“Pumped-up propulsion” during front crawl swimming

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ABSTRACT

TOUSSAINT, H. M., C. VAN DEN BERG, and W. J. BEEK. “Pumped-up propulsion” during front crawl swimming. *Med. Sci. Sports Exerc.*, Vol. 34, No. 2, pp. 314–319, 2002. **Purpose:** It is currently held that propulsion in human front crawl swimming is achieved by lift and drag forces predominantly generated by the hands. Calculation of these propulsive forces relies on the quasi-steady assumption that the fluid dynamic behavior of a hand model in a flow channel (constant velocity and orientation) is similar to that of a hand of a real swimmer. However, both experimental and theoretical analyses suggest that this assumption is questionable and that unsteady and rotational propulsion mechanisms play a significant role. Theoretical considerations suggest that arm rotation could lead to a proximodistal pressure gradient, which would induce significant axial flow along the arm toward the hand. **Methods:** To gain insight into such mechanisms, we used tufts to study the flow directions around the arm and hand during the front crawl, which consists of a glide, an insweep, and an outsweep phase. In a second experiment, we measured pressure during the stroke at various points along the arm and hand. **Results:** It was observed that 1) the flow during insweep and part of the outsweep was highly unsteady; 2) the arm movements were largely rotational; 3) a clear axial flow component, not in the direction of the arm movement, was observed during insweep and outsweep; and 4) both the V-shaped “contracting” arrangement of the tufts during the outsweep and pressure recordings point to a pressure gradient along the direction of the arm during the outsweep, as predicted on theoretical grounds. **Conclusion:** Our results demonstrate the reality of the predicted rotational and unsteady effects during front crawl swimming. We hypothesize that the axial flow observed during the outsweep has a propulsion-enhancing effect by increasing the pressure difference over the hand. Further investigation is required to establish more accurately the role of axial flow on propulsion. **Key Words:** PROPULSION, HUMAN FRONT CRAWL SWIMMING, FLOW ANALYSIS, QUASI-STEADY ANALYSIS, AXIAL FLOW

Propulsion is one of the key factors determining performance in human competitive front crawl swimming. It is therefore no surprise that the fluid dynamic mechanism of propulsion has been the subject of scientific inquiry. At present, the dominant view (see Rushall et al. (15) for debate on the relative importance of the lift and drag component in swimming propulsion) is that the hand acts as a hydrofoil, generating both lift (L) and drag (D) (7). The fluid dynamic forces acting on an object are usually described as a function of its velocity relative to the fluid (u, $m \cdot s^{-1}$), its surface area (plan area S, m^2) and the density of the fluid (ρ, $kg \cdot m^{-3}$) according to the following equations:

$$L = \tfrac{1}{2}\rho u^2 C_l S \tag{1}$$

$$D = \tfrac{1}{2}\rho u^2 C_d S \tag{2}$$

where C_l and C_d are the lift and drag coefficients. The values of these coefficients are characteristic for the object tested and are a function of the angle of attack, α, and the sweep-back angle, ψ (Fig. 1).

Schleihauf (17) investigated the hydrofoil characteristics of the human hand in a flow channel. In this manner, he calculated C_l and C_d as a function of u, α and ψ (with maximum values of about 1 and 1.2, respectively), showing that lift forces might indeed play a significant role in propulsion. By combining these data with hand velocity data collected from film analysis of swimmers, the magnitude and direction of the net propulsive force throughout the stroke cycle could be calculated with equations 1 and 2 (18,20). This approach is known as quasi-steady analysis, which crucially depends on the assumption that the flow under steady conditions (constant velocity, angle of attack, and sweep-back angle) is comparable to the flow during the actual swimming stroke.

