SUBMARINE DYNAMICS MODEL

INTRODUCTION

When a submarine is deeply submerged, many of its maneuvering characteristics can be determined from application of Morison's equation to model test data. A series of trials, often done with a planar motion mechanism (PMM), give the damping and inertia coefficients for small maneuvers in each of the six degrees of freedom. This method is not without limits. For trials done in the horizontal and vertical planes only, nonlinear cross coupling effects are ignored. The hydrodynamic coefficients work poorly for prediction of high speed maneuvers and control surface casualties. Here the large crossflow velocities, vortex hull interaction, and flow separation all have effects which are not predicted by the hydrodynamic coefficients. It is possible, however, to include some of these effects as additional nonlinear terms.

As the submarine approaches the free surface, several complexities are introduced into the hydrodynamic coefficient approach. First, the inertia terms change as an acceleration will no longer act upon an effectively infinite region. Second, an inviscid form of damping exists near the free surface. This comes about from the generation of waves by the body, and depends on the body depth and character of motion. Finally, the interaction between the incident waves and the submarine introduces added forces and moments. These effects combine to make designing for periscope depth vexing for engineers and operating at periscope depth an art for the ship's crew.

The approach in this thesis will be to first establish a dynamics model appropriate for a deeply submerged submarine at low to moderate speeds. The forces and moments resulting from the seaway will then be superimposed on this model to provide a reasonable approximation to the submarine motion beneath waves.

DEEPLY SUBMERGED EQUATIONS OF MOTION

Definition of coordinate system and states

The coordinate system defined in Figure 1 will be used. The origin of the global coordinate system is fixed at the ocean surface. The *z* axis is positive downward, towards the ocean bottom. The *x* axis is positive in the direction of intended submarine motion. The body fixed coordinates are rotated from the global coordinates by the angle   . Body fixed velocities *w* (heave) , *u* (surge), and *q* (pitch) are shown. The control surface deflections, δ_b (bow planes) and δ_s (stern planes) are also defined.

