Robust Control Of Underwater Vehicles: Sliding Mode Control Vs. Mu Synthesis.

Giampiero Campa* Mario Innocenti***
Department of Electrical System and Automation, Pisa University, Pisa Italy.

Francesco Nasuti**
Department of Mechanical and Aerospace Engineering, West Virginia University, WV USA.

Abstract: **This paper presents the results of a controller synthesis where a robust control of an AUV is developed using two different methods. The problem statement requires the design of a position and attitude control system for the vehicle in order to achieve precise trajectory following. Firstly, a detailed nonlinear model of the vehicle was derived. Secondly an operating point for nominal design was selected, and a multivariable linear model of the vehicle was obtained by linearization around the operating point. The presence of structured uncertainties due to errors in the computation of hydrodynamic coefficients, dynamic linearization and truncation, unknown disturbances, were considered. Two robust controllers were designed, the first one linear, using standard Mu analysis and synthesis techniques, and the second one nonlinear, using a Sliding Mode approach. The performance of the two controllers were extensively evaluated and compared in simulation with a full nonlinear model of the vehicle.**

I. Introduction

In the future, autonomous underwater vehicles (AUV's) will play an important role in the oceans exploration. An AUV will be required to move in adverse environments showing good performance and a great deal of autonomy under a variety of load conditions and unknown sea currents.

Developing a control system that can achieve the aforementioned goal is challenging for a variety of reasons:

1) The nonlinear behaviour of a vehicle subjected to hydrodynamic forces and moments.
2) The multivariable character of a 6dof vehicle, which exhibit an high degree of coupling among different command channels.
3) The consistent amount of uncertainty in both high and low frequencies, due for example to the addition and removal of work packages, to the lack of precise knowing of hydrodynamic drag coefficients, or to unknown disturbances introduced by linearization of the nonlinear dynamics.

* PhD Student.
** Research Assistant Professor, Member IEEE.
*** Associate Professor, Member IEEE.

Control strategies that address some of this issues have been reported in the literature, see for example [1] for an H_∞ based multivariable design

Variable Structure Control (VSC) with sliding modes is a well established approach for dealing with deterministic uncertainties in both linear and nonlinear systems. Mu synthesis along with H_∞ design, have become a popular method in control engineering when dealing with robustness issues in multivariable linear systems.

The purpose of this paper is to compare these two methods exploring their different weapons to deal with uncertainties giving some insights to the possible connections, and finally pointing out their major vantages and disadvantages.

The problem statement requires the design of an attitude control system for a linearized model of a missile-like AUV.

Of course this is only a part of the overall design problem since the control will work only in a range near the operating point, nevertheless it is a very important part since the same procedure could be applied for every interesting operating point, and then a gain scheduling (or self tuning) mechanism can provide a nonlinear controller that will achieve the required performance specification over a broad range of operating point.

The organisation of the paper reflects the natural sequence of steps in the design of a trajectory following system for an autonomous vehicle.

Section II introduces the nonlinear and linearized model of the vehicle used in our design, in sections III and IV the Sliding mode and Mu control synthesis is carried out, in section V we evaluate the performances of the two controllers on the full nonlinear model of the vehicle.

II. Modelling Considerations

This section reviews briefly the dynamic properties of both the nonlinear and linearized plant, and further model reduction carried out to achieve the linear system used in control synthesis.

A. Nonlinear Model

The following 6 DOF AUV model considered is [2] :

$$M\dot{v}_r + C(v)v + C_A(v_r)v_r + D(v_r,\delta)v_r + g(\eta) = \tau \\ \dot{\eta} = J(\eta)v \tag{2.1}$$

where $\eta = [x,y,z,\varphi,\theta,\psi]^T$, is the earth-fixed position and attitude vector, $v = [u,v,w,p,q,r]^T$, is a vector of body fixed linear and angular velocities, $v_c(\eta) = [u_c,v_c,w_c,0,0,0]^T$, is the current speed with respect to the body fixed frame, $v_r = v - v_c(\eta)$ is the relative speed of the vehicle, τ is a vector of control forces and moments, δ is the vector of the rotation angles of the 8 fins.
M is the inertia matrix , C and C_A are the matrices for Coriolis forces, D is the matrix for the damping forces, (note that it depends on the shape of the vehicle and hence on δ), finally, $g(\eta)$ is the vector of the restoring forces and moments, due to gravity and buoyancy.
Equations (2.1) describe a dynamic system with 12 states (η , v) and 20 inputs (δ , τ , first 3 components of v_c and their derivatives).