The quasi-steady analysis is appealing for its relative simplicity. Furthermore, it is a big step forward from the original “rowing” theory of propulsion, as it can account for sculling movements of the hand. However, recently some doubt was cast on whether the quasi-steady analysis can account quantitatively for high propulsive forces required for high-speed swimming. When swimming at a constant speed, the average propulsive force should equal the average drag force. In a replication of Schleihauf’s work, Berger et al. (5) found similar values of C_l and C_d of a model hand, but the calculated propulsive forces were 17% lower than the measured active drag using the MAD system (21), when u was set equal to the velocity of the middle of the palm of the hand, as suggested by Schleihauf (19). When setting u to the velocity of the middle of the third finger which, because of sculling, is higher than the mean hand velocity, the calculated forces were still 5% lower than the measured forces

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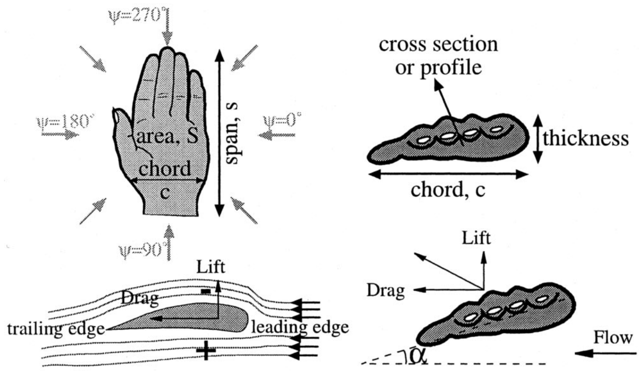

FIGURE 1—Relevant parameters describing the hand as hydrofoil (*top*). A hydrofoil subjected to flow (*lower left*) experiences a lift and drag force. The same is true for the human hand (*lower right*) when subjected to constant flow in a flow channel. The magnitude of the lift and drag forces depends on the angle of attack α and on the sweep-back angle ψ.

(3). Thus, the calculated propulsive forces are too low to account for the observed active drag forces. To make matters even worse, Sanders (16) recently reported much (about 50%) lower coefficients of lift and drag for the hand. He suggests that it is likely that towing the hand near the water surface resulted in an overestimation of Berger's and Schleihauf's drag and lift coefficients. Apart from these worrying experimental results, there are some theoretical problems with the quasi-steady approach: 1) it ignores unsteady effects and 2) it ignores the effect of limb rotation (as occurs in sculling).

Kinematic analysis shows that the arm movements are complicated combinations of rotation and translation. In the frame of the body, the limb segments clearly only rotate relative to one another. In an earthbound frame (which is more relevant in terms of the fluid dynamics), the forward translation of the body is superimposed on these rotations. The resulting movements of the arm (e.g., the hand) are far from steady, in terms of both velocity and direction of movement (4,16,19). Therefore, unsteady effects have to be taken into account. In order to proceed, some further fluid dynamic theory is presented. One way to look at the total fluid flow around a (hydro)foil is to decompose it in a circulating flow around the hydrofoil with velocity (v) and a uniform flow field at velocity (u) (Fig. 2). Note that the fluid particles do not actually circle around the hydrofoil; rather, the flow pattern can be thought of as a superposition of translation and circulation as indicated in Figure 2. The circulating flow component is known as the bound vortex (9). The strength or circulation Γ of the bound vortex is proportional to v (see fluid dynamics textbooks such as

FIGURE 2—Streamlines around a hand-hydrofoil in a flow from right to left. The flow past the hand can be decomposed in a uniform flow field with velocity u and a circulating flow around the hydrofoil with velocity v. The circulating flow is known as the bound vortex.

Prandtl and Tietjens (14) for a formal definition of circulation). The higher the circulation, the greater the velocity differential above and below the hand with, according to Bernoulli's equation, a concomitant greater pressure differential. When a hydrofoil is accelerated impulsively to a constant velocity, the bound vortex needs time to develop to its final, steady-state strength. This gradual buildup of the bound vortex is called the Wagner effect (for an excellent description, see Dickinson (8)); when a hydrofoil is accelerated impulsively to a constant velocity, the flow around the foil, responsible for the generation of lift, needs time to develop to its final, steady-state strength. It may take six chord lengths of travel for the lift force to reach only 90% of its steady-state value (9). Now, even though the hand is by no means a perfectly shaped hydrofoil, it must by definition create a bound vortex in order to create lift. The principle of a gradual buildup of the bound vortex is likely to apply to the hand as much as to a perfect hydrofoil. Therefore, it may be expected that about 0.6 m of travel of the hand (chord of hand $\approx$ 0.1 m, see Fig. 1) is required before lift reaches 90% of its steady-state value, which makes it unlikely that steady-state circulation is reached at all during the front crawl stroke, given that velocity and orientation of the hand are never constant. Consequently, the quasi-steady analysis overestimates the lift force, making the discrepancy between calculated and measured forces (5) more poignant. So, how does propulsion in front crawl swimming work? We seem to be further away from understanding the fluid dynamics of swimming than ever. A logical way to proceed would be to assume that there must be unsteady propulsion-enhancing mechanisms, which are as yet unknown. The challenge now is to identify such mechanisms.

