbol{\mu} + \boldsymbol{g}(\boldsymbol{\eta},\boldsymbol{q}) = \boldsymbol{\tau}, \tag{25}$$

where

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{M}_{11}(\boldsymbol{q}) & \boldsymbol{M}_{12}(\boldsymbol{q}) \\ \boldsymbol{M}_{21}(\boldsymbol{q}) & \boldsymbol{M}_{22}(\boldsymbol{q}) \end{bmatrix}, \tag{26}$$

$$\boldsymbol{C}(\boldsymbol{q},\boldsymbol{\mu}) = \begin{bmatrix} \boldsymbol{C}_{11}(\boldsymbol{q},\boldsymbol{\mu}) & \boldsymbol{C}_{12}(\boldsymbol{q},\boldsymbol{\mu}) \\ \boldsymbol{C}_{21}(\boldsymbol{q},\boldsymbol{\mu}) & \boldsymbol{C}_{22}(\boldsymbol{q},\boldsymbol{\mu}) \end{bmatrix}, \tag{27}$$

$$\boldsymbol{D}(\boldsymbol{q},\boldsymbol{\mu}) = \begin{bmatrix} \boldsymbol{D}_{11}(\boldsymbol{q},\boldsymbol{\mu}) & \boldsymbol{D}_{12}(\boldsymbol{q},\boldsymbol{\mu}) \\ \boldsymbol{D}_{21}(\boldsymbol{q},\boldsymbol{\mu}) & \boldsymbol{D}_{22}(\boldsymbol{q},\boldsymbol{\mu}) \end{bmatrix}, \tag{28}$$

$$\boldsymbol{g}(\boldsymbol{q},\boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{g}_1(\boldsymbol{\eta},\boldsymbol{q}) \\ \boldsymbol{g}_2(\boldsymbol{\eta},\boldsymbol{q}) \end{bmatrix}, \tag{29}$$

$$\boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{\tau}_v \\ \boldsymbol{\tau}_m \end{bmatrix}. \tag{30}$$

Notice that the subscripts of matrices are different from those of using previous section.

For the advanced and systematic control algorithm, the equations have to be divided by reasonable way. So, at the following, the dynamic decomposition will be treated. Firstly, the vehicle and manipulator dynamics and those interaction dynamics can be regarded. And, the added dynamics to the vehicle ones by the manipulator dynamics also included.

The vehicle and manipulator dynamic effects are simply obtained from many references and calculations. Those are shown as following:

$$\begin{bmatrix} \boldsymbol{M}_v & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \dot{\boldsymbol{\mu}} + \begin{bmatrix} \boldsymbol{C}_v(\boldsymbol{\nu}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{D}_v(\boldsymbol{\nu}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{g}_v(\boldsymbol{\eta}) \\ \boldsymbol{0} \end{bmatrix} = \boldsymbol{\tau}_1, \tag{31}$$

$$\begin{aligned} \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_m(\boldsymbol{q}) \end{bmatrix} \dot{\boldsymbol{\mu}} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C}_m(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D}_m(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix} \boldsymbol{\mu} \\ + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{g}_m(\boldsymbol{\eta},\boldsymbol{q}) \end{bmatrix} = \boldsymbol{\tau}_2. \end{aligned} \tag{32}$$

And, the effect of the added dynamics to the vehicle by manipulator has to be considered. This dynamic effect can be introduced as following:

$$\begin{aligned} \begin{bmatrix} \boldsymbol{M}_s(\boldsymbol{q}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \dot{\boldsymbol{\mu}} + \begin{bmatrix} \boldsymbol{C}_s(\boldsymbol{q},\boldsymbol{\nu}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{D}_s(\boldsymbol{q},\boldsymbol{\nu}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} \\ + \begin{bmatrix} \boldsymbol{g}_s(\boldsymbol{\eta},\boldsymbol{q}) \\ \boldsymbol{0} \end{bmatrix} = \boldsymbol{\tau}_3. \end{aligned} \tag{33}$$