B. Linear Model

The nonlinear model has then been linearized, about the operating point determined by $\eta = [\ 0,\ 0,\ 0]^T$, $v = [3\text{m/s}\ ,\ 0,\ 0]^T$, $\delta = [\ 0, 0, 0, 0, 0,0, 0, 0]^T$, $\tau = [\ 416\text{N}\ , 0, 0, 0, 0\ ,0]^T$, v_c w.r.t E and its derivatives w.r.t E both equal to $[\ 0, 0, 0, 0, 0\ ,0]^T$. Once the linear model was extracted, some simplifications were carried out, keeping in mind that we are mainly interested in attitude dynamic and control about the chosen operating point.
Specifically, the first three states which represent the positions x, y, z, (and correspond to 3 zero eigenvalues of the A matrix) have not been considered, leading to the following 9 dimensional state vector : $[\ \varphi, \theta, \psi, u, v, w, p, q, r\ ]^T$, also, only the first 8 of the 20 inputs (fins position) have been considered, $[\ \varphi, \theta, \psi\ ]$ have been taken as outputs.
Close examination of the structure of the linear model reveals the following. The system matrix A has a zero eigenvalue corresponding to ψ, instead φ and θ do not exhibit an integrator effect, due to a restoring torque generated by the combination of gravity and buoyancy forces (the center of mass is supposed to be under the buoyancy forces application point).
There are 3 unstable modes related to θ, ψ, v, w, and weakly to q, r, they are caused by the geometry of the vehicle which tends to be pushed to the side and up or down when moving forward, (again there is no exact symmetry between the steering and diving dynamics because of the combination of buoyancy and gravity forces).

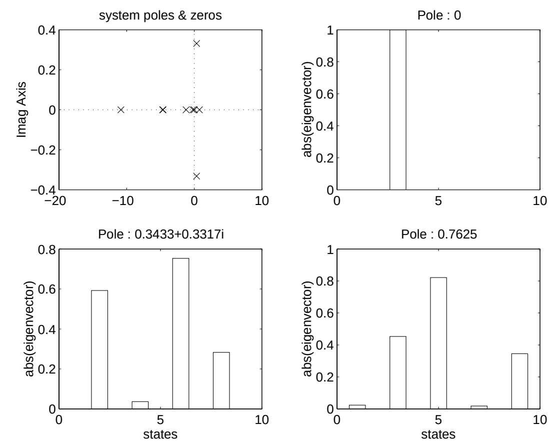

Fig. 1 Eigenvalues and Eigenvectors

These results are very similar to that obtained in [1], and are summarised in Fig. 1 where the location of every pole in the complex plane is shown and for the poles in zero and in the right half plane the norm of the corresponding eigenvector is plotted as a bar graph, showing the relationships between poles and states.
The singular values analysis shows that the system bandwidth is about 0.3 rad/sec, at lower frequencies the highest singular value (which is exactly associated with vertical fins and yaw motion) has a roll off of 20 dB/dec., (this is to be expected since the yaw has a zero eigenvalue associated with it) and the other two singular values, (which are exactly associated with pitch and roll) have a constant gain of 2.2 and 1.7.

III. Sliding Mode Control

A. Sliding mode basic concepts

A variable structure control is a non linear (state) feedback control which has a discontinuity on one or more manifolds in the state space. Thus the structure of the feedback system is altered or switched as its state crosses each discontinuity surface, in consequence of which, the closed loop system is described as a variable structure control system [3].
The central feature of VSC is sliding motion. This occurs when the system state repeatedly crosses and immediately re-crosses a switching manifold, because all motion is directed inwards (i.e. towards the manifold). In the sliding mode the motion of the system is effectively constrained to lie within a certain subspace of the full state space, and thus