It is interesting to note that the study of insect flight has faced similar discrepancies between quasi-steady calculations and measured forces (see Ellington (9) and Vogel (22)). Given this situation, it was recognized that unsteady lift-enhancing mechanisms must play a crucial role in insect flight. Recently, an unsteady mechanism was described involving a strong three-dimensional leading-edge vortex, that was stabilized by the wing rotations (10). Thus, it was shown that both unsteady and rotational effects are essential to understand the fluid dynamics of insect flight.

The fluid dynamics of limb rotation may be dramatically different from the fluid dynamics of limb translation, as measured in a flow tank. For example, Pai and Hay (13) found that the fluid dynamic forces generated by a rotating cylinder, which was used as a simple hydrodynamic model of the human swimming stroke, deviated dramatically from the quasi-steady prediction. Could the rotation of the cylinder be responsible for the observed higher fluid dynamic forces?

Theoretical considerations on the fluid dynamics of limb rotation. Let us consider the simple model of a rotating stiff arm. This rotation will lead to a velocity gradient along the arm, so that the (tangential) velocity near the hand will be higher than near the elbow (Fig. 3). It seems likely that the (tangential) velocity gradient along the limb

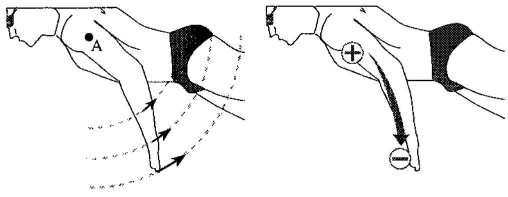

FIGURE 3—The rotational movement of the arm about the instantaneous center of rotation (A) during the outsweep (*left frame*) creates a tangential velocity gradient along the arm. This will induce a pressure gradient, leading to an accelerating axial flow component toward the hand (*arrow* in *right frame*).

will induce a (tangential) velocity gradient of the affected fluid close to the limb, which in turn will induce a pressure gradient, where local pressure close to the limb decreases in the direction of the fingertips. (For wind turbine blades, a formal derivation of how rotation creates a pressure gradient was recently published by Corten (6)). This pressure gradient will induce an axial fluid flow along the arm and hand toward the fingertips. Thus, the limb rotation leads to a pressure gradient pumping fluid along the arm toward the hand, not unlike the axial flow observed above the rotating wing of a hovering insect. Of course, in reality the picture will be complicated because of rotations within the arm (elbow extension) and of the body (roll), forward movement of the shoulder, and angular accelerations.

Translation of the arm through a fluid results in a high pressure at the leading side and a low pressure at the trailing side; this pressure difference is the basis of propulsion by paddling. Rotation of the limb will induce an axial pressure gradient on both leading and trailing sides. The interaction between the circumferential pressure gradient (because of translation) and axial pressure gradient (because of rotation) is not immediately clear. However, at the instantaneous center of rotation, the velocity of the limb relative to the water is zero and thus the pressure difference between trailing and leading sides is zero. Therefore, it seems probable that the axial pressure gradient at the trailing side is steeper than at the leading side. The resulting pressure differential across the propelling surface (the hand) would increase the propulsive force. This hypothetical propulsion-enhancing mechanism, which we dubbed "pumped-up propulsion," may be summarized as follows: the rapidly rotating arm during the outsweep acts as a rotational displacement pump, transporting water along the trailing side of the arm toward the hand, thus boosting the suction (low pressure) of the wake of the arm, which aids propulsion.

