# Chapter 3 - Introduction to Multibeam Sonar: Projector and Hydrophone Systems

The previous chapter examined how multibeam sonar can be used to make up for many of the short comings of single-beam sonar. It introduced the concept of directivity and narrow projector beams. This chapter describes how

- groups of projectors, called *projector arrays*, and groups of hydrophones, called *hydrophone arrays*, can be used to produce narrow transmit and receive beams, a process called *beam forming*
- these narrow beams can be targeted at specific angles using *beam steering* processes
- a hydrophone array can be used to simultaneously record sound from many steered beams
- projector and hydrophone arrays are combined in a *Mills Cross* arrangement
- all of these techniques are employed in the SEA BEAM 2100 system

## Projector Arrays and Beam Forming

Recall from the section, "A Single-Beam Depth Sounder," in Chapter 2, that a ping from a simple single-beam echo sounder expands spherically with uniform amplitude as it propagates through water, spreading its acoustic energy equally in all directions. This symmetric spreading is called an *isotropic expansion*, and the projector that produces it is called an *isotropic source*. A good example of a wave with isotropic expansion is the circular pattern produced when a small stone is dropped in a quiet pond (see Figure Chapter 3 - -1).

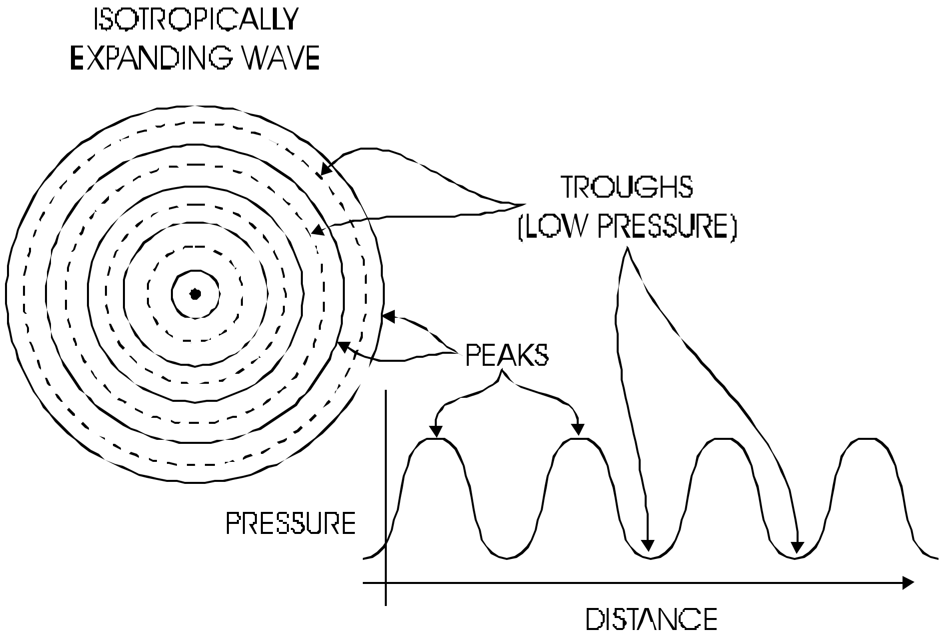


**Figure Chapter 3 - -1: Isotropic Expansion**

An isotropic source is not ideal for a depth-sounding sonar for two reasons:

- The spherically expanding pulse strikes the ocean floor in all directions. There is no way to determine the direction of the return echoes, so no detailed information about the bottom can be discerned.
- The power of the transmitted pulse is sent equally in all directions, so much of it is squandered, ensonifying areas that may not be interesting.

Fortunately, groups of isotropic sources, called *projector arrays*, can be used to transmit non-isotropic waves or sound waves whose amplitude varies as a function of angular location (still spreading spherically), allowing projected pulses to have a degree of directivity. Directed pulses can be used to ensonify specific areas on the ocean floor, causing stronger echoes from these locations. Ranges can then be found to those locations, generating more detailed information about the bottom.

Recall from the section, "The Physics of Sound in Water," in Chapter 2, that a sound wave is composed of a series of pressure oscillations. The circular solid lines in Figure Chapter 3 - -1 represent high pressure *peaks*. Spaced half-way between these lines are low pressure *troughs* represented by dashed lines. Alone, an ideal single-point projector always produces an isotropically expanding wave. Operating at a constant frequency, it creates a continuous series of equally spaced, expanding peaks and troughs, which look similar to what is pictured in Figure Chapter 3 - -1.

