REPORT No. 111

THE VARIATION OF AEROFOIL LIFT AND DRAG COEFFICIENTS WITH CHANGES IN SIZE AND SPEED

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THE VARIATION OF AEROFOIL LIFT AND DRAG COEFFICIENTS WITH CHANGES IN SIZE AND SPEED.

By Walter S. Diehl.

This report, prepared by Walter S. Diehl at the request of the National Advisory Committee for Aeronautics, contains the results of an investigation of existing scale correction data and the derivation of an original method for making these corrections rapidly and accurately. The following summary outlines briefly the subject as treated in the report.

SUMMARY.

1. General statement of the principle of dynamic similarity as applied to the problem of determining the variation of the lift and drag of an aerofoil with (variations in) the size and speed.

2. Interchangeability of v and l. Notes on limitations as found by experiment.

3. Review of existing scale correction data. Criticism and comments on the method employed by the N. P. L.

4. Determination of the variation of D_0 with scale. It is shown that the minimum drag varies as $(vl)^{1.84}$ in a number of tests and must therefore be due almost entirely to a viscosity effect. Assuming that any increase in the drag coefficient, over the minimum, is due to inertia effects, the relation between the drag coefficients at any angle for the values of vl is

$$D_{c2} = D_{c1} - D_{c0}\ [1 - \left(\frac{v_2 l_2}{v_1 l_1}\right)^{-0.16}]$$

where
$\quad D_{c2} =$ drag coefficient at $v_2 l_2$
$\quad D_{c1} =$ drag coefficient at $v_1 l_1$

and
$\quad D_{c0} =$ minimum drag coefficient at $v_1 l_1$

The formula is checked by test results.

5. Variation of L_0 with scale. It is found that at any given angle of attack the lift coefficients for the two values of vl bear the relation

$$L_{c2} = L_{c1} + \Delta L_0$$
$$= L_{c1} + .057\ \log_{10}\left(\frac{v_2 l_2}{v_1 l_1}\right)$$

where
$\quad L_{c2} =$ lift coefficient at $v_2 l_2$
and
$\quad L_{c1} =$ lift coefficient at $v_1 l_1$

The value of the constant 0.057 was determined from existing experimental data.

6. Applications and limitations. Method of applying formula. Discussion of limitations.

7. Conclusions.—It is recommended that the formulæ be checked by accurate tests made for this purpose and extending over a large range of vl.

INTRODUCTION.

A general expression for the resistance or reaction due to relative motion between a fluid and an immersed body may be written by application of the principle of dimensional homo-

geneity. If the motion be uniform and the fluid incompressible it is found that the reaction is given by

$$R = \rho v^2 l^2 f \left(\frac{vl}{\nu} \right)$$
(1)

where

ρ = density of the fluid,
v = relative velocity,
l = some linear dimension of the body,
μ = viscosity of the fluid,

and

$\nu = \frac{\mu}{\rho}$, the kinematic viscosity.

The derivation of the expression is due to Lord Rayleigh[1] and may be found in any treatise on aerodynamics. (See Bairstow, "Applied Aerodynamics," Ch. VIII.)

In most aeronautical engineering computations it is customary to neglect the variations of ν and to consider only variations of v and l. This is justified since model tests and flying are usually carried out under conditions which render ν substantially constant. The product of the velocity, v, in feet per second, and the chord, l, of an aerofoil in feet is then referred to as the "vl" or "scale" of the test or flight.

This investigation is concerned with the determination of the functions of vl which express the variation of lift and drag coefficients of an aerofoil, with particular reference to the application of model tests to full-size airplane performance.

Before discussing the previous work in this field it seems desirable to call attention to certain phenomena connected with the limitations of the interchangeability of v and l.

INTERCHANGEABILITY OF V AND L.

It should be noted that the condition of dynamic similarity, which may be expressed $v_1 l_1 = v_2 l_2$, presupposes geometrical similarity. This is equivalent to saying that geometrically similar aerofoils will give identical characteristic curves when tested at speeds inversely proportional to their chords.

This interchangeability of v and l and the dependence of aerofoil coefficients upon their product has been accepted for many years as being necessary from a physical standpoint. The validity of the assumption has sometimes been challenged but never disproved. On the other hand the results of various tests such as those made at N. P. L. on two geometrically similar aerofoils (Br. A. C. A., R. and M. No. 148) and at Göttingen on several series of geometrically similar aerofoils. (Kumbruch——Zeitschrift für Flugtechnik und Motorluftschiffahrt, May 31, 1919) are to be taken as positive proof.