it becomes completely insensitive (invariant) with respect to perturbations or parameter variations which are normal with respect to the surface where the sliding occurs.
Specifically, given a linear system, it is always possible to write it in the form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_1 \\ y \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & B_2 \\ C_1 & C_2 & D \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u \end{bmatrix} \qquad (3.1)$$

where rank(B_2) = rank(B), during the sliding mode the state is constrained to lie in the subspace defined by $Sx=S_1x_1+S_2x_2=0$, thus the dynamic become:

$$\dot{x}_1 = (A_{11} + A_{12} S_2{}^{-1} S_1) x_1 \qquad (3.2)$$

which is completely independent by A_{21}, A_{22}, and B_2, and therefore from any uncertainty on these matrices or any perturbation belonging to the space generated by the columns of B_2.
From a "modern robust control" standpoint, one can say that (Fig. 2), when the VSC block is active the transfer function from w to z approaches 0, so the uncertainty attached between z and w "sees" a zero transfer function and therefore it cannot affect the controlled system, no matter how "big" this uncertainty is.

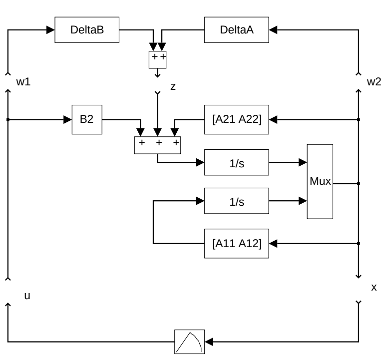

Fig. 2 VSC from a modern standpoint.

It should be noted anyway that the complete invariance happens as long as the feedback mechanism is able to drive the state to the same surface no matter the values of A_{21}, A_{22}, and B_2, in other words it must be completely independent of these 3 matrices. We will point out later that unfortunately this behaviour (which requires nonlinearity), cannot be obtained in the whole state space but only in a certain region surrounding the surface, moreover the greater is the uncertainty on A_{21}, A_{22}, and B_2, the smaller is the region.

B. Sliding mode Control Design

The design of a sliding mode control follows two main steps. First one must design a switching surface $Sx=0$ to represent a desired system dynamics, then, design a variable structure control law $u(x)$ to drive the whole dynamic toward the switching surface in a finite time, several methods have been proposed in the literature for both steps [3].
The switching surface was designed to minimise a cost functional in which the integrand is a quadratic function of the state ([3] third chapter).

$$J = \int_{t_s}^{\infty} x(t)^T Qx(t)dt \qquad (3.3)$$

where t_s is the time at which the sliding mode starts.
The chosen Q was a diagonal matrix containing 100 in the first three entries and one the other entries.
For what concerns the control law, the adopted scheme is the "unit vector control", which offers several advantages with respect to the others and is also easy to implement, because of its simple structure:

$$u(x) = Lx + \frac{\rho}{\|Mx\| + \delta} Nx \qquad (3.4)$$

In formula (3.4), the matrices L, M, N depend totally on the chosen surface S, (and thus in our design are automatically determined by the choice of Q), while ρ and δ are free parameters to be regulated by the designer. In particular, δ is a small positive constant needed to "smooth" the discontinuity about the surface and thus avoiding chattering and stressing of actuator mechanism, while ρ is a positive constant that determine the "strength" used by the control to drive the state to the surface, and consequently it affect both the reaching time to the surface and the "convergence to the surface" region in presence of uncertainty.
A precise evaluation of shape and extension of this region is beyond the scope of this paper, anyway it is not hard to show [3] that convergence to the sliding surface is assured if

$$\rho > \|D(\Delta A_{21}, \Delta A_{22,} \Delta B_2, B_2)x\| \qquad (3.5)$$

so it is straightforward to see that this region is greater the greater is ρ, moreover it can be shown that the region is greater the smaller are the uncertainty ΔA_{21}, ΔA_{22}, ΔB_2.

The standard VSC is a state feedback, so it requires all the states to be measured or estimated, since direct measure of all 12 states is unrealistic, a Kalman state estimator has been used, the chosen state white noise covariance matrix W is a diagonal matrix containing 100 in the first three entries and 10 on the other entries, the chosen output white noise covariance matrix V is an identity matrix multiplied by 0.1.

