CONTROLLING AN OUTPUT WAVE SHAPE DURING UNDULATORY SWIMMING

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Abstract

We study the undulatory swimming of a rubber fish (nektor) with a focus on the output body-wave shape. We use a coupled fluid-structural model to simulate the dynamics of the system. The inviscid, irrotational flow is modeled with an unsteady vortex panel method. The structure of the nektor is modeled as linear elastic two-dimensional beam with small deflections. Our goal is to obtain an optimal set of forcing and structural parameters to best match the midline shape of a swimming fish. We use a multi-dimensional Newton's method to obtain the combination of parameters that produce the closest match. We hope to better emulate the body-wave shape of high efficiency fish utilizing a simple elastomer foil.

Introduction

Through evolution, aquatic animals have developed efficient ways to propel themselves. Most fish, for instance, can maintain fast speeds, accelerate quickly, and have excellent maneuverability [1]. They swim with undulatory motion in which waves created by the bending of the body propagate from the head to the tail [2]. Their movement is regulated by various characteristics such as tail-beat frequency and amplitude. Armed with a better understanding of how fish swim, the affect of these parameters on the control of speed can be determined.

McHenry *et al* [3] suggested body stiffness, tail-beat frequency, and driving amplitude are the control variables for swimming speed. The theory was tested with an experimental apparatus in which model sunfish were actuated by a mechanical bending couple located just posterior to the neurocranium. The wave input signal was sinusoidal and tests were run at various frequencies and amplitudes. Results of the experiment showed that the predicted control variables did indeed dictate the swimming speed for the model. Data from live fish allowed for predictions of how their swimming differed from that of the model.

For our work we hope to emulate the body-wave shape of a swimming fish utilizing a simple elastomer foil that is actuated in pitch. We use a two-dimensional vortex lattice method coupled to a linear elastic structural model to simulate the dynamics of the foil. To best emulate the motion we optimize a simple parameter set of elasticity, oscillation amplitude, and oscillation phase by utilizing Newton's method to minimize the L_2 norm over one period of oscillation. The goal of this work is to demonstrate that a simple elastomer foil can more closely emulate the body motion of a fish by optimizing parameters of the foil.

Numerical Modeling

Solving this problem required the use of hydrodynamic techniques and solid mechanics. The velocity field and the pressure exerted by the fluid on the nektor must be calculated as the nektor moves in time. The equation of bending of the foil is also needed in order to find the new position of the foil at each time step. The pressure affects the movement of the foil, which then changes the circulation in the fluid.

For the computational model, the nektor body is approximated as a slender, elastic beam. The leading quarter chord of the two-dimensional beam is assumed inflexible. The trailing three-quarter chord is compliant and allowed to bend. The structural equation is

$$r_s'' \frac{\partial^2 h}{\partial t^2} + EI' \frac{\partial^4 h}{\partial x^4} = \Delta p$$

where h is the vertical displacement of the beam from its nondeformed position and Δp is the hydrodynamic pressure difference between the top and bottom of the foil. E is the modulus of elasticity, I' is the moment of inertia per unit span, and r_S'' is the mass per unit chord. We assume that the beam is made of a homogeneous material with a constant thickness so the modulus of elasticity, moment of inertia, and density are constant over the chord.

We assume the fluid is incompressible, inviscid, and irrotational over the entire flow field. These assumptions are valid for large Reynolds numbers, such as those found in the biological system we seek to emulate [1]. The equation that describes the flow field is

$$\nabla^2 \Phi = 0$$

where Φ is the velocity potential whose gradient is the flow velocity. The relationship between the velocity potential and pressure can be derived from the momentum equation. Assuming small disturbances and linear theory, the pressure difference between the upper and lower surfaces of the foil is

$$\Delta p = r\left(U_\infty \frac{\partial}{\partial x}\Delta\Phi + \frac{\partial}{\partial t}\Delta\Phi\right)$$

where r is the fluid density, U_∞ is the mean free stream velocity, and $\Delta\Phi$ is the velocity potential difference between the and bottom of the beam.

An unsteady discrete vortex lattice model is used to describe the hydrodynamic portion of the problem. In this discrete model the pressure difference at any panel becomes

$$(\Delta p)_j = \frac{rU_\infty}{\Delta x}\Gamma_j + r\frac{\partial}{\partial t}\sum_{i=1}^{j}\Gamma_i$$

where Γ_i is the discrete vortex strength at panel i.