However it is well known to everyone who has had occasion to study the results of many aerofoil tests that there are certain limits within which it is necessary to keep both v and l, if the data are to be reliable. For instance if the velocity of the wind during a test be less than 30 f. p. s., or if the chord of the model be less than 3 inches, the flow is determined not only by the aerofoil section but also by the method of supporting the model and the quality of the air flow, or turbulence present in air stream. The upper limit to velocity depends chiefly upon compressibility and may arbitrarily be set at 200 f. p. s., at which speed the effect is of the order of 1 per cent.

It may therefore be said with some confidence that the laws connected with dynamic similarity apply to aerofoils subject to the limitations just mentioned.

N. P. L. SCALE CORRECTIONS.

By far the greatest amount of testing for scale effect has been done at the National Physical Laboratory. In a series of three reports of the British Advisory Committee for Aeronautics (R. and M. Nos. 72, 110, and 148) monoplane aerofoil characteristics for the R. A. F.–6 section

[1] Br. A. C. A., R. and M. No. 16.

and modifications are given for values of vl from 2.5 to 40. In another report (Br. A. C. A., R. and M. No. 196) biplane R. A. F.–6 characteristics are given for values of vl from 5 to 16.5. In addition to these systematic tests a large number of aerofoils have been tested at two or more speeds.

Although the data on scale effect thus accumulated are comparatively extensive, it appears that but little attention has been given to the actual determination of the laws involved. It has been customary since R. and M. No. 72 was published to plot scale tests with the aerofoil characteristics as ordinates and vl (or log vl) as abscissae, drawing a line for each angle of attack through the values of L_c, D_c, or L/D at the corresponding values of vl. This method will give satisfactory results only so long as the curves are used on similar aerofoils. On account of the great gap between the highest vl obtainable in the present wind tunnel tests and the vl of the average machine in flight, it leads to a conclusion which is in error. Figures 1, 2, and 3 are taken from a report by Mr. E. F. Relf, "An Empirical Method for the Prediction of Wing Characteristics from Model Tests, Compiled from Existing Experimental Data" (Br. A. C. A., R. and M. No. 450, June, 1918), and presumably represent the latest N. P. L. scale correction data.

In the summary to this report the following conclusion is given: "With regard to the $\frac{vl}{v}$ correction it appears that the model results can be directly applied without any great error, if the wing vl of the test is greater than 25 in ft.^{2}/sec." It had been stated in a previous report (Br. A. C. A., R. and M. No. 72, Sec. VI) that there was no scale effect above a vl of 40. Referring to figures 1, 2, and 3 it appears on first sight that there is ample basis for this conclusion. However, if the curves from figures 1 and 3 be replotted on a logarithmic scale as in figures 4 and 5, it immediately becomes evident that the effect of scale is operating according to the same law at $vl = 40$ as at lower values of vl. It is in order to mention that this effect has been noted in a more recent report (Br. A. C. A., R. and M. No. 656, November, 1919), which states that "The drag coefficient of the model is still changing with speed at the highest speed of the experiments. The same is true of the lift coefficient to a very small extent."

The greatest difficulty experienced in applying the correction curves of figures 1, 2, and 3 occurs with high lift aerofoils, such as the RAF–19. In general it is found that the method is rather unsatisfactory on account of its limitations.

VARIATION OF D_e WITH SCALE.

An inspection of figure 5 will reveal two outstanding features. First, that the minimum drag coefficient, or rather the drag coefficient at an angle attack corresponding very nearly to the minimum drag, decreases as vl is increased in such a manner that all of the values lie on a straight line which has the slope −0.14. This indicates that the minimum drag varies as $(vl)^{1.86}$ instead of $(vl)^2$, and is consequently due almost entirely to skin friction, or more properly, perhaps, to a "viscosity effect." Second, that the higher values of D_e do not follow the same law, since the successive values of D_o, as vl is increased, lie on lines which are concave upward. Before drawing any conclusions from these observations it is desirable to examine a number of tests to see if the phenomena are universal.