IV. Mu Control

A. Mu Analysis and Synthesis basic concepts

Given a plant P (Fig. 3) with w and u as inputs (generally used as disturbance and control inputs) and z and y as outputs (generally used as error and measured outputs), the goal is to find the linear controller *K* such that when it is attached to the plant, the structured singular value (with respect to a certain Δ structure) of the transfer function from w to z is minimised [4].

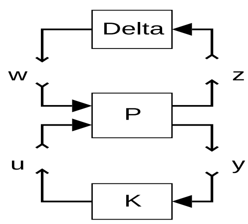

Fig. 3 Standard stability robustness scheme

Since the structured singular value of a transfer function *Tzw(K)* is defined as:

$$\mu(\Delta, T_{zw}) = \frac{1}{\min(\overline{\sigma}(\Delta) : \det(I - T_{zw}\Delta) = 0)} \qquad (4.1)$$

we have that if $max(\sigma(\Delta))\mu(\Delta,Tzw) < 1$ for every $s=j\omega$, then the uncertainty cannot destabilise the plant.

In other words, if we have a certain structured Δ, by applying the controller which minimise $\mu(\Delta,Tzw)$ we reduce the possibility by Δ to affect the stability of the plant, moreover, if the max singular value of Δ is bounded by a known function *w(s)* then we can perform an easy analysis ($w(s)\ \mu(\Delta,Tzw) < 1$), to infer if the controlled system is stable under all the admissible uncertainties or not.

For what concerns performance requirements, they can easily been incorporated in the above framework (Fig. 4) let us suppose that the performance requirement is to minimise a transfer function *Tkh* , then defining

$$T_p = \begin{bmatrix} T_{zw} & T_{zh} \\ T_{kw} & T_{kh} \end{bmatrix}, \quad \Delta_p = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_f \end{bmatrix} \qquad (4.2)$$

where Δ_f is a full block which sees *Tkh*, we have that if $max(\sigma(\Delta_p))$ is bounded by a known function $w_p(s)$ then $\mu(\Delta_p,T_p)w_p(s) < 1$ implies both [4]:

1. $\mu(\Delta,T)w_p(s) < 1$ hence stability for every Δ attached from z to w such that its max singular value is less than $w_p(s)$ (robust stability).
2. $max(\sigma(Tkh))w_p(s) < 1$ when Δ is attached from z to w, (robust performance).

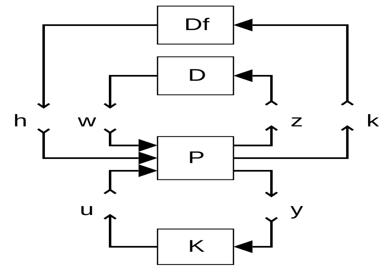

Fig. 4 Standard performance robustness scheme

In simpler words the less is $\mu(\Delta_p,T_p)$ the less are Tzw and Tkh, and the greater must be the attached uncertainty Δ to either destabilise the plant and to make Tkh on the uncertain plant greater than a given bound.

B. Control Design.

The control design follows these steps:

Firstly, one must decide which functions to minimise, the chosen functions to minimise are: output control sensitivity (*Mo*) as robust stability requirement and output sensitivity (*So*) as robust performance requirement.

As a result of the first step we have a plant in the form of Fig. 4 , eq. (4.2), with *Tp* to have its structured singular value minimised.

The second step is to express upper bounds on the functions to minimise, this involve the decision on their desired shape versus frequency, W_S the chosen upper bound on *S* has a roll-on rate of 20dB/dec until ω=10^1.29 rad/sec and a constant value of 3 dB at higher frequencies, while W_M, the chosen upper bound on *M* has a constant value of 50 dB until ω=10^1.3 rad/sec and then a roll off rate of 20 dB/dec.

This express the intention to achieve robust performance at low frequencies and robust stability at high frequency, since the two requirements are in opposition.

As a result of this step we have two weighting functions.

In the next step, which does not involve any decision, we construct an augmented plant multiplying the inverse of the weighting function by the plant obtained as a result of the first step, then a Mu minimisation algorithm can be applied to this augmented plant to obtain the controller.