In addition, the interaction dynamics between two subsystems are expressed in the form:

$$\begin{aligned} \begin{bmatrix} \boldsymbol{0} & \boldsymbol{M}_{12}(\boldsymbol{q}) \\ {\boldsymbol{M}_{12}}^T(\boldsymbol{q}) & \boldsymbol{0} \end{bmatrix} \dot{\boldsymbol{\mu}} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{C}_{12}(\boldsymbol{q},\boldsymbol{\mu}) \\ \boldsymbol{C}_{21}(\boldsymbol{q},\boldsymbol{\mu}) & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} \\ + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{D}_{12}(\boldsymbol{q},\boldsymbol{\mu}) \\ \boldsymbol{D}_{21}(\boldsymbol{q},\boldsymbol{\mu}) & \boldsymbol{0} \end{bmatrix} \boldsymbol{\mu} = \boldsymbol{\tau}_4, \end{aligned} \tag{34}$$

where the inertia matrix is symmetric, because the characteristics of Lagrangian system, and it can be easily proved. Finally, the Eq. (25) can be expressed as below:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + \boldsymbol{\tau}_3 + \boldsymbol{\tau}_4. \tag{35}$$

And, the Eqs. (26) – (29) can be rewritten by meaningful terms in the form:

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{M}_v + \boldsymbol{M}_s(\boldsymbol{q}) & \boldsymbol{M}_{12}(\boldsymbol{q}) \\ \boldsymbol{M}_{12}{}^T(\boldsymbol{q}) & \boldsymbol{M}_m(\boldsymbol{q}) \end{bmatrix}, \tag{36}$$

$$\boldsymbol{C}(\boldsymbol{q},\boldsymbol{\mu}) = \begin{bmatrix} \boldsymbol{C}_v(\boldsymbol{\nu}) + \boldsymbol{C}_s(\boldsymbol{q},\boldsymbol{\nu}) & \boldsymbol{C}_{12}(\boldsymbol{q},\boldsymbol{\mu}) \\ \boldsymbol{C}_{21}(\boldsymbol{q},\boldsymbol{\mu}) & \boldsymbol{C}_m(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix}, \tag{37}$$

$$\boldsymbol{D}(\boldsymbol{q},\boldsymbol{\mu}) = \begin{bmatrix} \boldsymbol{D}_v(\boldsymbol{\nu}) + \boldsymbol{D}_s(\boldsymbol{q},\boldsymbol{\nu}) & \boldsymbol{D}_{12}(\boldsymbol{q},\boldsymbol{\mu}) \\ \boldsymbol{D}_{21}(\boldsymbol{q},\boldsymbol{\mu}) & \boldsymbol{D}_m(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix}, \tag{38}$$

$$\boldsymbol{g}(\boldsymbol{q},\boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{g}_v(\boldsymbol{\eta}) + \boldsymbol{g}_s(\boldsymbol{\eta},\boldsymbol{q}) \\ \boldsymbol{g}_m(\boldsymbol{\eta},\boldsymbol{q}) \end{bmatrix}. \tag{39}$$

Now, it will be discussed that the calculation method of each terms of above equations. We already considered $\boldsymbol{M}(\boldsymbol{q})$, $\boldsymbol{C}(\boldsymbol{q},\boldsymbol{\mu})$ and $\boldsymbol{g}(\boldsymbol{q},\boldsymbol{\eta})$ in the previous subsections. Hence, only unknown terms are drag forces part.

Using the Newton-Euler method or the Lagrangian method to obtain the manipulator dynamics, the vehicle may have a initial velocity and acceleration, $\boldsymbol{\nu}$ and $\dot{\boldsymbol{\nu}}$, then:

$$\begin{aligned} &\boldsymbol{M}_m(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}_m(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{D}_m(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}_m(\boldsymbol{\eta},\boldsymbol{q}) + \\ &\quad \boldsymbol{M}_{12}{}^T(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{21}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}_{21}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau}_m. \end{aligned} \tag{40}$$

Eq. (40) gives $\boldsymbol{D}_{21}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})$.