Upon plotting to a logarithmic scale, as in figure 5, the drag coefficients from available tests, there is obtained in every case a group of lines very similar to those in figure 5. It is to be noted that the slope of the line representing minimum drag is slightly greater in the groups obtained from test data. This slope is quite uniform and of the average value −0.16. A specimen group is given in figure 6 to illustrate the general nature of all.

It can therefore be shown that the minimum drag of an aerofoil is almost entirely due to a viscosity effect, which is to say that

minimum drag $\propto (vl)^{1.84}$

or minimum $D_e \propto (vl)^{-0.16}$ _(2)

Now so long as the flow over the aerofoil is nonturbulent, the magnitude of the viscosity effect

can not change by any great amount. But it has been shown by Betz (Technische Berichte I–4, for translation see N. A. C. A., T. N. No. 41) that

$$D_{ol} = \frac{2L_o^2}{\pi}\left(\frac{S}{b^2}\right)$$ ---(3)

where $S =$ the area of the aerofoil,

   $b =$ the span,

and $D_{ol} =$ the coefficient of the "induced drag." That is D_{ol} is a measure of the inertia re-action, in the direction of flight, experienced by aerofoil in imparting to the encountered air the downward deflection which produces the lift represented by L_o. Since D_{ol} is an inertia effect it must vary as $(vl)^2$. Consequently the total drag, which is assumed to vary as $(vl)^2$, has two components, the one varying as $(vl)^{1.84}$, the other as $(vl)^2$. The scale correction to the coefficient of total drag, D_o, must therefore be concerned only with that part of the drag which varies as $(vl)^{1.84}$, and since this part is due to an effect which renders it practically constant over the range of angles corresponding to steady flow it follows that the effect of a change in vl is to add to or subtract from each value of D_o a constant amount. This may be expressed in symbols as:

at $v_1 l_1,$ $D_{o1} = D_{ol} + D_{ov}$ ---(4)

at $v_2 l_2,$ $D_{o2} = D_{ol} + D_{ov} \pm \Delta D_{ov}$ ---(5)

or $D_{o2} - D_{o1} = \pm \Delta D_{ov}$--(6)

where $D_{ol} =$ that part of D_o due to inertia effects and varying as $(vl)^2$.

   $D_{ov} =$ that part of D_o due to viscosity effects and varying as $(vl)^{1.84}$.

   $\Delta D_{ov} =$ the correction to D_{ov} necessary to allow for the fact that D_{ov} varies as $(vl)^{1.84}$ instead of $(vl)^2$.

In order to obtain a definite check upon the above conclusions it is necessary to com-pare at each angle the drag coefficients obtained from tests on an aerofoil at two values of vl. There should be a difference between the two coefficients at each angle (within the limits pre-viously stated), of ΔD_{ov}, which is given by

$$\Delta D_{ov} = D_{ov} - D_{ov}\left[\frac{(v_2 l_2)^{1.84}}{(v_1 l_1)^2}\right]$$

$$= D_{ov}\left[1 - \left(\frac{v_2 l_2}{v_1 l_1}\right)^{-0.16}\right]$$ --(7)

Since D_{ov} is substantially equal to the minimum value of D_o, which may be denoted by D_{oo}, the above expression may be written

$$\Delta D_o = D_{oo}\left[1 - \left(\frac{v_2 l_2}{v_1 l_1}\right)^{-0.16}\right]$$ ---(8)

A number of tests have been compared on this basis in the manner illustrated by Table I, the results being tabulated in Table II. It is found that the values of ΔD_o are not only very nearly constant but that they check very closely with values given by equation 8. It is par-ticularly to be noted that the aerofoil sections listed in Table II include every type from the double cambered RAF–20 to the deeply cambered RAF–19. The RAF–20 has a very low D_{co} and the RAF–19 a very high D_{oo}, yet the calculated and observed values of ΔD_o agree very well in each case. This agreement is to be interpreted as a strong confirmation of the formula, which appears to be a very satisfactory approximation applying equally well to all aerofoils.

For convenience in making corrections and comparisons the expression $\left(\frac{v_2 l_2}{v_1 l_1}\right)^{-0.16}$ has been plotted in Fig. 8.

VARIATION OF L_o WITH SCALE.

There is very little to be learned from an inspection of figure 4 in regard to the variation of L_o with vl. Experimental data when plotted on logarithmic scales agree very well with

figure 4 but are disappointing on account of the low ranges of vl. A typical plot is given in figure 7 to illustrate the general appearance of test data.