 

If two neighboring projectors are emitting identical isotropically expanding signals, their wave patterns will overlap and *interfere* with each other. This situation is depicted in Figure Chapter 3 - -2. At some points in the surrounding water, the peaks of the pattern from one projector will coincide with peaks from the other, and will add to create a new, stronger peak. Troughs that coincide with troughs will create new, deeper troughs. This is called *constructive* interference. At other points, peaks from one projector will coincide with troughs of the other and will effectively cancel each other. This is called *destructive* interference.

In general, constructive interference occurs at points where the distances to each projector are equal, or where the difference between the two distances is equal to an *integer* number of wavelengths. Destructive interference occurs at positions where the difference between the distances to the projectors is half a wavelength, or half a wavelength plus an integer number of wavelengths (1.5, 2.5, 3.5, and so forth). If a hydrophone is placed at the positions of constructive interference, a combined wave would be measured with an amplitude twice that of the signals emitted by each projector individually. A hydrophone placed at a position of destructive interference would measure nothing at all. Where are these places?

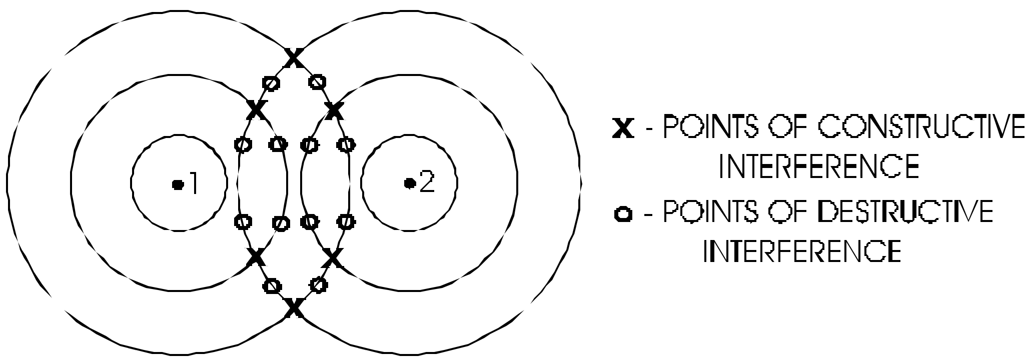


**Figure Chapter 3 - -2: Constructive and Destructive Interference**

In Figure Chapter 3 - -3, two projectors $P_1$ and $P_2$ are separated by a distance d (referred to as the *element spacing*). Consider a point located distance $R_1$ from $P_1$ and $R_2$ from $P_2$. If this point is located anywhere on the perpendicular bisector of line $P_1P_2$, then $R_1$ and $R_2$ are equal. Any point along this line will witness constructive interference.