The unsteady nature of the problem is introduced to the otherwise non-time dependent hydrodynamic equation through the unsteady boundary conditions. This manifests itself in the vortex lattice model through the coupled equations that describe the induced and prescribed downwash at each panel. This equation is

$$U_\infty \left(a - \frac{\partial h}{\partial x}\bigg|_i \right) - \frac{\partial h}{\partial t}\bigg|_i = \sum_{j=1}^{N} \Gamma_j \frac{z_j - z_i}{2pr_j^2} + \sum_{k=1}^{N_W} \Gamma_k \frac{z_k - z_i}{2pr_k^2}$$

where a is the forced angle of attack of the leading edge quarter chord. The two summations on the right hand side of the equation account for the induced wash due to the bound vorticity of the structure and the vorticity shed into the wake. The vorticity shed into the wake is specified by the Kelvin circulation theorem. For the discrete vortex lattice model the strength of the shed vortex is given as

$$\Gamma_{shed} = \sum_{i=1}^{N} \left(\Gamma_i^n - \Gamma_i^{n+1} \right)$$

where the superscripts represent the time step for each of the panel's vorticity value. The vorticity shed into the wake is allowed to convect with free stream and induced velocity utilizing standard numerical models [8].

To best match the midline shape of a swimming fish, we wish to minimize the L_2 norm between the time dependent centerline displacements of our model and experimentally determined data. For this work we do not allow our model to heave. We place the point of pivot for the forced pitching at near quarter chord of the nektor which is the experimental observed point of minimum centerline displacement for swimming mackerel [7]. To minimize the L_2 norm and achieve the best match we allow the amplitude of forced oscillation, structural modulus of elasticity, and forcing phase to be varied. We utilize a Newton method to minimize the L_2 norm by obtaining the optimum combination of amplitude, elasticity, and phase [6].

Results

We choose to use the data obtained by Katz and Shadwick [7] as the experimentally determined midline position used for comparison to our model. To mathematically represent the time dependent midline position obtained experimentally we choose

$$h = g(x)\sin 2p\left(\frac{x}{l} + f\,t\right)$$

where $g(x)$ is the amplitude envelope of oscillation, l is the propulsive wavelength of the body, and f is the frequency of oscillation. The values for wavelength and frequency are obtained directly from Katz and Shadwick [7]. These values are f = 4.6Hz and l =1.2L, where L is the body length which is equal to 25.5cm. The representation for the amplitude envelope is obtained by fitting a curve to the experimental values given by Katz and Shadwick [7]. Figure 1 shows the experimental data obtained by Katz and Shadwick for the amplitude envelope of body undulations. Superimposed on this graph are exponential and cubic curve fits.

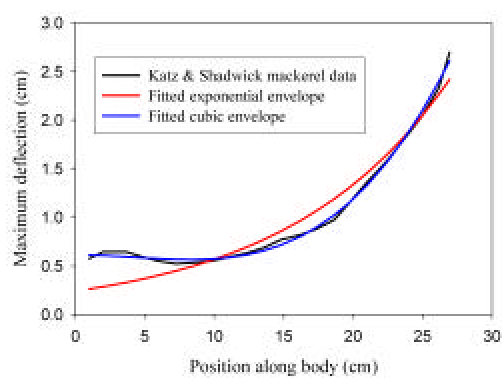

Figure 1: Comparison of Plots for Amplitude Envelope of Body Undulations

Although it is more standard to have the amplitude envelope in the form of an exponential, we have chosen to use a cubic because it is a much better approximation of the experimental data, also Katz and Shadwick use a fourth order polynomial to describe the envelope of much of their experimental data. The cubic equation of our curve fit is

$$g(x) = 0.0002x^3 - 0.0025x^2 + 0.0017x + 0.6137$$

where x is the position along the body in centimeters.

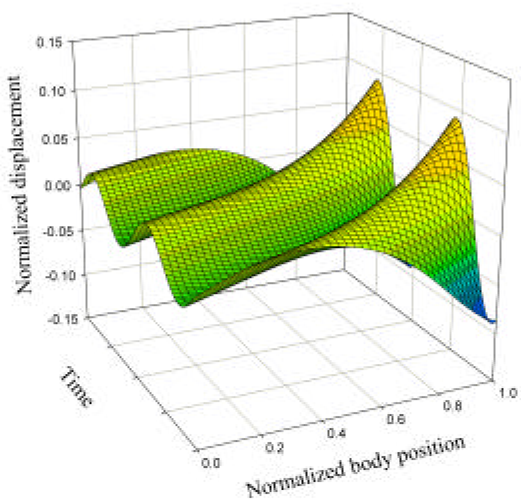

Figure 2: Midline Displacement Surface for Experimental Data Fit

Figure 2 shows two cycles of the midline displacement for the time dependent experimental data utilizing the curve fit equations.

Utilizing our coupled model of a flexible nektor harmonically pitched at the quarter chord, we minimize the L_2 norm of the error between the model and experimental data by varying a limited number of nektor parameters. For this work we optimize the parameter set of elasticity, oscillation amplitude, and oscillation phase by using Newton's method.