If a careful study be made of the various tests it will be observed that the effect of increasing vl is to increase by a small amount each lift coefficient within the range of angles corresponding to nonturbulent flow. It will also be observed that the average increase in L_c is the same absolute quantity when vl is increased, for example, from 5 to 10 as from 10 to 20. That is to say, the average value of $\triangle L_c$ is proportional to the ratio $\left(\frac{v_2 l_2}{v_1 l_1}\right)$ and increases arithmetically as vl increases geometrically. $\triangle L_c$ should therefore be given by an expression of the form

$$\triangle L_c = K \cdot \log \left(\frac{v_2 l_2}{v_1 l_1}\right) \tag{9}$$

where K is a constant to be determined from test data.

Table III contains data from a series of tests on two RAF-6a aerofoils with the corresponding values of $\triangle L_c$. The same method was employed on other tests to obtain the values of $\triangle L_c$ given in Table IV. The results are surprisingly consistent when consideration is given to the fact that $\triangle L_c$ is obtained as the differences between two nearly equal values of L_c, each subject under the best of conditions to an error of 2 per cent or more. The average of a number of readings should eliminate such errors, however.

Table IV contains all data used in the determination of K, which is found to be

$$K = .057$$

The value of the lift coefficient L_{c2} at a given angle of attack and $v_2 l_2$ is therefore given by

$$L_{c2} = L_{c1} + .057 \log_{10} \left(\frac{v_2 l_2}{v_1 l_1}\right) \tag{10}$$

where L_{c1} is the lift coefficient at the same angle and $v_1 l_1$.

It is to be noted that the value of K seems independent, not only of the type of aerofoil section but also of the arrangement, i. e., monoplane or biplane.

The method has been applied to tests on a complete model airplane (Br. A. C. A., R. and M. No. 656) with satisfactory results. It is unfortunate that free flight test data available for comparison are too erratic to be used, except at large angles where it checks very well with that calculated by 10.

APPLICATIONS AND LIMITATIONS.

In applying these corrections it is necessary to employ data obtained at a vl sufficiently high to eliminate all uncertainty in regard to steadiness of flow. There are so many factors which influence steadiness of flow that it is difficult to specify a lower limit to vl although in general it may be said that the results obtained from tests on a model of 3'' chord at 40 f. p. s. are reliable, but neither velocity nor chord should ever be less than these figures.

The application of the corrections should also be limited to the range of angles corresponding to steady flow roughly from zero lift to maximum lift.

In regard to limitations, there have been tests on certain double cambered aerofoils in which the lift coefficient was found to decrease as vl was increased. Data are lacking to indicate the cause of this reversal, but since other double cambered aerofoils behave in the usual manner it is possible that the phenomena may be due to some special condition or type of flow.

In a few cases, the lift curves for tests at two or more values of vl on the same aerofoil coincide over a range of several degrees in the angle of attack. Special tests are required to indicate whether or not an individual correction of the same general form as equation 10 should be applied at each angle of attack in such cases.

CONCLUSIONS.

It has been shown that the value of the drag coefficient varies with vl according to the expression

$$D_{c2} = D_{c1} - D_{c0}\ [1 - \left(\frac{v_2 l_2}{v_1 l_1}\right)^{-0.16}]$$ (11)

where D_{c1} is the drag coefficient at a given angle of attack and $v_1 l_1$.

D_{c0} is the minimum drag coefficient at $v_1 l_1$

and D_{c2} is the drag coefficient at the same angle of attack as D_{c1} but at $v_2 l_2$.

It has also been shown that the lift coefficient at $v_2 l_2$ is given by

$$L_{c2} = L_{c1} + .057\ \log_{10}\left(\frac{v_2 l_2}{v_1 l_1}\right)$$ (10)

where L_{c1} is the lift coefficient at $v_1 l_1$ for the particular angle of attack under consideration.

The most obvious criticism of these formulæ is that they are based on low values of vl. The only tests at high values of vl, available for inclusion in this study were those made at Göttingen and reported by Kumbruch in the Zeitschrift für Flugtechnik und Motorluftschiffahrt, of May 31, 1919. Unfortunately the forces involved in the Göttingen tests were so large that the models deflected until the angles of attack were uncertain. The models were also of aspect ratio 2.5 and end plates were used to eliminate the tip losses. Although corrections were made for aspect ratio and the interference between the model and the walls of the tunnel it is felt that the effect of scale is not given by the final results.