As suggested by the study of hovering insects, the fluid dynamics of human swimming may need revision, taking into account both unsteady effects (be they detrimental, such as the Wagner effect, or propulsion-enhancing) and rotational effects. Thus, the aim of the current study was to extend the theory on propulsion during front crawl swimming, taking into account the dynamic effects of arm rotation. The second aim was to experimentally visualize the direction of fluid flow during front crawl swimming by tufts in order to test whether unsteady and rotational effects could be demonstrated. As argued above, rotational effects are expected to be reflected not only in axial flow but also in axial pressure gradients. We present a preliminary study of pressure recordings along the arm (at the shoulder, elbow, wrist, dorsal side of the hand, and palm of the hand) to provide experimental evidence relating to this second point.

METHODS

Flow visualization. Quantitative measurement of flow conditions around the hand and forearm is quite difficult. For a first exploration of the flow, we used tufts (similar to Hay and Thayer (11)). Black woolen tufts about 0.07 m long and 0.02 m apart were attached to the skin of the left forearm and hand with Leukoplast® tape (Beiersdorf AG, Germany), which fully surrounded the limb. Such tuft clusters were positioned below the elbow (10 tufts), halfway between the elbow and wrist (10 tufts), at the wrist (eight tufts), and just above the knuckles of the hand (six tufts only at the dorsal side of the hand). Individual tufts were attached to the tip of the thumb, index finger, and little finger.

The tufts responded promptly to changes in the direction of flow (see Merzkirch (12) for further background on the use of tufts to visualize flow). The tufts proved to be clearly visible in the video recordings, allowing for identification of their free end. The tufts had no noticeable effect on the front crawl performance. In a pilot study, it was observed that tuft length (0.05 to 0.10 m) had no perceptible effect on the observed general flow directions that are presented here. Note that careful frame-by-frame analysis is required to interpret the flow direction from the tuft orientation in highly unsteady conditions, such as occur during swimming: the frame-to-frame changes in tuft curvature and orientation are more important than their orientation *per se*. Furthermore, note that we want to elucidate the global flow pattern rather than local details of the flow.

Four competitive swimmers gave written informed consent to serve as subjects in this study, which was approved by the local committee on ethics in research on humans. The motions of the left arm (with tufts) during the pull were recorded with two gen-locked SVHS cameras operating at 25 Hz, one positioned to the left and one in front of the swimmer (for set-up, see Berger (3)). The distance from camera to swimmer was a compromise between capturing a complete stroke and maintaining good visibility of the tufts. The subjects were instructed to swim at a range of speeds (approximately 0.9, 1.3, and 1.6 $m{\cdot}s^{-1}$). Each subject performed about nine trials, depending on whether the side camera recorded (part of) the pulling pattern of the left arm successfully.

The SVHS recordings were transformed to digital video and downloaded onto an Apple Macintosh computer (Apple Computer, Inc., Cupertino, CA). All recordings were trimmed and synchronized using the time base recorded in each video field. Front and side views were combined in one frame and the outlines of swimmer and tufts were carefully traced. By comparing consecutive frames, the average between-frames direction of motion and the movement paths of the index finger tip, the elbow, and the shoulder were determined. The displacement from frame to frame also gave a rough impression of the velocity. The movement path of the index finger was used to divide the underwater motion of the arm into a glide phase (from hand entry to the most frontal position of the finger); an insweep phase that lasts until the finger reaches the most medial position; and the outsweep phase, which is the remainder of the stroke to hand exit. For each phase, the movement direction of the hand and forearm was determined simultaneously with the flow direction around the limb, as revealed by analysis of the tuft movements.