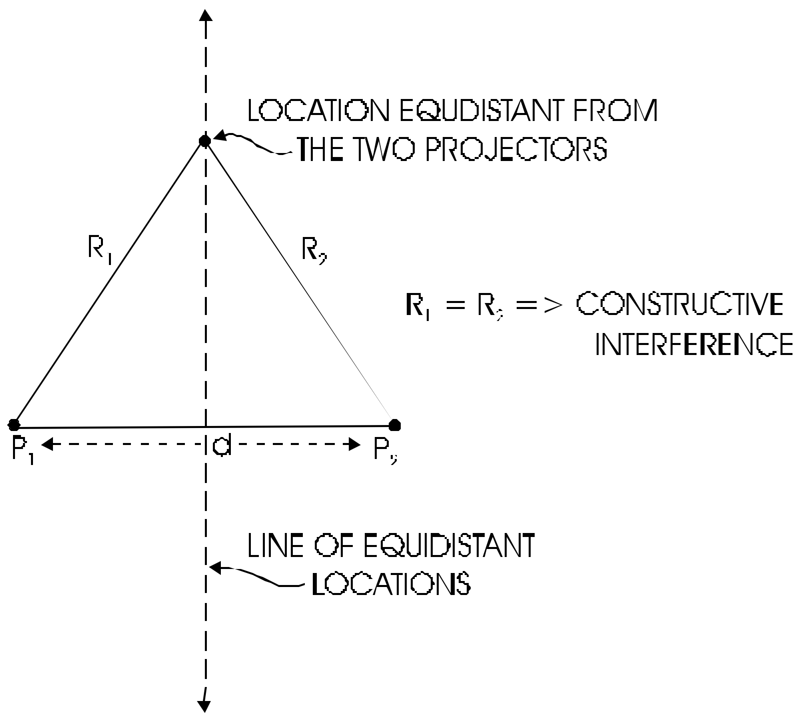


**Figure Chapter 3 - -3: Positions of Constructive Interference (Example 1)**

The locations of other constructive interference are less obvious, but they can be found with some simple geometry. In Figure Chapter 3 - -4, two projectors $P_1$ and $P_2$ again have a spacing $d$. Consider a point at a location $R_1$ from $P_1$ and $R_2$ from $P_2$. The direction to this location (labeled $R_0$ in the figure) intersects a line perpendicular to the spacing d with an angle $\theta_0$. Next, assume that the point you are considering is very far away compared to the spacing of the projectors—meaning that $R_1$ and $R_2$ are much larger than d. For a typical operating environment for a sonar, this is a good approximation—projectors are spaced centimeters apart ($d$ = cm) and the ocean floor they are ensonifying is hundreds or thousands of meters away ($R_0$ = 100 m to 1000 m, meaning $R_0/d$ = 1000 to 10000). This is called the *far field* approximation, and it is necessary to keep computations simple. In this situation, the lines $R_0$, $R_1$, and $R_2$ are treated as parallel, and all intersecting angles $\theta_0$, $\theta_1$, and $\theta_2$ as equal.

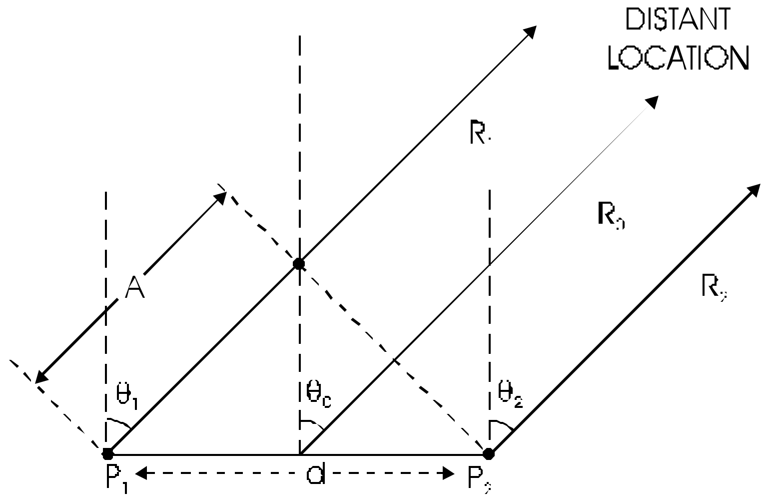


**Figure Chapter 3 - -4: Positions of Constructive Interference (Example 2)**

The difference between $R_1$ and $R_2$ is the line segment labeled *A* in Figure Chapter 3 - -4. If all angles are equal, this distance is:

$$A = d \times \cos(90 - \theta_0), \tag{3.1}$$

or more simply:

$$A = d \times \sin \theta_0. \tag{3.2}$$

Recall that constructive interference occurs when *A* is an integer number of wavelengths:

$$A/\lambda = 0, 1, 2, 3, 4, \ldots\ldots\ldots\ldots \text{etc.}, \tag{3.3}$$

where $\lambda$ represents wavelength.

Substituting Equation 3.2 for *A*:

$$(d/\lambda) \times \sin \theta_0 = 0, 1, 2, 3, 4, \ldots\ldots\ldots \text{etc.} \tag{3.4}$$

Similarly, destructive interference will occur where:

$$(d/\lambda) \times \sin \theta_0 = .5, 1.5, 2.5, 3.5, \ldots\ldots\ldots \text{etc.} \tag{3.5}$$

From these equations you can see that locations of constructive and destructive interference are dependent on the projector spacing *d*, the wavelength of the sound emitted $\lambda$, and the angle $\theta_0$ to the location. Both *d* and $\lambda$ remain constant for a typical sonar installation—the only remaining variable is $\theta_0$. This indicates that two projectors in the configuration described will transmit constructively interfering (that is, high amplitude) waves in certain *directions*, while in others it will transmit nothing due to destructive interference. Knowing *d* in terms of $\lambda$, you can determine which directions will have constructive and destructive interference.

A typical sonar spacing $d$ is $\lambda/2$—half a wavelength—because the angles at which destructive and constructive interference occur are most advantageous. Using $d/\lambda$ = l/2 in the Equations 3.4 and 3.5 yields:

Constructive interference: $\theta$ = 0, 180

Destructive interference: $\theta$ = 90, 270

So, the two projector arrays will transmit the highest energy sound in the directions $\theta$ = 0, 180, and no sound at all in the directions $\theta$ = 90, 270 (see Figure Chapter 3 - -5). The emission of this system can be measured with a hydrophone located at different angles around it. At 0 and 180, it measures maximum amplitude, while at 90 and 270 it measures nothing. At angles in between, there is a mix of constructive and destructive interference leading to intermediate amplitudes. By positioning a hydrophone at a set radius and many different angles around a projector array, you can record the amplitude of emitted sound in different directions. Figure Chapter 3 - -6 plots measured amplitude as a function of angle for a two projector array. Figure Chapter 3 - -6 is a polar plot—amplitudes are measured along radial lines from the center. The amplitude plot is what is known as a *beam pattern*, or *array pattern* (and sometimes a *power pattern*). It clearly shows that the bulk of the energy emitted by the two projector array propagates perpendicular to the axis of separation of the projectors.