It is recommended that the formulæ 10 and 11 be checked by tests extending over a large range of vl. Such tests should be made with more than usual care in measuring the angle of attack and wind velocity. The results so obtained should be checked with reliable free flight performance data.

TABLE I.—*Determination of D_c, monoplane RAF-6c.*

[Data from Br. A. C. A., R. and M. No. 110.]

α	D_c $v_1 l_1 = 7.5.$	D_c $v_2 l_2 = 12.5.$	$-\Delta D_c.$	
−6	0.0377	0.0370	0.0007	
−4	.0255	.0250	.0005	$\left(\frac{v_2 l_2}{v_1 l_1}\right)^{-0.16} = (1.67)^{-0.16} = 0.923.$
−2	.0183	.0170	.0013	
0	.0152	.0138	.0014	
2	.0152	.0135	.0017	
4	.0190	.0173	.0017	Minimum $D_{c0} = .0152.$
6	.0244	.0235	.0009	
8	.0331	.0313	.0018	$\Delta D = D_{c0}[1-0.923].$
10	.0418	.0398	.0020	.0152×0.077.
12	.0514	.0510	.0004	
Average			.00124	= .00117.

TABLE II.—*Comparison of D_c, calculated and observed.*

(See Table I.)

$v_1 l_1$	$v_2 l_2$	$\frac{v_2 l_2}{v_1 l_1}.$	Min. $D_{c0}.$	ΔD_c calculated.	Average ΔD_c observed.	Section.	Arrangement.	Reference.
7.5	12.5	1.67	0.0152	0.0012	0.0012	RAF-6c	Monoplane	R. and M. 110.
5	12.5	2.5	.0157	.0020	.0022	do	do	Do.
5	10	2.0	.0158	.0016	.0012	RAF-6	do	Do.
5	12.5	2.5	.0158	.0020	.0023	do	do	Do.
5	20	4.0	.0142	.0027	.0031	RAF-6A	do	R. and M. 148.
15	30	2.0	.0119	.0012	.0013	do	do	Do.
5	16.5	3.3	.0143	.0023	.0020	RAF-6	Biplane	R. and M. 190.
7	12.5	1.67	.0142	.0010	.0013	do	do	Do.
20	40	2.0	.0117	.0012	.0012	Propeller	Monoplane	R. and M. 362.
30	40	1.33	.0101	.0005	.0006	do	do	Do.
5	14.6	2.92	.0410	.0061	.0058	RAF-19	do	R. and M. 415.
5	10	2.0	.0410	.0041	.0047	do	do	Do.
5	10	2.0	.0083	.0008	.0007	RAF-20	do	Do.

TABLE III.—*Test data from R. and M. No. 148, showing method of obtaining ΔL_c.*

α	$vl=5$ L_c	$vl=10$ L_c	$vl=15$ L_c	$vl=20$ L_c	$vl=30$ L_c	$vl\ 30-vl\ 5$ ΔL_c	$vl\ 30-vl\ 10$ ΔL_c	$vl\ 30-vl\ 15$ ΔL_c	$vl\ 30-vl\ 20$ ΔL_c
−4	−0.066	−0.064	−0.074	−0.066	−0.048	0.018	0.016	0.026	0.018
−2	−.051	+.010	+.022	+.037	+.060	.061	.050	.038	.023
0	+.068	.102	.127	.136	.145	.077	.043	.018	.009
2	.182	.207	.215	.214	.218	.056	.011	.003	.004
4	.269	.284	.283	.286	.293	.024	.009	.010	.007
6	.346	.355	.357	.358	.365	.019	.010	.008	.007
8	.413	.424	.431	.434	.441	.028	.017	.010	.007
10	.472	.489	.500	.505	.513	.041	.024	.013	.008
12	.537	.551	.563	.572	.585	.048	.034	.022	.013
14	.577	.584	.615	.616	.627	.050	.043	.012	.011
Average ΔL_c						.0422	.0257	.0160	.0107