For the vehicle, the weight and motion of the manipulator will give interaction effects. These forces and moments, $\boldsymbol{\sigma}_m = [\boldsymbol{\sigma}_f{}^T, \boldsymbol{\sigma}_n{}^T]^T$, which are derived from the last iteration of the manipulator of the Newton-Euler method, can be calculated as:

$$\begin{aligned} \boldsymbol{\sigma}_f &= {}^B\boldsymbol{R}_0{}^0\boldsymbol{f}_0, \\ \boldsymbol{\sigma}_n &= {}^B\boldsymbol{R}_0{}^0\boldsymbol{n}_0 + {}^B\boldsymbol{p}_0 \times {}^B\boldsymbol{R}_0{}^0\boldsymbol{f}_0. \end{aligned} \tag{41}$$

And, it can be categorized as follows:

$$\begin{aligned} &\boldsymbol{M}_s(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{C}_s(\boldsymbol{q},\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}_s(\boldsymbol{q},\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}_s(\boldsymbol{\eta},\boldsymbol{q}) + \\ &\quad \boldsymbol{M}_{12}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}_{12}(\boldsymbol{q},\boldsymbol{\mu})\dot{\boldsymbol{q}} + \boldsymbol{D}_{12}(\boldsymbol{q},\boldsymbol{\mu})\dot{\boldsymbol{q}} = \begin{bmatrix} \boldsymbol{\sigma}_f \\ \boldsymbol{\sigma}_n \end{bmatrix}. \end{aligned} \tag{42}$$

From Eq. (42), we can fill up the last unknown terms, $\boldsymbol{D}_s(\boldsymbol{q},\boldsymbol{\nu})$ and $\boldsymbol{D}_{12}(\boldsymbol{q},\boldsymbol{\mu})$.

Finally, the $\boldsymbol{\tau}_v$ can be expressed as follows:

$$\begin{aligned} &\boldsymbol{M}_v\dot{\boldsymbol{\nu}} + \boldsymbol{C}_v(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}_v(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}_v(\boldsymbol{\eta}) + \\ &\boldsymbol{M}_s(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{C}_s(\boldsymbol{q},\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}_s(\boldsymbol{q},\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}_s(\boldsymbol{\eta},\boldsymbol{q}) + \\ &\boldsymbol{M}_{12}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}_{12}(\boldsymbol{q},\boldsymbol{\mu})\dot{\boldsymbol{q}} + \boldsymbol{D}_{12}(\boldsymbol{q},\boldsymbol{\mu})\dot{\boldsymbol{q}} = \boldsymbol{\tau}_v. \end{aligned} \tag{43}$$

The equations of motion for the total system in the earth-fixed reference frame can be written in the form

$$\boldsymbol{M}_I(\boldsymbol{q},\boldsymbol{\eta})\ddot{\boldsymbol{\xi}} + \boldsymbol{c}_I(\boldsymbol{q},\boldsymbol{\eta},\boldsymbol{\mu}) + \boldsymbol{d}_I(\boldsymbol{q},\boldsymbol{\eta},\boldsymbol{\mu}) + \boldsymbol{g}_I(\boldsymbol{q},\boldsymbol{\eta}) = \boldsymbol{\tau}_I, \tag{44}$$

where $\boldsymbol{\xi} = [\boldsymbol{\eta}^T, \boldsymbol{p}_e^T]^T$ and let

$$\boldsymbol{\Lambda}(\boldsymbol{q},\boldsymbol{\eta}) \triangleq (\boldsymbol{J}(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{J}^T(\boldsymbol{q},\boldsymbol{\eta}))^{-1}, \tag{45}$$

then:

$$\begin{aligned} \boldsymbol{M}_I(\boldsymbol{q},\boldsymbol{\eta}) &= \boldsymbol{\Lambda}(\boldsymbol{q},\boldsymbol{\eta}), \\ \boldsymbol{c}_I(\boldsymbol{q},\boldsymbol{\eta},\boldsymbol{\mu}) &= \boldsymbol{\Lambda}(\boldsymbol{q},\boldsymbol{\eta}) \cdot \\ &\quad (\boldsymbol{J}(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{M}^{-1}(\boldsymbol{q})(\boldsymbol{C}(\boldsymbol{q},\boldsymbol{\mu})\dot{\boldsymbol{q}} - \dot{\boldsymbol{J}}(\boldsymbol{q},\boldsymbol{\eta})\dot{\boldsymbol{q}}), \\ \boldsymbol{d}_I(\boldsymbol{q},\boldsymbol{\eta},\boldsymbol{\mu}) &= \boldsymbol{\Lambda}(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{J}(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{D}(\boldsymbol{q},\boldsymbol{\mu})\dot{\boldsymbol{q}}, \\ \boldsymbol{g}_I(\boldsymbol{q},\boldsymbol{\eta}) &= \boldsymbol{\Lambda}(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{J}(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{g}(\boldsymbol{q},\boldsymbol{\mu}), \end{aligned}$$

and

$$\boldsymbol{\tau} = \boldsymbol{J}^T(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{\tau}_I.$$

TWO-TIME SCALE CONTROL

For the underwater manipulation, the vehicle control is not desirable, because the small movement of vehicle affect significant changes of the end-effector. And also, the vehicle actuator is small bandwidth system, so the error which comes from the interaction of manipulator movement cannot be completely compensated. Therefore, it is desirable that the vehicle remains passive system or apply simple P-type controller. In this case, the residual error can be compensated by manipulator. In this paper, the active damping control with two-time scale approach is proposed (Lew and Moon, 2001). And, it is formulated in the operational space. In this case, there is difficulty in control problem, that is, one control input has to control two system.

For the UVM systems, a passive vehicle approach is used by Chuang *et al.* (Chung, Eom, Yi, Suh, Oh, Chung, and Kim, 2001). They applied disturbance observer(DOB) method. However, they controlled the vehicle position by manipulator directly, so it has a lot possibility actuator saturation. Also, they do not consider the characteristics of the UVM systems.

We proposed here an efficient control scheme for UVM systems considering the system characteristics. It is used that a passive vehicle approach or simple P-type controller for the vehicle and the two-time scale controller for the manipulator to control the total system. In case of vehicle P-type controller, we can simply apply just adding the spring terms in the vehicle dynamics. Then, even if the vehicle is lying under control, we can deal as passive vehicle system. To implement this controller on a passive system, we also need the controllability definition (Spong, 1994, Mahindrakar and Banavar, 2002, Arai and Tachi, 1991).

Partially Linearized Form

To control the whole UVM systems as decentralized system, a partially linearized form of the dynamics system is useful (Arai, Tanie, and Tachi, 1993). In this subsection, we explain the procedure and derive the equations.

The overall system can be represented as

$$\begin{bmatrix} \boldsymbol{M}_{11}(\boldsymbol{q}) & \boldsymbol{M}_{12}(\boldsymbol{q}) \\ \boldsymbol{M}_{12}{}^T(\boldsymbol{q}) & \boldsymbol{M}_{22}(\boldsymbol{q}) \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\nu}} \\ \ddot{\boldsymbol{q}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{h}_1(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu}) \\ \boldsymbol{h}_2(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\tau}_v \\ \boldsymbol{\tau}_m \end{bmatrix}, \tag{46}$$

where the $\boldsymbol{h}_1(\boldsymbol{\eta},\boldsymbol{q},\mu)$ and $\boldsymbol{h}_2(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu})$ are the nonlinear functions which contain the Coriolis and Centripetal, drag forces and total buoyancy forces. The detail equations are as following:

$$\begin{aligned} \begin{bmatrix} \boldsymbol{h}_1(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu}) \\ \boldsymbol{h}_2(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu}) \end{bmatrix} &= \begin{bmatrix} \boldsymbol{C}_{11}(\boldsymbol{q},\boldsymbol{\nu}) & \boldsymbol{C}_{12}(\boldsymbol{q},\boldsymbol{\mu}) \\ \boldsymbol{C}_{21}(\boldsymbol{q},\boldsymbol{\mu}) & \boldsymbol{C}_{22}(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix} \boldsymbol{\mu} \\ &\quad + \begin{bmatrix} \boldsymbol{D}_{11}(\boldsymbol{q},\boldsymbol{\nu}) & \boldsymbol{D}_{12}(\boldsymbol{q},\boldsymbol{\mu}) \\ \boldsymbol{D}_{21}(\boldsymbol{q},\boldsymbol{\mu}) & \boldsymbol{D}_m(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix} \boldsymbol{\mu} + \begin{bmatrix} \boldsymbol{g}_1(\boldsymbol{\eta},\boldsymbol{q}) \\ \boldsymbol{g}_2(\boldsymbol{\eta},\boldsymbol{q}) \end{bmatrix}. \end{aligned} \tag{47}$$

And, if the P-type vehicle controller in the operational space is given as

$$\boldsymbol{\tau}_v = -\boldsymbol{J}_v{}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_v\boldsymbol{\eta}, \tag{48}$$

where $\boldsymbol{K}_v$ is the proportional gain matrix, then the Eq. (47) is rewritten as the form:

$$\boldsymbol{M}_{11}(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{M}_{12}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}_1(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu}) + {\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_v\boldsymbol{\eta} = \mathbf{0}, \tag{49}$$

$$ {\boldsymbol{M}_{12}}^T(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{M}_{22}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}_2(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu}) = \boldsymbol{\tau}_m. \tag{50}$$

For the passive vehicle system, we just set the proportional gain matrix as $\mathbf{0}$.

And, we can invoke $\dot{\boldsymbol{\nu}}$ like following:

$$\dot{\boldsymbol{\nu}} = -{\boldsymbol{M}_{11}}^{-1}(\boldsymbol{q})\left(\boldsymbol{M}_{12}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}_1(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu}) + {\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_v\boldsymbol{\eta}\right). \tag{51}$$

Substituting Eq. (51) to Eq. (50), then

$$\boldsymbol{\tau}_m = \boldsymbol{M}^*\ddot{\boldsymbol{q}} + \boldsymbol{h}^*, \tag{52}$$

where

$$\begin{aligned} \boldsymbol{M}^* =& \boldsymbol{M}_{22}(\boldsymbol{q}) - {\boldsymbol{M}_{12}}^T(\boldsymbol{q}){\boldsymbol{M}_{11}}^{-1}(\boldsymbol{q})\boldsymbol{M}_{12}(\boldsymbol{q}),\\ \boldsymbol{h}^* =& \boldsymbol{h}_2(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu})\\ &- {\boldsymbol{M}_{12}}^T(\boldsymbol{q}){\boldsymbol{M}_{11}}^{-1}(\boldsymbol{q})\{\boldsymbol{h}_1(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu}) + {\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_v\boldsymbol{\eta}\}. \end{aligned} \tag{53}$$

Then, for the manipulator controller, a feedback linearizing control law is introduced as follows:

$$\boldsymbol{\tau}_m = \boldsymbol{M}^*\boldsymbol{u} + \boldsymbol{h}^*, \tag{54}$$

where $\boldsymbol{u} \in \Re^m$ is an additional control input to be yet specified. The substitution of Eq. (54) into Eq. (50) yields a partially linearized system as follows:

$$\boldsymbol{M}_{11}(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{h}_1(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu}) + {\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_v\boldsymbol{\eta} \quad = \quad -\boldsymbol{M}_{12}(\boldsymbol{q})\boldsymbol{u}, \tag{55}$$

$$\ddot{\boldsymbol{q}} \quad = \quad \boldsymbol{u}. \tag{56}$$

Controllability Condition

To ensure the manipulator control input to be able to control the vehicle, we have to check the controllability condition for the vehicle control input, $-\boldsymbol{M}_{12}(\boldsymbol{q})\boldsymbol{u}$. To impact effective acceleration to the passive vehicle from the active manipulator, a *strong inertial coupling* between the joints is required (Spong, 1994).

$$rank(\boldsymbol{M}_{12}(\boldsymbol{q})) = l \qquad \text{for all } \boldsymbol{q} \in \Re^n, \tag{57}$$

where l is the dimension of vehicle state, $\boldsymbol{\eta} \in \Re^l$. Note that strong inertial coupling requires $n \geq l$, i.e., that the number of manipulator degrees of freedom is at least as great as the number of vehicle degrees of freedom.