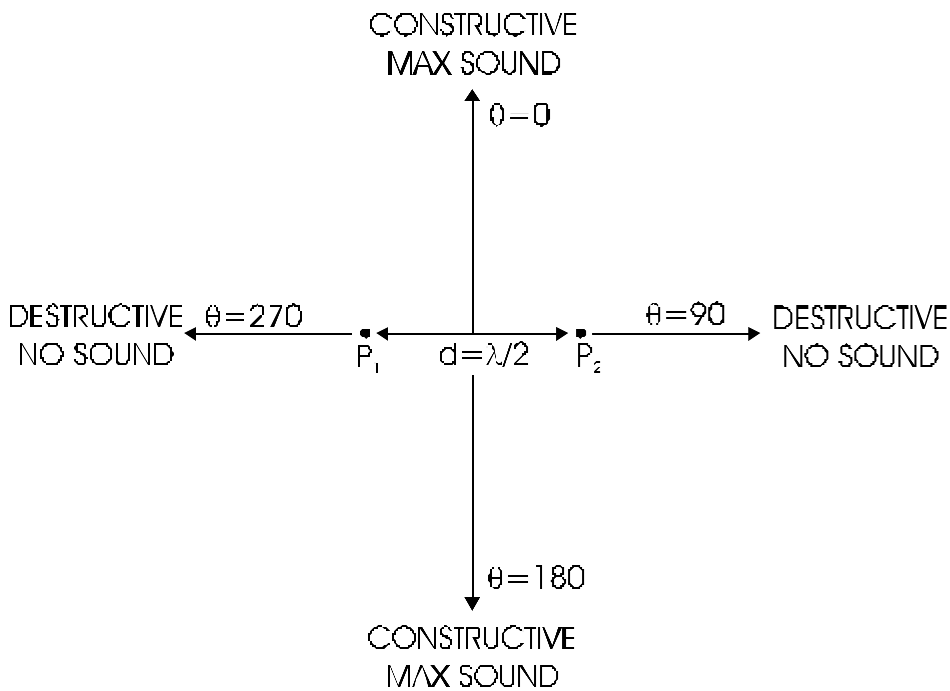


**Figure Chapter 3 - -5: Directions of Constructive and Destructive Interference for Two Projectors with Spacing l/2**

 

