MBARI Memorandum

Dorado-007

TO:	Dorado/Altex Team	DATE: February 10, 2001
FROM:	Rob McEwen	
SUBJECT:	Vehicle Range Versus Navigation Errors	
REVISION:	1.0	

Summary

Figure 1 shows how far the vehicle can travel before the cross-track navigation error exceeds 1km in an Earth-Fixed reference frame.

This is a deterministic, worst-case analysis with the following assumptions:

1. Planar, Cartesian kinematics.
2. The heading reference has a constant angular drift rate and a constant misalignment angle, in the Body-Fixed reference frame.
3. The ice drifts at a constant rate in an Earth-Fixed reference frame

Not included are:

1. Gyro random walk (or any stochastic effects)
2. Effects of reversing course
3. Scale factor error
4. Unknown errors

A stochastic analysis of this problem is also appropriate, but is not included here.

Derivation

The heading is given by

$$\psi = \omega_D t + \psi_0 \tag{1}$$

where ω_D is the angular drift rate, and ψ_0 is the misalignment angle. The velocity of the vehicle with respect to an inertial frame, coordinatzed in the inertial frame, is

$$\vec{v}_N = \begin{bmatrix} 0 \\ v_i \end{bmatrix}_N + T_{N/B} \begin{bmatrix} v \\ 0 \end{bmatrix}_B \tag{2}$$

where v_i is the speed of the ice along the inertial y axis, v is the speed of the vehicle along the body x axis, and $T_{N/B}$ is the direction cosine matrix given by

$$T_{N/B} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \tag{3}$$

The vehicle location $\vec{r}^{N_0 B_0}$ in the inertial frame is then

$$\vec{r}^{N_0 B_0} = \int_0^t \vec{v}_N \, dt \tag{4}$$

$$= \begin{bmatrix} \frac{v}{\omega_D}\left(\sin\left(\omega_D t + \psi_0\right) - \sin\psi_0\right) \\ \frac{v}{\omega_D}\left(-\cos\left(\omega_D t + \psi_0\right) + \cos\psi_0\right) + v_i t \end{bmatrix}_N \tag{5}$$

where Eqn. 5 is obtained by substituting Eqn. 1 into Eqn. 3, and then Eqn. 3 into 2, and finally 2 into 4.

The second element of $\vec{r}^{N_0 B_0}$ is the cross-track error in the Earth-Fixed frame. We desire to determine how far the vehicle can travel in the Earth-Fixed x direction before the cross-track error exceeds a specified distance, d. The distance d represents the homing beacon (or system) capture radius. Thus, we desire first to find the time, t_f, at which

$$d = \frac{v}{\omega_D}\left(-\cos\left(\omega_D t_f + \psi_0\right) + \cos\psi_0\right) + v_i t_f \tag{6}$$

This equation is transcendental and cannot be solved in closed-form for t_f, but can be solved numerically. Once t_f has been found, it can be substituted into the first element of $\vec{r}^{N_0 B_0}$ in Eqn. 5 to obtain the range.

Values of $v_i = .05$, $v = 1.5$, and $d = 1$km yield the family of curves in Figure 1. All values are in the mks system unless otherwise indicated.

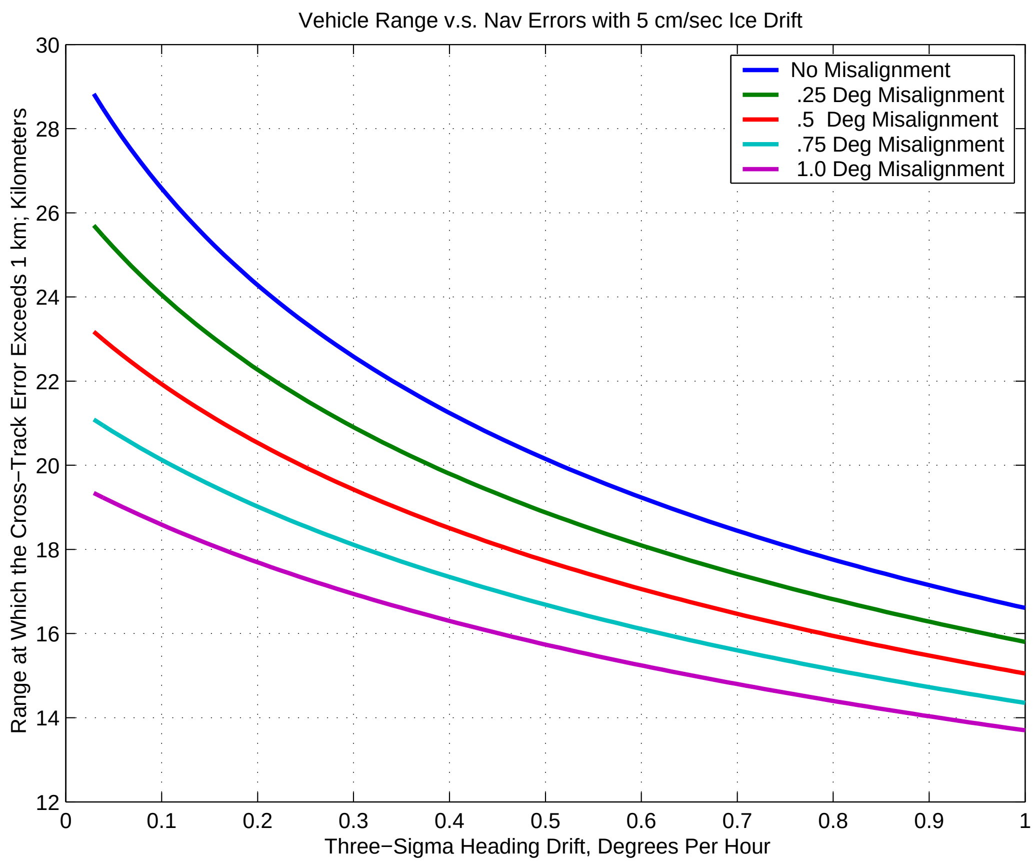

Figure 1: Vehicle Range Versus Navigation Errors