Under the assumption of strong inertial coupling, we may compute a Moore-Penrose pseudo-inverse ${\boldsymbol{M}_{12}}^{\#}(\boldsymbol{q})$ for $\boldsymbol{M}_{12}(\boldsymbol{q})$ according to

$${\boldsymbol{M}_{12}}^{\#} = {\boldsymbol{M}_{12}}^T(\boldsymbol{q})(\boldsymbol{M}_{12}(\boldsymbol{q}){\boldsymbol{M}_{12}}^T(\boldsymbol{q}))^{-1}. \tag{58}$$

Two-time Scale Control

A composite (fast and slow time scale) controller is designed based on the partially decoupled model Eqs. (55) and (56). Recall that the control objective is to determine the control input $\boldsymbol{u}$ that stabilizes both Eqs. (55) and (56). Define the control input, $\boldsymbol{u}$, to have two parts

$$\boldsymbol{u} = \boldsymbol{u}_{slow} + \boldsymbol{u}_{fast}. \tag{59}$$

Using two-time scale theory, $\boldsymbol{u}_{fast}$ and $\boldsymbol{u}_{slow}$ are chosen in two different time scales. It is intended that $\boldsymbol{u}_{slow}$ affects mainly on Eq. (55), slow vehicle motion and $\boldsymbol{u}_{fast}$ mainly on Eq. (56), fast decoupled manipulator dynamics. For $\boldsymbol{u}_{fast}$, a typical linear tracking controller is proposed with high gains such that the control bandwidth is much higher than the natural frequency of the vehicle motion.

From the Eqs. (9) and (10), we can obtain the manipulator joint acceleration as following:

$$\begin{aligned}\ddot{\boldsymbol{q}} = {\boldsymbol{J}_m}'^{\#}(\boldsymbol{q})[{\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})\{\ddot{\boldsymbol{p}}_e - (\dot{\boldsymbol{J}}_v(\boldsymbol{\eta})\boldsymbol{J}_1(\boldsymbol{\eta},\boldsymbol{q}) + \boldsymbol{J}_v(\boldsymbol{\eta})\dot{\boldsymbol{J}}_1(\boldsymbol{\eta},\boldsymbol{q}))\boldsymbol{\nu}\\ - (\dot{\boldsymbol{J}}_v(\boldsymbol{\eta}){\boldsymbol{J}_m}'(\boldsymbol{q}) + \boldsymbol{J}_v(\boldsymbol{\eta}){\dot{\boldsymbol{J}}_m}'(\boldsymbol{q}))\dot{\boldsymbol{q}}\}\\ - \boldsymbol{J}_1(\boldsymbol{\eta},\boldsymbol{q})\dot{\boldsymbol{\nu}}].\end{aligned} \tag{60}$$

And, $\boldsymbol{u}_{fast}$ can be chosen as

$$\begin{aligned}&\boldsymbol{u}_{fast} =\\ &{\boldsymbol{J}_m}'^{\#}(\boldsymbol{q})[{\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})\{(\ddot{\boldsymbol{p}}_e)_{ref} - (\dot{\boldsymbol{J}}_v(\boldsymbol{\eta})\boldsymbol{J}_1(\boldsymbol{\eta},\boldsymbol{q}) + \boldsymbol{J}_v(\boldsymbol{\eta})\dot{\boldsymbol{J}}_1(\boldsymbol{\eta},\boldsymbol{q}))\boldsymbol{\nu}\\ &\qquad - (\dot{\boldsymbol{J}}_v(\boldsymbol{\eta}){\boldsymbol{J}_m}'(\boldsymbol{q}) + \boldsymbol{J}_v(\boldsymbol{\eta}){\dot{\boldsymbol{J}}_m}'(\boldsymbol{q}))\dot{\boldsymbol{q}}\}\\ &- \boldsymbol{J}_1(\boldsymbol{\eta},\boldsymbol{q})\dot{\boldsymbol{\nu}}],\end{aligned} \tag{61}$$

where

$$(\ddot{\boldsymbol{p}}_e)_{ref} = (\ddot{\boldsymbol{p}}_e)_d + \boldsymbol{K}_D((\dot{\boldsymbol{p}}_e)_d - \dot{\boldsymbol{p}}_e) + \boldsymbol{K}_P((\boldsymbol{p}_e)_d - \boldsymbol{p}_e). \tag{62}$$