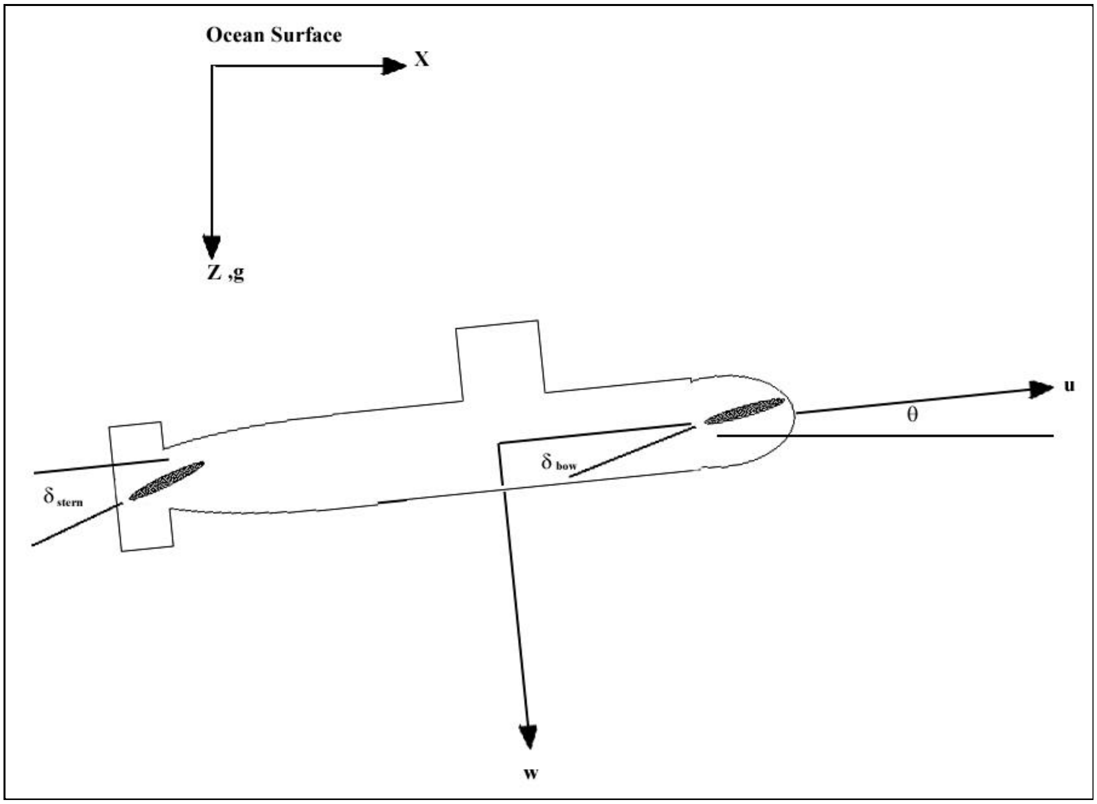

Figure 1. Coordinate System Definition

Hydrodynamic coefficients review

For a deeply submerged submarine, small motions can be analyzed using the concept of hydrodynamic coefficients.   These represent a Taylor series expansion of the functional relationship between body movements and the resulting fluid forces. For example, given the deeply submerged body in Figure 2 undergoing pure heave, resulting body forces can be expressed in the following manner:

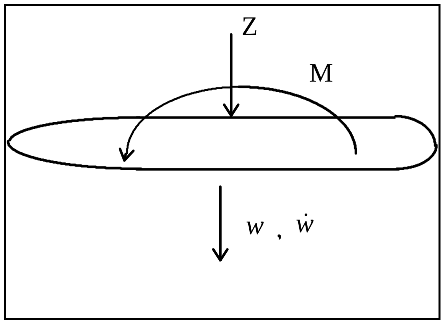

Figure 2. Submerged body in pure heave

$$M = M_w w + M_{w|w|} w|w| + M_{\dot{w}} \dot{w} \tag{1}$$

$$Z = Z_w w + Z_{w|w|} w|w| + Z_{\dot{w}} \dot{w} \tag{2}$$

This method is extended to the six degrees of freedom of the body, and done for velocity and acceleration components of the movement. This includes representations of added mass, viscous drag, and square law drag.

Vertical plane equations of motion

By using this system of notation, and applying Newton's second law to the body fixed coordinates, and transforming to global coordinates, the equations of pitch and heave may be obtained in the vertical plane. The general case is quite complex, having centers of mass and buoyancy that are separate from each other and the coordinate system origin. This, along with cross coupled hydrodynamic coefficients, results in a nonlinear, coupled set of differential equations.

These equations of pitch and heave may be simplified considerably by several reasonable assumptions. Assuming that the submarine motion is constrained to the vertical plane, the equations of motion for heave and pitch are (Smith, Crane, and Summey (1978)):

$$\begin{aligned} m[\dot{w} - uq - x_G \dot{q} - z_G q^2] = \\ & Z_{\dot{q}} \dot{q} + Z_{\dot{w}} \dot{w} + Z_q uq + Z_w w \\ & + u^2 (Z_{\ b} \ b + Z_{\ s} \ s) \end{aligned} \tag{3}$$

$$I_y\dot{q} - m[x_G(\dot{w} - uq) - z_G(\dot{u} + wq)] = \tag{4}$$

$$\begin{aligned} &M_{\dot{q}}\dot{q} + M_{\dot{w}}\dot{w} + M_q uq + M_w uw \\ &+ u^2(M_{\ \ _b}\ _b + M_{\ \ _s}\ _s) \\ &-(x_G mg - x_B B)\cos(\ \ ) \\ &-(z_G mg - z_B B)\sin(\ \ ) \end{aligned}$$

It is apparent that Equations (3) and (4) are nonlinear, coupled differential equations in *w* and *q* and *u*. To reduce this coupling, terms involving the derivatives of *w* and *q* can be collected, resulting in a mass matrix.

$$\overline{M} = \begin{bmatrix} m - Z_{\dot{w}} & -Z_{\dot{q}} \\ -M_{\dot{w}} & I_y - M_{\dot{q}} \end{bmatrix} \tag{5}$$

The mass matrix can be readily inverted:

$$\overline{M}^{-1} = \frac{\begin{bmatrix} I_y - M_{\dot{q}} & Z_{\dot{q}} \\ M_{\dot{w}} & m - Z_{\dot{w}} \end{bmatrix}}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}} \tag{6}$$