The flow direction around the hand and arm in relation to the movement direction of the fingertip, elbow, and shoulder during a complete stroke is exemplified with data for one subject swimming at a slow pace (about 0.94 m·s^{-1}). The low-swimming-velocity sequence was chosen for detailed analysis because, when swimming at a higher speed, more bubbles were dragged along with the hand, occasionally obscuring the tufts, and the bubbles produced with the leg kick reflected more ambient light into the lens, which frequently led to overexposure of the front view. Nevertheless, for each subject it was possible to determine the arrangement of the tufts and the movement direction of the fingertip, elbow, and shoulder for the complete stroke, albeit not in one complete recording. All subjects showed qualitatively similar patterns as presented below. In general, there was good repeatability across trials of the flow phenomena discussed below, particularly for trials of the same subject. Note that in this preliminary study we tried to determine in a qualitative way the general flow pattern. A further quantitative flow analysis must await a future study.

Pressure measurements. To further investigate the suggested pumped-up propulsion mechanism, pressure was measured at the shoulder, elbow, wrist, dorsal side of the hand, and palm of the hand (Fig. 4 for the location of the sensors) using modified disposable blood pressure transducers (Truwave, Baxter Healthcare Corporation, Irvine, CA). Pressure signals were low pass filtered at 25 Hz. The sensors were calibrated by measuring pressure signals at 11 different depths (0 to 1.00 m) in steps of 0.1 m (980.67 Pa). One competitive swimmer gave informed consent to participate in an experiment. Pressure signals were recorded swimming at slow, intermediate, and sprint speed. The recordings were not corrected for hydrostatic pressure. Thus, without the proposed effect of limb rotation, the highest pressure would be at the tip of the limb when the hand reaches the greatest depth.

RESULTS

In contrast to the glide phase (Fig. 5), the flow during the insweep phase (Fig. 6) and to a lesser extent the outsweep phase (Fig. 7), was highly unsteady, as exemplified by a complete reversal of the flow direction along the hand and arm. As a result of complicated limb rotations, complex gradients in flow direction occurred along the forearm and hand (see especially front views at t = 0.96 and 1.04). Furthermore, the flow direction of each tuft changed rapidly from frame to frame.

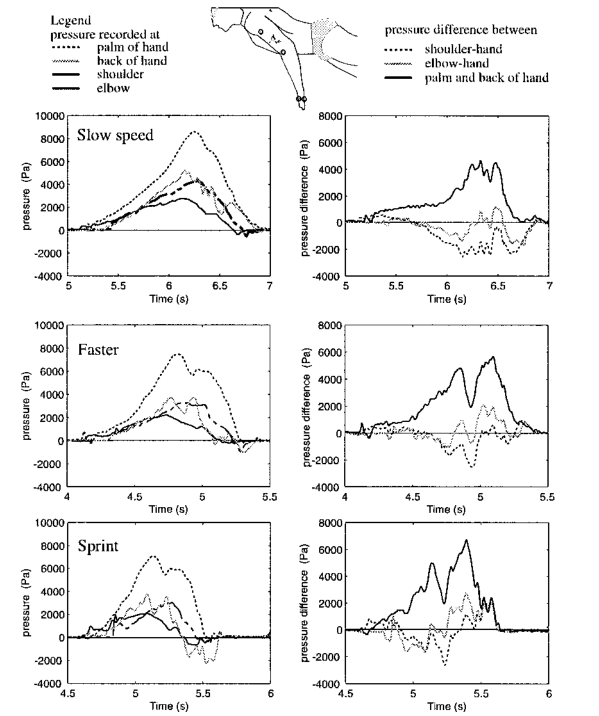

FIGURE 4—Pressure was recorded at the shoulder, elbow, wrist, dorsal side of the hand, and palm of the hand as indicated in the *top panel* swimming slow, faster, and at sprint speed. *Left panels* present pressure not corrected for hydrostatic effects; *right panels* give pressure difference relative the dorsal side of the hand.