$\boldsymbol{K}_P$ and $\boldsymbol{K}_D$ are the proportional and derivative gain matrix, respectively. Then, the closed loop error dynamics for the manipulator in the operational space becomes

$${\boldsymbol{J}_m}'^{\#}(\boldsymbol{q}){\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})\{\ddot{\boldsymbol{e}}_p + \boldsymbol{K}_D\dot{\boldsymbol{e}}_p + \boldsymbol{K}_P\boldsymbol{e}_p\} = -\boldsymbol{u}_{slow}, \tag{63}$$

where $\boldsymbol{e}_p$ is defined by the position error, $(\boldsymbol{p}_e)_d - \boldsymbol{p}_e$. If $\boldsymbol{u}_{slow}$ changes slowly compared to the fast error dynamics, it can be treated as more like a constant. Then the error dynamics of Eq. (63) converge to be very small with positive definite matrices $\boldsymbol{K}_P$ and $\boldsymbol{K}_D$.

Now, the slow control input, $\boldsymbol{u}_{slow}$, is defined for the vehicle in Eq. (55) as

$$\begin{aligned}&\boldsymbol{u}_{slow} =\\ &\quad {\boldsymbol{M}_{12}}^{\#}(\boldsymbol{q})\{\boldsymbol{M}_{11}(\boldsymbol{q}){\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})((\ddot{\boldsymbol{\eta}})_{ref} + \dot{\boldsymbol{J}}_v(\boldsymbol{\eta})\boldsymbol{\nu}) - \boldsymbol{h}_1(\boldsymbol{\eta},\boldsymbol{q},\boldsymbol{\mu})\},\end{aligned} \tag{64}$$

where

$$(\ddot{\boldsymbol{\eta}})_{ref} = \boldsymbol{K}_d\dot{\boldsymbol{\eta}}. \tag{65}$$

Only the derivative feedback of the vehicle with gain $\boldsymbol{K}_d$ is added to increase the collapsing the vehicle velocity.

Then, after applying the composite controller and let ${\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_v$ as $\boldsymbol{M}_{11}(\boldsymbol{q}){\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})\boldsymbol{K}_p$, Eq. (55) becomes

$$\begin{aligned}\boldsymbol{M}_{11}(\boldsymbol{q}){\boldsymbol{J}_v}^{-1}(\boldsymbol{\eta})\{\ddot{\boldsymbol{e}}_\eta + \boldsymbol{K}_d\dot{\boldsymbol{e}}_\eta + \boldsymbol{K}_p\boldsymbol{e}_\eta\} &= -\boldsymbol{M}_{12}(\boldsymbol{q})\boldsymbol{u}_{fast},\\ &\approx -\boldsymbol{M}_{12}(\boldsymbol{q})\ddot{\boldsymbol{q}}_{ref}.\end{aligned} \tag{66}$$

It can be easily shown that the second line of Eq. (66) is bounded since $\ddot{\boldsymbol{q}}_{ref}$ is bounded. Therefore, as long as K_d and K_p are chosen as a positive definite matrix, Eq. (66) remains stable.

Figure 3 shows the composite controller schematically.

SIMULATION AND EXPERIMENT

Figure 4 shows the ODIN in the University of Hawaii, which is well developed and verified UUV system, especially omni-directional actuator. And, its symmetric structure enable us to easily get the dynamic parameters with little modeling uncertainties. Now, we have made an effort to design the attached underwater manipulator as Fig. 5. For the future works, various kinds of simulation and experiment will be performed using new developing ODIN system and the proposed control algorithm in a moment.