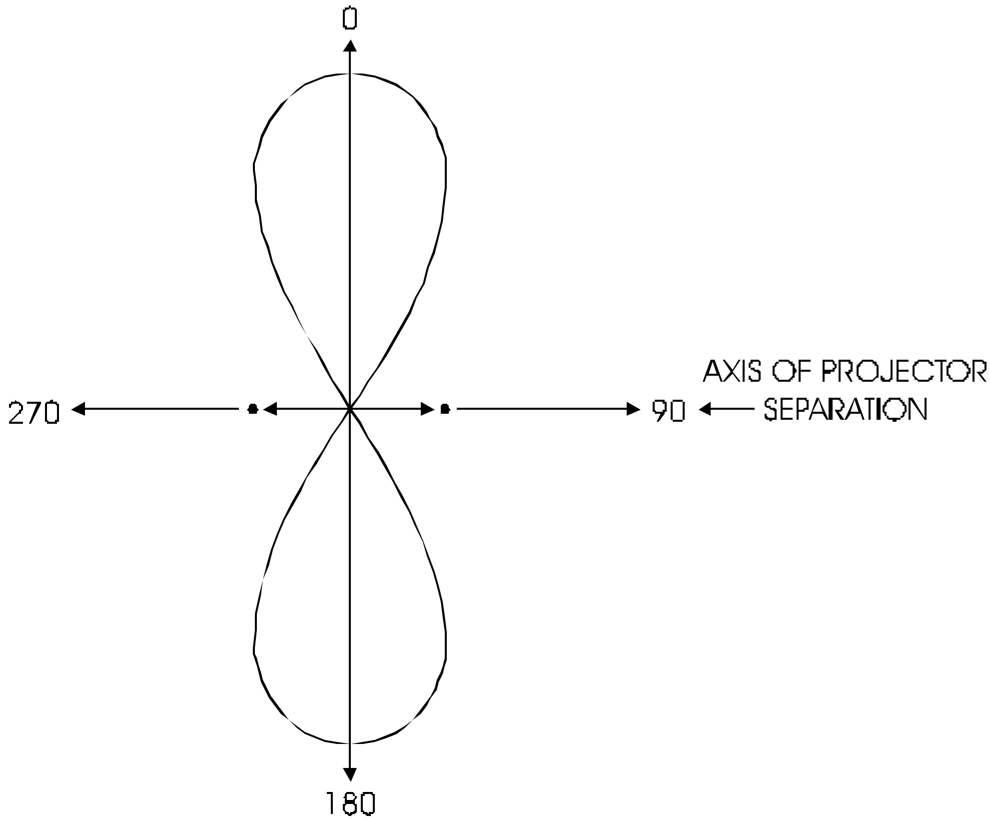


**Figure Chapter 3 - -6: Beam Pattern for Two Hydrophones with Spacing l/2**

Remember that projectors operate in a three-dimensional environment. To accurately represent the beam pattern of a two-projector array, Figure Chapter 3 - -6 should be rotated around the axis of separation of the projectors, about which it is symmetric (see Figure Chapter 3 - -7). Because three-dimensional drawings of beam patterns are difficult to interpret, the patterns are usually drawn in two-dimensions. For those beam patterns with an axis of symmetry, a two-dimensional drawing provides a complete description. More complex patterns that are not symmetric about an axis are usually represented by multiple drawings showing the cross-section of the pattern at different angles.

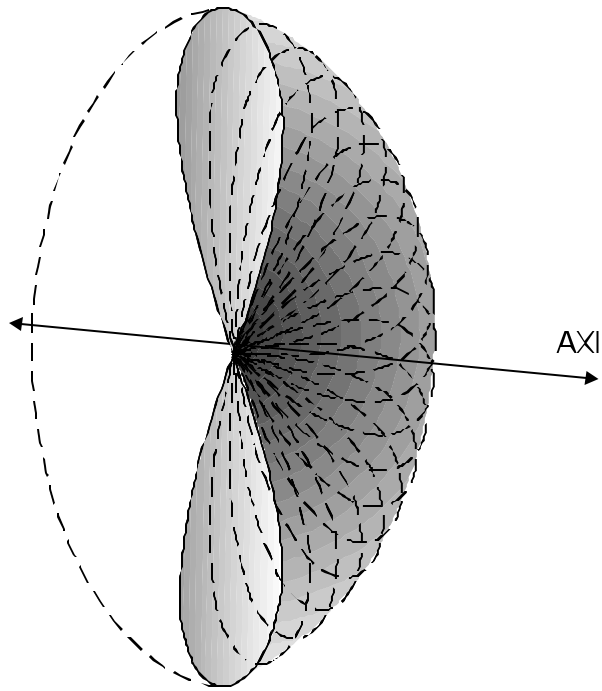


**Figure Chapter 3 - -7: Three-Dimensional Beam Pattern for Two Projectors with Spacing l/2**

One useful way of interpreting Figure Chapter 3 - -6 is as a plot of the angles at which the energy of an acoustic pulse is directed. In the figure, the highest level of acoustic energy is directed perpendicular to the axis of projector separation. Objects in these directions are ensonified with the most energy and return the strongest echoes. Objects in other directions return significantly weaker echoes. This selective projection of energy is called *directivity*. The beam pattern provides a measure of the directivity of a system—projector arrays that direct the bulk of their energy at a very narrow range of angles are said to have a high directivity.

Real projector arrays generally have more than two projector elements and have complex beam patterns. One common array configuration is a simple extension of the two projector array—an arrangement of many projectors in a straight line called a *line array*. The beam pattern of a multiple-element line array is pictured in Figure Chapter 3 - -8. Although detailed mathematics are required to compute this pattern, it is essentially just an extension of the reasoning used to find the two-projector array pattern. This complex beam pattern can be used to point out the features of all such patterns.

 