Unlike the insweep, during the outsweep most tufts pointed consistently toward the hand during several consecutive frames. This shows that the flow was not in the local direction of the arm movement, but in an axial direction. We found further evidence for a pressure gradient along the direction of the arm during the outsweep: the tufts near the trailing side of the forearm (especially those of the tuft

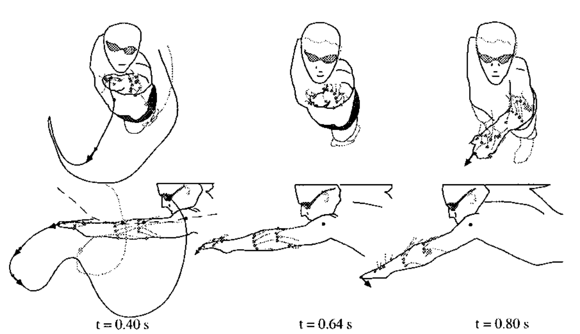

FIGURE 5—In the glide phase, the arm was kept stretched in front of the gliding swimmer. The fairly constant tuft orientation during this phase, in a direction opposite to the movement direction of the arm, suggests that the flow during this phase is pretty much stationary. Two stages of the glide phase (at t = 0.40 and t = 0.64) and transition to the insweep are given (t = 0.80): the tufts (fixed end indicated with *small dot*) show a regular flow pattern opposite to the movement direction of the arm and hand. Superimposed traces show the trajectory of the index fingertip (in *black*), the elbow (in *gray*), and the shoulder (*dashed*) during the complete stroke. The glide phase lasted approximately 0.80 s. At t = 0.80 s, the start of the insweep, the movement direction of fingertip, elbow, and shoulder is indicated.

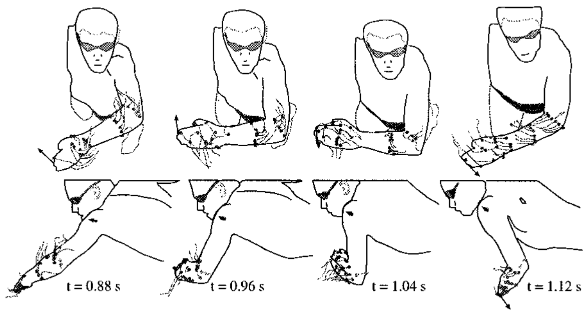

FIGURE 6—The insweep (t = 0.88 to 0.96) and transition to outsweep (t = 1.04 to 1.12): the direction of the changes in tuft orientation generally corresponded to the movement of the forearm and hand. However, starting at t = 0.96, a strong axial flow (toward the fingertips) developed near the wrist while the hand was still moving up and back. Next, this axial flow became evident along the whole arm, especially at the trailing side (t = 1.12). Note that the flow direction at the end of the insweep was still not exactly in the direction of the hand movement, but had a distinct axial component. The tufts changed orientation frame by frame, indicating highly unsteady flow during this phase of the stroke. At t = 1.04, first signs of the development of axial flow becomes visible.

cluster halfway along the forearm) showed V-shaped convergence of the free ends in seven consecutive frames (t = 1.24 to 1.48). This tuft arrangement suggests contraction of the flow such as is associated with flow acceleration along the arm (like a liquid column coming from a spigot that contracts as it accelerates downward; see Vogel (22)). This convergence was observed in *all* subjects, especially during the last part of the outsweep. At the wrist, the flow splits in two paths where part of the tufts (just behind the leading side) had a proximo distal axial arrangement, whereas tufts close to the thumb (trailing side) and those on the fingertips aligned to the movement direction of the hand. Clearly, the flow during this phase was dominated by its axial component along the arm, but separated at the trailing edge of the hand, so that the tufts at that location aligned to the direction of the hand. This suggests that the local flow around the hand is influenced by the rotation of the attached upper and forearm, such that fluid is pumped in the direction of the hand. Thus, it may not be appropriate to view the hand as a separate propelling surface for which the force-producing characteristics can be determined in isolation in a flow channel.

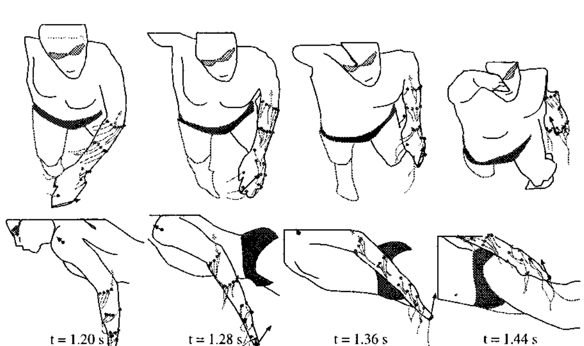

FIGURE 7—During the outsweep, the axial flow established during the insweep became very prominent. The free ends of the tufts that belong to the same tuft cluster show a V-shaped arrangement, indicative of an accelerating axial flow along the trailing side of the arm toward the hand. The outsweep lasted 0.44 s.