By applying Equation (6), the cross coupling of terms in $\dot{w}$ and $\dot{q}$ can be removed from Equations (3) and (4). To allow the introduction of external forces and moments, the system was augmented by force and moment disturbances acting at the origin of the body fixed coordinates. They were multiplied by the cosine of the pitch angle for conversion to the body fixed coordinate system. These disturbances can be used to input external effects, such as changes in trim and wave forces. By further assuming that the center of buoyancy is at the body fixed coordinate system origin, the center of mass is directly below, and that the forward speed *u* is constant, the equations of motion can be reduced to the following:

$$\dot{w} = a_{11}uw + a_{12}uq + a_{13}\sin(\ \ ) + b_{11}u^2\ \ _b + b_{12}u^2\ \ _s + F_d\cos(\ \ ) + e_{11}q^2 + e_{12}qw \tag{7}$$

$$\dot{q} = a_{21}uw + a_{22}uq + a_{23}\sin(\ \ ) + b_{21}u^2\ \ _b + b_{22}u^2\ \ _s + M_d\cos(\ \ ) + e_{21}q^2 + e_{22}qw \tag{8}$$

$$\dot{\ } = q \tag{9}$$

$$\dot{z} = w\cos(\ \ ) - u\sin(\ \ ) \tag{10}$$

$$\dot{x} = w\sin(\ \ ) + u\cos(\ \ ) \tag{11}$$

where:

$$a_{11} = \frac{Z_w(I_y - M_{\dot{q}}) + Z_{\dot{q}} M_w}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$a_{12} = \frac{(Z_q + m)(I_y - M_{\dot{q}}) + Z_{\dot{q}} M_q}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$a_{21} = \frac{M_{\dot{w}} Z_{\dot{w}} + (m - Z_{\dot{w}}) M_w}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$a_{22} = \frac{M_{\dot{w}}(Z_q + m) + (m - Z_{\dot{w}}) M_q}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$a_{13} = \frac{Z_{\dot{q}} z_G m g}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$a_{23} = \frac{(m - Z_{\dot{w}}) z_G m g}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$b_{11} = \frac{(I_y - M_{\dot{q}}) Z_{b} + Z_{\dot{q}} M_{b}}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$b_{21} = \frac{M_{\dot{w}} Z_{b} + M_{b}(m - Z_{\dot{w}})}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$b_{12} = \frac{(I_y - M_{\dot{q}}) Z_{s} + Z_{\dot{q}} M_{s}}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$b_{22} = \frac{M_{\dot{w}} Z_{s} + M_{s}(m - Z_{\dot{w}})}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$e_{11} = \frac{(I_y - M_{\dot{q}}) z_G m}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$e_{12} = \frac{Z_{\dot{q}} z_G m}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$e_{21} = \frac{M_{\dot{w}} z_G m}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

$$e_{22} = \frac{(m - Z_{\dot{w}})z_G m}{(m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - Z_{\dot{q}} M_{\dot{w}}}$$

Equations (7) through (11) are the governing equations of motion for this thesis. It is of note that the disturbance force and moment terms represent accelerations due to the disturbances. To provide ease of use, the equations of motion were implemented in the SIMULINK® model shown in Figure 3. This building block approach was very effective for conducting studies on the effectiveness of different types of controllers.

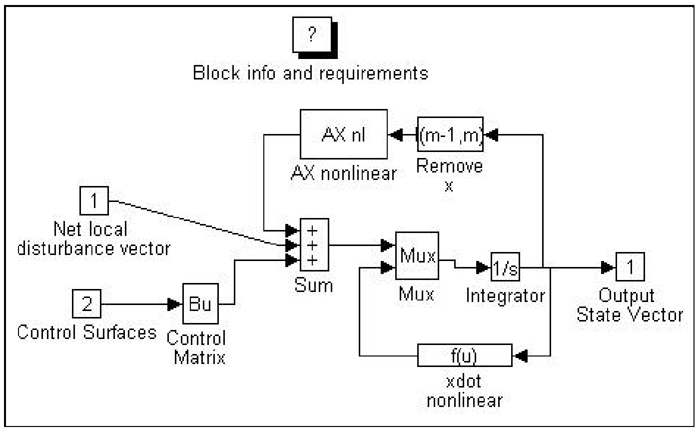

Figure 3. SIMULINK® model of vertical plane submarine dynamics

For control design, it is convenient to use a linear state space representation of the system. This allows the use of a variety of controller design tools including pole placement and linear quadratic regulator algorithms. Equations (7) through (11) can be linearized about a level flight condition. This results in the linear state space representation:

$$\dot{w} = a_{11}uw + a_{12}uq + a_{13} \quad + b_{11}u^2 \ _b + b_{11}u^2 \ _s + F_d \tag{12}$$

$$\dot{q} = a_{21}uw + a_{22}uq + a_{23} \quad + b_{21}u^2 \ _b + b_{22}u^2 \ _s + M_d \tag{13}$$

$$\dot{} = q \tag{14}$$

$$\dot{z} = w - u \tag{15}$$

$$\dot{x} = w \quad + u \tag{16}$$

Equations (12) through (15) can be rewritten in matrix form. This form of the linear submarine vertical plane dynamics equations will be used for controller design. For controller design, Equation (16) was excluded from the matrix form. Because of the constant forward speed u assumption, there was no direct means of control for x.