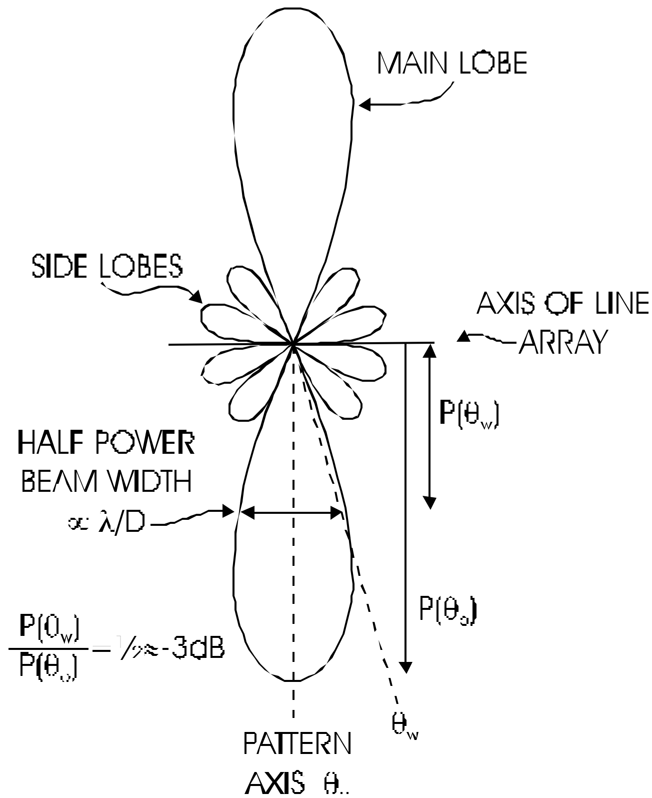


**Figure Chapter 3 - -8: Beam Pattern of a Multiple-Element Line Array**

The bulk of the energy in the line array beam pattern is in what is called the *main lobe*. The direction of the peak energy projection—the center of the main lobe—is called the *maximum response axis* (MRA) of the beam pattern. The *width* or *beam solid angle* of the main lobe, which is a measure of the pattern's directivity, is twice the angle from the axis to the *half power point* on the pattern—the angle at which the projected power is exactly half that of the axis. This is referred to as the beam width measured to the -3 dB point, where the projected power is -3 dB that of the axis (these points are almost equal, as -3 dB corresponds roughly to 1/2).

The main lobe of the line array pattern is narrower than that of the two-projector array pattern. In general, the larger an array or projector system is in a dimension, the narrower the main lobe of the beam it projects in perpendicular directions will be. A good first approximation for the width of the main lobe of a pattern of a system with size $D$ (which can be the physical size of the transducer in a single-projector system, or $(N - 1) \times d$, the total length of $N$ elements in an array with spacing $d$) transmitting at a wavelength $\lambda$ is:

$$\text{half power beam width (degrees)} = 50.6 \times \lambda/D \qquad (3.6)$$

Equation 3.6 illustrates some of the design considerations involved in building a sonar system. If a high-resolution sonar system is desired, a narrow beam is needed. The width of the beam can be reduced by building larger projectors and arrays, but there are physical limits on both. Alternatively, shorter wavelengths can be used. However, shorter wavelength sound, which is also higher frequency, suffers greater attenuation in water as discussed in the section "The Physics of Sound in Water." The attenuation can be compensated for by raising the transmitted power, but there are limits to the amount of power a single projector can cleanly transmit into water.

On either side of the main lobe are a series of *side lobes* where partial constructive interference takes place. In general, the purpose of the array is to ensonify targets in the direction of the main lobe. The side lobes are an annoyance—not only is some of the projector energy being squandered in these directions, but there might be echoes from them as well, and these may be confused with the echoes from the target in the main lobe. The nearest set of side lobes on either side of the main lobe are called the *first side lobes*, and subsequent sets are called the second side lobes, third side lobes, and so on. The strength of side lobes is measured as a fraction of the power projected into them divided by the power projected into the main lobe, and is called the *side lobe level*. These numbers are given in dB. For an array in which all projectors emit the same power level, the first side lobe level is roughly -13 dB.

Side lobes in the beam patterns of projector arrays are unavoidable, although the energy that is projected into them can be reduced by projecting stronger signals from the individual elements in the center of an array than from those on the edges. This technique is called *shading*, and the fraction of energy that is projected by each projector element divided by the energy projected by the peak element is called its *shading value*. There are a variety of algorithms used to determine what shading values should be applied to each array element. Different combinations of shading values produce different side lobe structures. One popular shading scheme, called *Dolph-Chebyshev* shading, can be used to bring all side lobe levels to a uniform value. In theory, this side lobe level can be any value, but practical considerations limit side lobe reduction generally to a maximum of -40 dB. Although it can be used to reduce the side lobes, shading also causes the width of the main lobe to be larger, decreasing the directivity of the system (and nullifying some of the advantage of the large array). A comparison of the approximate widths of the main lobes of different array sizes both unshaded (-13 dB first side lobe levels) and with Chebyshev shading to obtain -35 dB side lobe levels is given in Table Chapter 3 - -1. Discovering the proper balance of shading, array size, and array elements that yields high directivity with minimal side lobes is a complex art.

**Table Chapter 3 - -1: Main Lobe Width Comparisons**

| | Beamwidth | |
|---|---|---|
| **Array Elements** | **Unshaded** | **Chebyshev (-35 dB)** |
| 20 | 5.1° | 6.8° |
| 40 | 2.5° | 3.3° |
| 48 | 2.1° | 2.8° |
| 80 | 1.3° | 1.6° |
| 96 | 1.1° | 1.4° |

# Hydrophone Arrays

The previous section described how arrays of projectors can be used to generate narrow beams of sound that will ensonify small patches of the sea floor. Hydrophones as well as projectors can be formed into arrays. Where projector arrays transmit sound into narrow beams, hydrophones arrays are used to receive sound from narrow beams.