TABLE IV.—*Determination of K in the equation.*

$$\left[L_{c2}=L_{c1}+K.\log_{10}\left(\frac{v_2 l_2}{v_1 l_1}\right)\cdot\right]$$

| No.	$v_1 l_1$	$v_2 l_2$	Average ΔL_c	$\dfrac{(v_2 l_2)}{(v_1 l_1)}$	$\log_{10}\dfrac{(v_2 l_2)}{(v_1 l_1)}$	K.	Section.	Reference.
1	2.5	5	0.0174	2	0.3010	0.0578	} Monoplane R A F. 6	R. and M. No. 110.
	5	10	.0170	2	.3010	.0564		
	2.5	12.5	.0394	5	.6990	.0564		
2	5	30	.0422	6	.7782	.0542	} Monoplane RAF. 6c	R. and M. No. 148.
	10	30	.0257	3	.4771	.0538		
	15	30	.0160	2	.3010	.0532		
	20	30	.0107	1.5	.1761	.0608		
3	5	10	.0173	2	.3010	.0572	} Biplane RAF–6	R. and M. No. 196.
	5	16.5	.0329	3.3	.5185	.0635		
	7.5	12.5	.0138	1.67	.2219	.0622		
	7.5	16.5	.0217	2.2	.3424	.0632		
4	40	35	−.0034	.875	−.0581	.0585	} Propeller aerofoil: $x/c=0.5$ $\frac{h}{c}=.075$	} R. and M. No. 362.
	40	30	−.0063	.750	−.1239	.0510		
	40	25	−.0116	.625	−.2041	.0568		
	40	20	−.0176	.50	−.3010	.0585		
5	5	10	.0176	2	.3010	.0585	} Monoplane RAF–19	R. and M. No. 415.
	5	14.6	.0238	2.92	.4654	.0511		
6	5	14.6	.0267	2.92	.4654	.0574	Monoplane RAF–20	R. and M. No. 415.
Average						.0570		
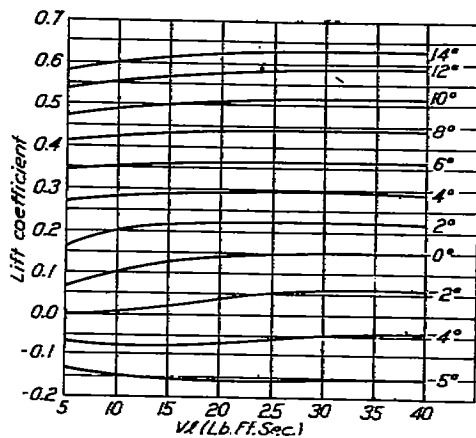
FIG. 1.—Correction to lift coefficient for scale effect. Reproduction of figure 19, B. A. C. A. R. & M. No. 450.
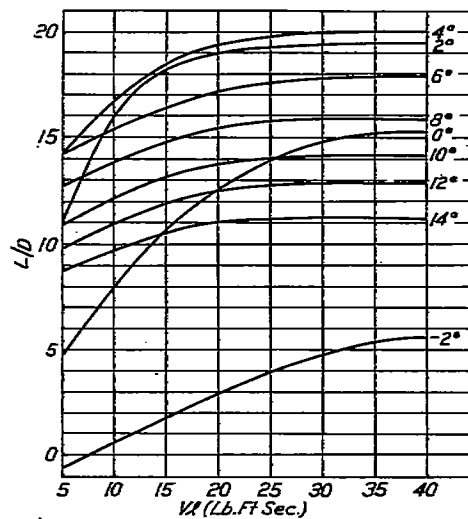
FIG. 2.—Corrections to L/D for scale effects. Reproduction of figure 20, B. A. C. A. R. & M. No. 415.

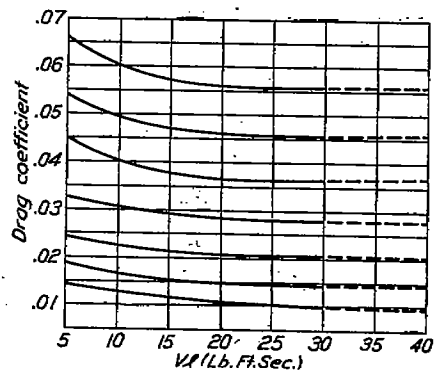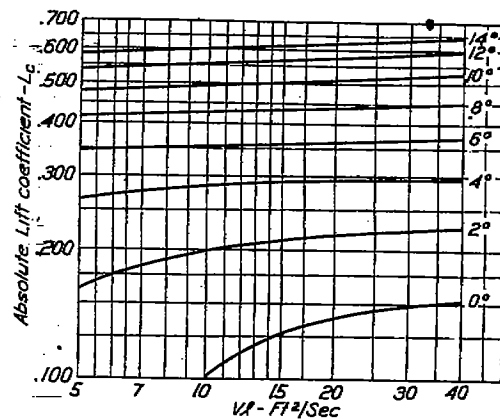FIG. 3.—Correction to drag coefficient for scale effect Reproduction of figure 21, B. A. C. A. R. & M No. 450.