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} a_{11}u & a_{12}u & a_{13} & 0 \\ a_{21}u & a_{22}u & a_{23} & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & u & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \\ z \end{bmatrix} + \begin{bmatrix} b_{11}u^2 & b_{12}u^2 \\ b_{21}u^2 & b_{22}u^2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} {}_b \\ {}_s \end{bmatrix} + \begin{bmatrix} F_d \\ M_d \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

EXTENSION TO VERTICAL PLANE PATHKEEPING

Equations (7) through (10) and the corresponding SIMULINK® model are linearized around a constant commanded depth, or level flight. They can be extended to a two dimensional pathkeeping simulation by a coordinate transformation. After coordinate rotation by an angle β (positive in the same direction as), the resulting system is:

$$\dot{w} = a_{11}uw + a_{12}uq + a'_{13}\sin(\ ') + b_1u^2 \quad + F_d{}' \tag{18}$$

$$\dot{q} = a_{21}uw + a_{22}uq + a'_{23}\sin(\ ') + b_2u^2 \quad + M_d{}' \tag{19}$$

$$\dot{}' = q \tag{20}$$

$$\dot{z}' = w\cos(\ ') - u\sin(\ ') \tag{21}$$

$$\dot{x}' = w\sin(\ ') + u\cos(\ ') \tag{22}$$

where:

$$' = \ - \tag{23}$$

$$x' = -z\sin(\ ) + x\cos(\ ) \tag{24}$$

$$z' = z\cos(\ ) + x\sin(\ ) \tag{25}$$

$$a_{13}{}' = a_{13}\cos(\ ) \tag{26}$$

$$a_{23}{}' = a_{23}\cos(\ ) \tag{27}$$

$$F_d' = F_d + a_{13}\cos(\ \ ')\sin(\ \ ) \tag{28}$$

$$M_d' = M_d + a_{23}\cos(\ \ ')\sin(\ \ ) \tag{29}$$

If the expected angular deviation from the planned path is small, Equations (28) and (29) can be simplified by assuming that $\cos(\ \ ')$ is equal to one. Then the rotated equation set, Equations (18) through (22), is identical in form to Equations (7) through (11).

Equations (23) through (29) allow any vertical plane path consisting of a series of straight line segments to be simulated one segment at a time.

THE DARPA SUBOFF

Background

For the purpose of this work, it was desired to have a vertical plane model of submarine dynamics which would give a similar response to a modern fast attack nuclear submarine (SSN). Several sets of unclassified hydrodynamic coefficients were available, these being for the swimmer delivery vehicle (SDV) detailed in Smith, Crane, and Summey (1978) and for the DARPA SUBOFF model detailed in Roddy (1990).

The SDV had a very complete set of hydrodynamic coefficients which have been used in a large number of Autonomous Underwater Vehicle (AUV) research projects. Among these is the Naval Postgraduate School (NPS) AUV sliding mode controller, Hawkinson (1990). Despite these advantages, the SDV hydrodynamic coefficients were not used because the wing like hull of the SDV bore little resemblance to an axisymetric submarine hull.

The SUBOFF hydrodynamic coefficients detailed in Roddy (1990) lacked some of the cross coupling coefficients. The documentation also lacked details on the models metacentric height. Because the SUBOFF represented a submarine hull form and most of the vertical plane coefficients and parameters were available, it was chosen as the model for this thesis.

SUBOFF known parameters and coefficients

The SUBOFF was developed to allow comparison between flow field predictions and model test data (Roddy, 1990). The available coefficients were based on planar motion mechanism tests conducted on the model.

Because the aim of the study was to examine full scale submarine motions, the model and its hydrodynamic coefficients were scaled to a length of 300 feet. After scaling, several parameters had to be modified or assumed to give control and response comparable to a modern fast attack submarine. The force coefficients of the stern planes was doubled to provide a more realistic level force. Bow planes were assumed to have one half the force and one quarter the moment authority of the stern planes. Finally, a metacentric height of one foot was assumed, as it provided a realistic point of stern planes reversal. The resulting parameters are shown in Table 1.