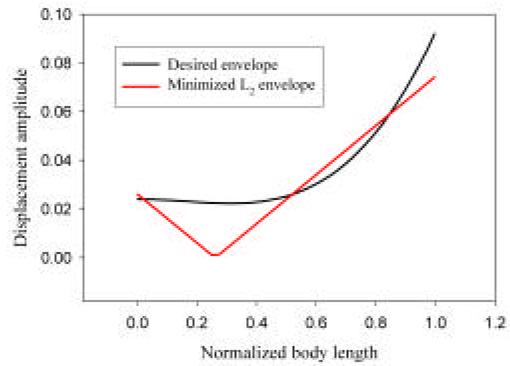

Figure 3: Amplitude Envelope Comparison of Desired Envelope and the Optimized Envelope

Figure 3 shows a comparison between the amplitude envelope for the experimental data and the envelope obtained when the L_2 is minimized. The optimum parameter set to obtain the minimum L_2 norm is an oscillation amplitude of $\pm 5.73°$, a phase angle of $-71.1°$, and an infinite modulus of elasticity. The minimized L_2 envelope of Figure 3 clearly shows the need to add a heaving variable to the parameter set to allow for a more closely matched set of data near the current "pinned" pitching position of the nektor.

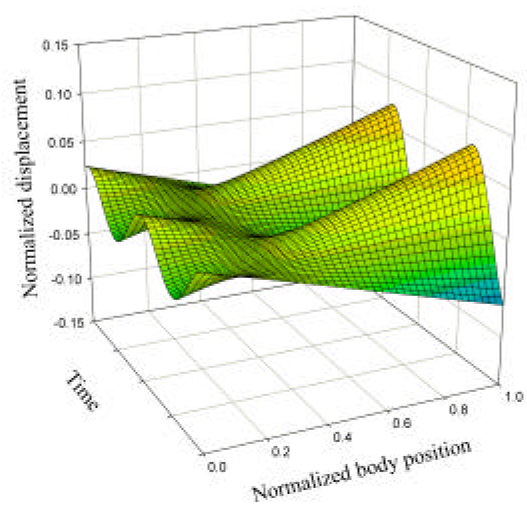

Figure 4: Midline Displacement Surface for Optimized Parameter Set

Figure 4 shows two cycles of the midline displacement for the nektor with the parameter set that minimizes the L_2 norm. Figures 2 and 4 show a graphical representation of the data that was compared to obtain the minimum L_2 norm.

Conclusions

In this work we demonstrated a method to maximize the emulation of a complex body-wave shape of an aquatic animal by optimizing a parameter set for a simply actuated compliant foil. The results for the parameter set we chose for this work did not allow enough degrees of freedom to simulate the experimental data well. In particular, the inability of our foil to heave causes a substantial inherent error that the chosen parameter set could not compensate for. We did, however, optimize the shape for the limited number of free parameters we chose.

We plan on using this work as a foundation for future work on this topic. We will utilize a larger nektor parameter set to more closely match the body-wave shape of aquatic animals. We will modify our model to include a heave component. The variables we plan on adding to our parameter set include magnitude and phase of heave, pitching point location, and thickness variations along the chord. We also plan on using this same methodology to optimize the propulsive efficiency of simply actuated oscillating foils.

Acknowledgements

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References

1. Vogel, S. *Life in Moving Fluids*. Princeton University Press, 1994.
2. Long, Jr., J.H., McHenry, M.J., and Boetticher, N.C. Undulatory swimming: how traveling waves are produced and modulated in sunfish (*Lepomis Gibbosus*). J. Exp. Biol. 192: 129-145, 1994.
3. McHenry, M., Pell, C.A., and Long, Jr., J.H. Mechanical control of swimming speed: stiffness and axial wave form in undulating fish models. J. Exp. Biol. 198: 2293-2305, 1995.
4. Knower, T. Shadwick, R.E., Katz, S.L., Graham, J.B., and Wardle, C.S. Red muscle activation patterns in yellowfin (*Thunnus Albacares*) and skipjack (*Katsuwonus Pelamis*) tunas during steady swimming. J. Exp. Biol. 202: 2127-2138, 1999.
5. Katz, J. and Plotkin, A. *Low-Speed Aerodynamics*. Cambridge University Press, 2001.
6. Kubicek, M. and Marek, M. *Computational Methods in Bifurcation Theory and Dissipative Structures*. Springer-Verlag, 1983.
7. Katz, S. and Shadwick, R.E. Curvature of swimming fish midlines as an index of muscle strain suggests swimming muscle produces net positive work. J. Theor. Biol. 193: 243-256, 1998.
8. Murray, M.M. and Howle, L.E., Response of compliant foils to dynamic loadings produced by oncoming vortices. *Proc. 12th Int. Symp. Unmanned Untethered Submersible Tech.*, Durham, NH.