FIG. 4.—Variation of lift coefficient with V1 Re-p ot of figure 19, B. A. C. A. R. & M. No. 450. (See Fig. 1.)
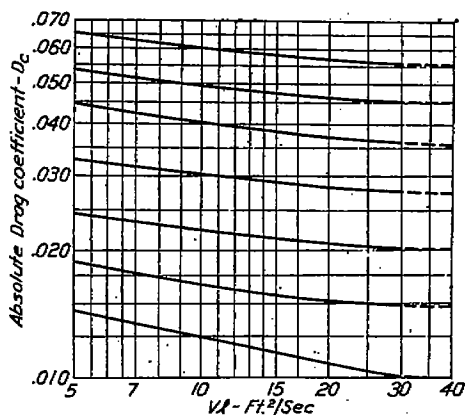

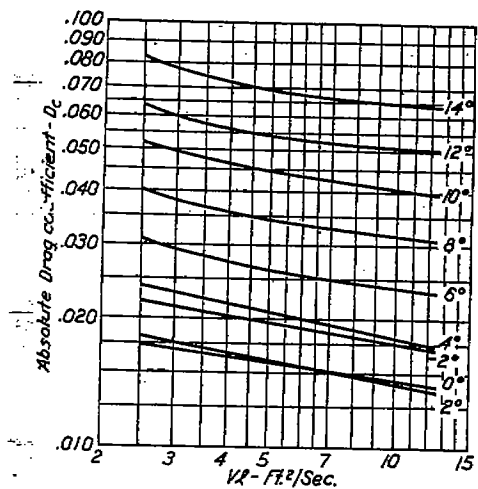FIG. 5.—Variation of drag coefficient with VI. Re-plot of figure 21, B. A. C. A. R. & M. No. 450. (See Fig. 3.)

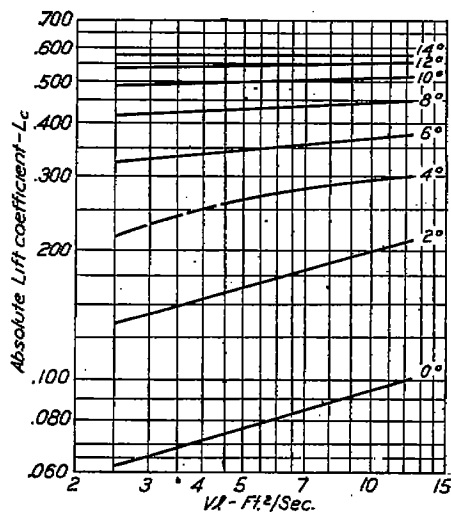FIG. 6.—Variation of drag coefficient with VI. Data from B. A. C. A. R. & M. No. 110. R. A. F. 6 Aerofoil.

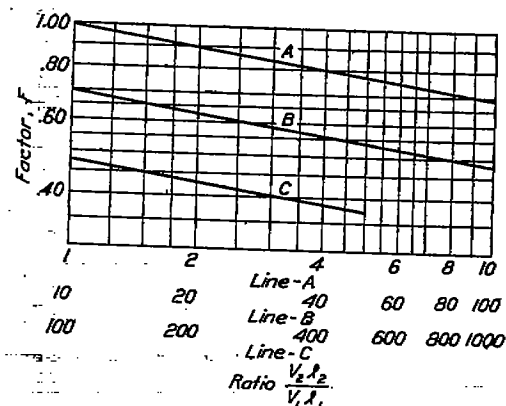FIG. 7.—Variation of lift coefficient with VI. Data from B. A. C. A. R. & M. No. 110. R. A. F. 6 Aerofoil.

FIG. 8.—Evaluation of $F = \left(\dfrac{V_2 l_2}{V_1 l_1}\right)^{-0.14}$