The pressure recordings (uncorrected for hydrostatic pressure) are supportive for the suggested pressure gradient along the arm dependent on the rotational kinematics of the arm/hand during the outsweep. Especially at sprint speed, the pressure at the dorsal side of the hand during the last part of the stroke is lower than the pressure at the shoulder and elbow (thus completely opposite the pressure gradient, given the difference in depth of hand, shoulder, and elbow). With increased speed and thus stroke frequency, the pressure difference over the hand increased (right panels in Fig. 4), which is expected to lead to increased propulsion. Interestingly, this increase arose because of a strong reduction of pressure on the dorsal side of the hand, rather than because of an increase of the pressure at the palm of the hand, which would be expected when the hand acts like a paddle. This finding supports our hypothesis concerning the propulsion-enhancing effect of the axial flow attributable to rotation of the arm. This suggests that swimming faster, with concomitant larger propelling forces, is a matter of decreasing the pressure at the dorsal side of the hand. It is important to note that this pumped-up component of propulsion is likely to complement the paddling action by the hand (drag) as well as the circulation-based lift forces. Thus, we expect that a new, composite theory will be needed in order for the study of the fluid dynamics of front crawl propulsion to progress.

DISCUSSION

The kinematics of arm and hand during the insweep and outsweep was highly unsteady; velocity and movement direction of fingertip and elbow changed considerably from frame to frame. Furthermore, the arm movements were largely rotational rather than translational (i.e., as is the case in the flow channel). The flow direction, as revealed by the tufts, reflected the unsteadiness of the limb movements. The flow direction varied strongly, particularly during the insweep, whereas an axial flow component, not in the direction of the arm movement, was observed during the late insweep and the outsweep. The V-shaped convergence of the free ends of tufts, belonging to the same tuft cluster, suggested that this axial flow was gaining strength toward the hand. The pressure recordings, especially during the sprint, show pressure on the dorsal side of the hand being lower than the pressure on the elbow and shoulder in the latter portion of the stroke. These latter two findings corroborate our prediction regarding the flow pattern generated by a rotating limb set up by a "pumping effect." These observations, together with earlier considerations such as the Wagner effect and the unpredicted high lift forces generated by an oscillating cylinder (13), strongly suggest that the rotational and unsteady aspects of front crawl propulsion may not be ignored.

The quasi-steady analysis of front crawl swimming has been a very important step forward in the study of human swimming, as it was the first quantitative analysis of propulsion. However, since it does not take into account the fluid dynamic effects of the unsteady and rotational aspects described in this article, the mechanism of propulsion needs to be reconsidered. The implications of the observed unsteady flow (insweep) for propulsion are as yet unclear. However, we may hypothesize about the possible propulsion-enhancing effect of the axial flow that was observed during the outsweep. In fact, this axial flow conforms to the hypothetical rotational pumped-up propulsion mechanism, which was put forward in the Introduction section. Indeed, our preliminary pressure recordings suggest that the predicted axial pressure gradients associated with this mechanism do exist.

The results of this study are reminiscent of studies of insect hovering flight, which demonstrated the failure of the quasi-steady approach and described a strong axial flow component along the rotating wing, stabilizing a leading-edge vortex, which could account for up to two thirds of the required lift force (1,2,10). Our simple analysis implies that axial flow may be a general phenomenon for rotational movements. Therefore, axial flow may also be relevant for the leg kick during swimming, which involves rotations in hip, knee, and ankle joints. In fact, in the natural world, rotation of propelling elements is far more common than translation. Therefore, axial flow may be crucial for understanding propulsion in many other forms of aquatic and aerial locomotion involving rotation of propelling elements, such as fins, paddles, wings, and webbed feet.

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