Parameter	SUBOFF Model	Scaled / Modified Result
Length (Feet)	14.2917	300
Displacement (tons)	0.7704	7,7145
Maximum Diameter (Feet)	1.667	35
Metacentric Height (Feet)	Not Provided	1
XG	0.00975	0
ZG	Not Provided	1
XB	-0.006669	0
ZB	Not Provided	0
Z'_s	-0.005603	-0.011206
M'_s	-0.002409	-0.004818
Z'_b	Not Provided	-0.005603
M'_b	Not Provided	0.0012045

Table 1. SUBOFF Assumed and modified parameters

CONCLUDING REMARKS

A simplified model of submarine vertical plane dynamics was derived. The coefficients for use in this model were obtained from the DARPA SUBOFF model, which is a representative axisymetric submarine hull form. The simplified nonlinear equations of motion were incorporated in a SIMULINK® model to allow easy integration with wave force models and different controllers.

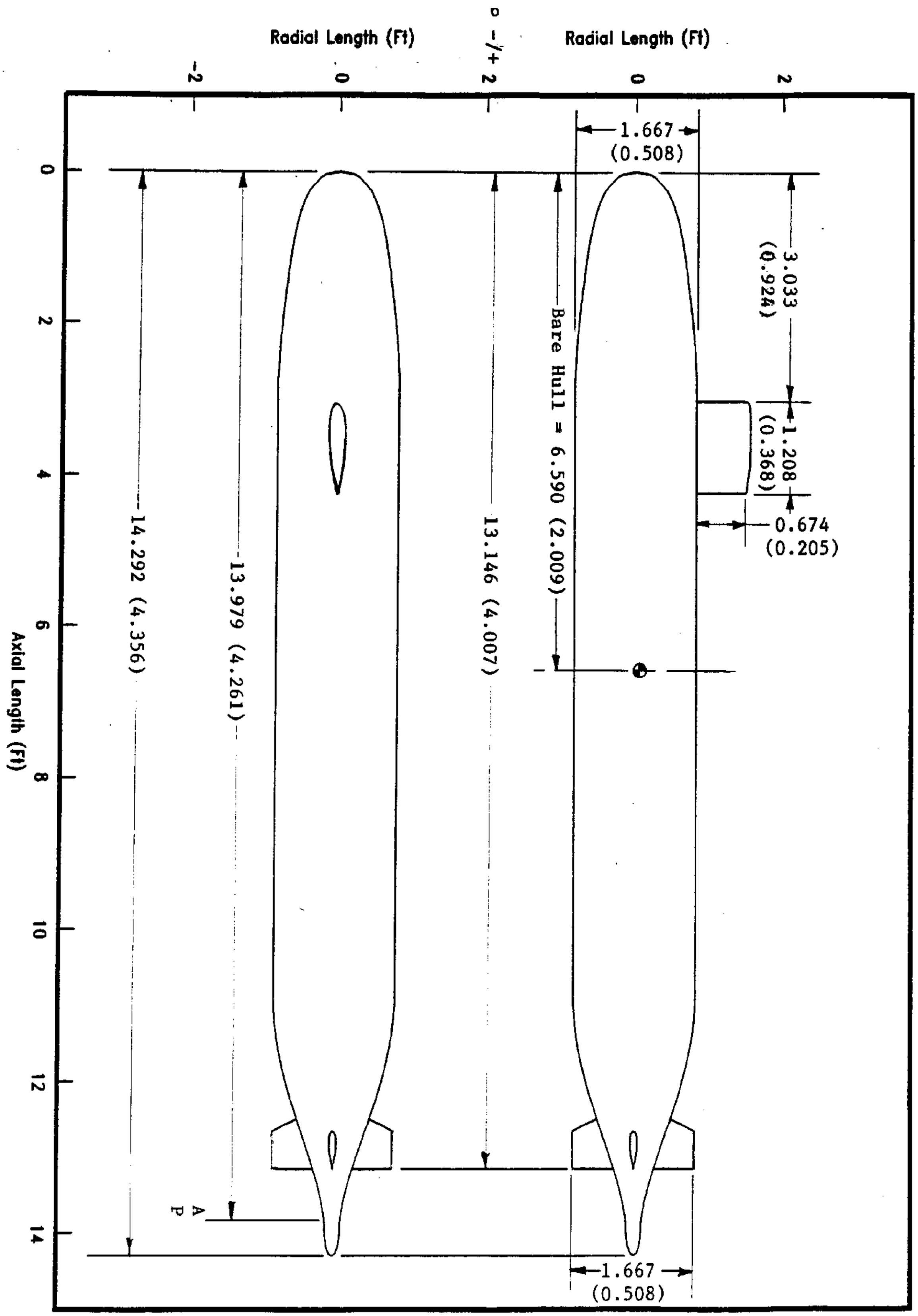

Figure 4. DARPA SUBOFF model, Roddy (1990)