It is easy to see how this works. In Figure Chapter 3 - -9 through Figure Chapter 3 - -14, a line array of hydrophones is examined in two situations. In Figure Chapter 3 - -9, a source perpendicular to the axis of the line array is producing sound waves. The source of these sound waves is in the far field, such that the wave fronts striking the hydrophone array are a series of parallel lines. Each hydrophone in the array is making an independent measurement of the wave. The signal measured by each hydrophone as a function of time can be plotted in what is called a signal *trace*. Figure Chapter 3 - -10 shows signal traces for three of the hydrophones. Because the parallel waves are striking these hydrophones at exactly the same time, these three plots are identical—measured peaks and troughs occur at the same times. The three measurements are said to be *in phase*. Summing the three hydrophone traces results in a trace with the same frequency but three times the amplitude (pictured in Figure Chapter 3 - -11)—the measurements of the three hydrophones add *constructively*. Collectively, the hydrophone array is highly sensitive to sounds from this direction.

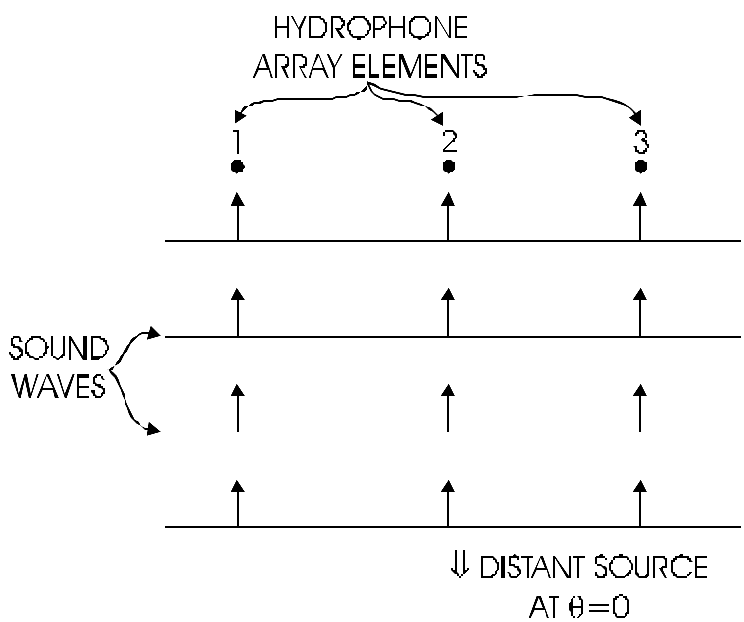


**Figure Chapter 3 - -9: Hydrophone Array with a Perpendicular Source**

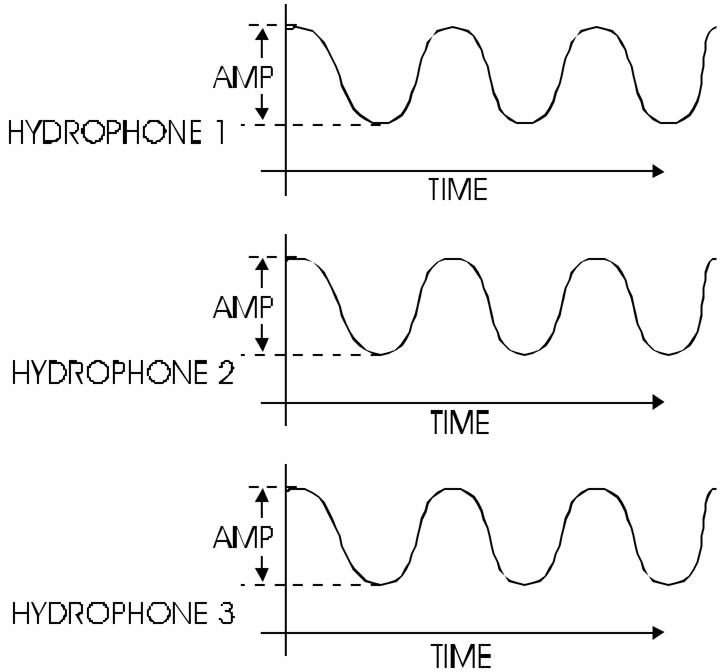