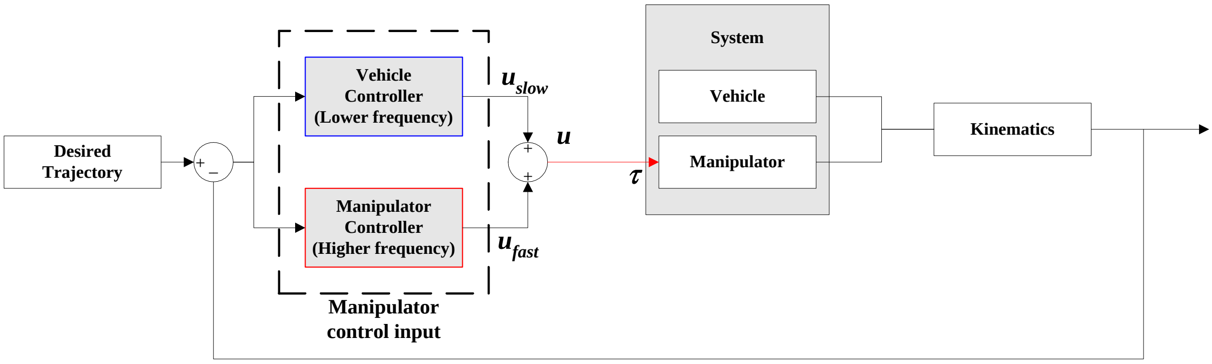

Fig. 3. Block diagram of the control system

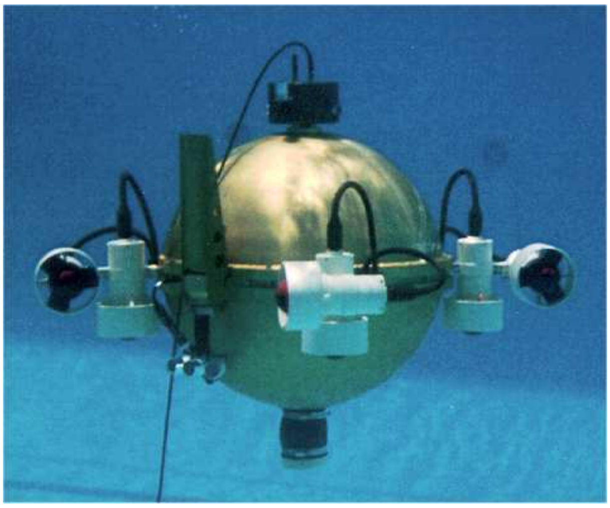

Fig. 4. ODIN in UH

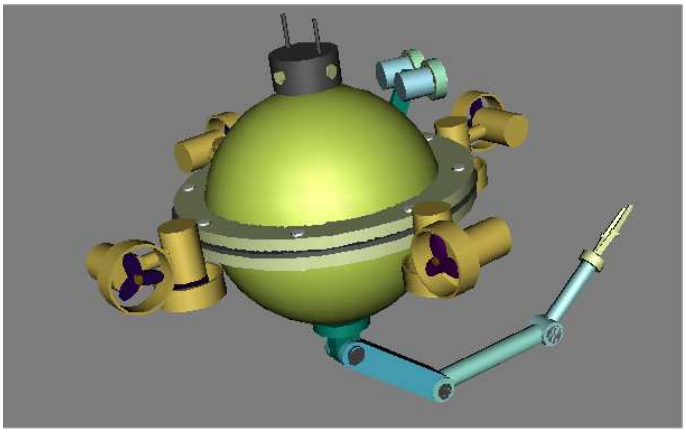

Fig. 5. Concept of ODIN and manipulator

CONCLUSIONS

In this paper, the kinematic and dynamic modeling for UVM systems are analyzed in efficient way. It can gives the advantage to design of decentralized controller with considering the interaction between the two subsystems. Nextly, the two-time scale controller is proposed for the passive vehicle system. Using different time scale controller, fast time scale controller is used for the manipulator and slow time scale controller for the vehicle according to the system bandwidth. Active damping controller for the vehicle can help the base of whole system not going far.

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