V. Results

These two controllers were applied to the full nonlinear plant, and their performances were evaluated by mean of a manoeuvre following sequence given by:

1. At time t=5 sec a 0.1 rate ramp in the yaw channel.
2. At time t=10 sec a 0.1 rate ramp in the pitch channel.
3. At time t=15 sec a 0.1 rate ramp in the roll channel.

A. Simple tracking

The controllers are asked simply to track the above commands with an amplitude of 0.1 radians, the result is that the Mu controller provides a perfect decoupling and can track the command better than VSC.

B. Range

The extension of the range surrounding the operating point in which a controller can work could be determined by the maximum amplitude of the ramps that can be tracked.

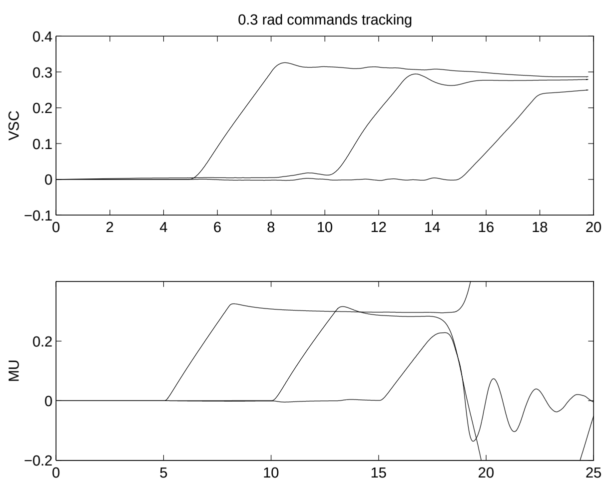

Fig. 5 Operating range extension

C. Noise and Uncertainty

The command to be tracked is the same as in case A (0.1 radians) but a white noise of power 1e-6 has been added to each output, moreover an output multiplicative static uncertainty consisting on an anti-diagonal matrix having 0.5 in its entries of has been considered.

VI. Conclusions

Both the methods have their vantages and their drawbacks.
The Mu design is rather attractive since it makes the design a standard process where one must only specify the desired sensitivity shape, unfortunately the time domain specifications like for example overshoot or settling time are not easy to specify and one often has to try several weightings, to find the most satisfying one.
In comparison, the sliding mode synthesis is longer since it requires several parameters to be adjusted, and an observer to be designed, nevertheless the algorithms involved are simpler and quicker and hence less subject to numerical problems.
For what concerns the results, while VSC outperforms Mu when the nonlinearity of the plant faces in, the last one shows better tracking and performances capabilities in a narrow range around the operating point.

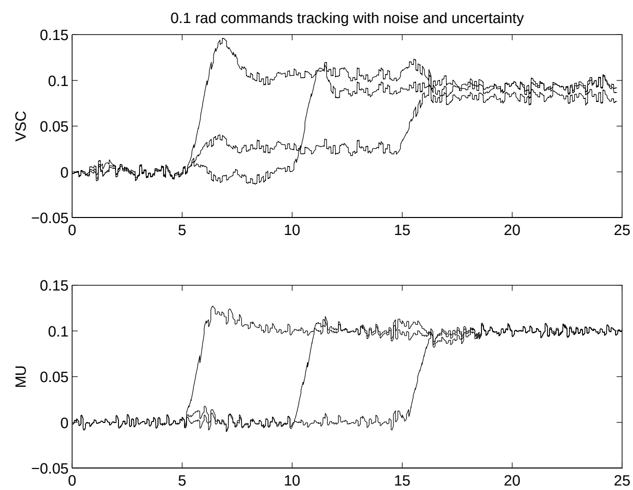

Fig. 6 Behaviour with noise and uncertainty

The reasons for this behaviour is that the Mu control is a linear control that uses all the poles enclosed in the weightings to shape sensitivities and hence to provide good performances (i.e. tracking) while trying to minimise uncertainty effects. The VSC control does not have any added pole (the addition of integrators should be straightforward anyway), but its force is a nonlinear mechanism which virtually eliminates, at least in a certain range, the uncertainty in the rank of B_2, which for a mechanical system is the uncertainty that corresponds to the dynamics as can be seen from (2.1).

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