**Figure Chapter 3 - -10: Hydrophone Traces for a Perpendicular Source**

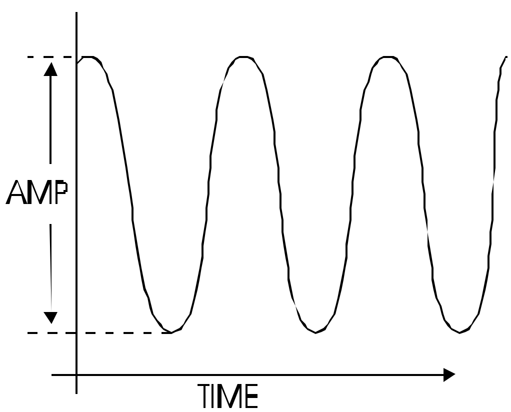


**Figure Chapter 3 - -11: Sum of Hydrophone Traces for a Perpendicular Source**

Figure Chapter 3 - -12 shows the same hydrophone array receiving sound waves from a source at an angle θ off the perpendicular. Due to the angle of the source, the parallel waves from this source strike the hydrophones in the array at different times. This causes the hydrophone traces to look different—peaks and troughs occur at different times as pictured in Figure Chapter 3 - -13. The three measurements are said to be *out of phase*. The sum of the three hydrophone traces has peaks and troughs eliminating each other—they add *destructively* (see Figure Chapter 3 - -14). Collectively, the hydrophone array is not sensitive to sounds from this direction.

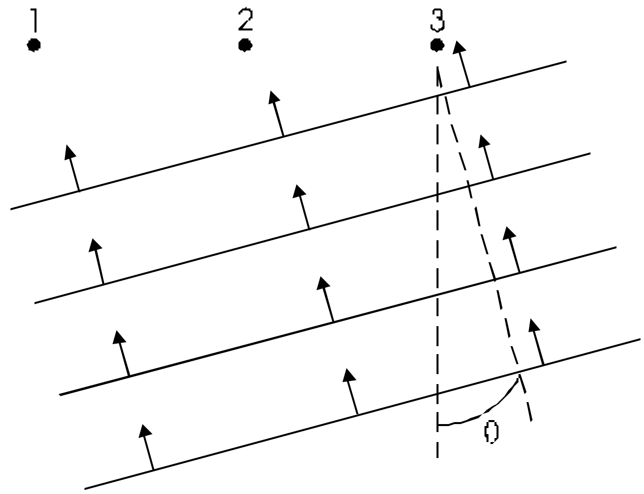


**Figure Chapter 3 - -12: Hydrophone Array with Waves from an Angled Source**

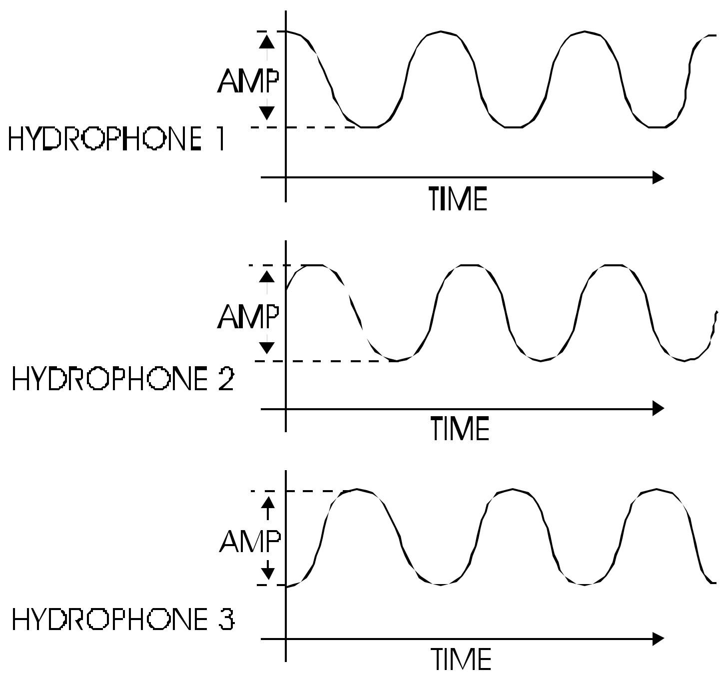


**Figure Chapter 3 - -13: Hydrophone Traces for an Angled Source**

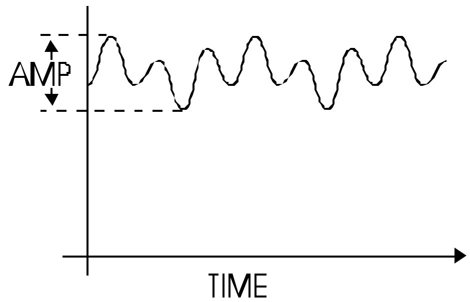


**Figure Chapter 3 - -14: Sum of Hydrophone Traces for an Angled Source**

Placing a projector at many distant (far field) points around the hydrophone array, and recording the sum of what the hydrophone elements measure, generates a familiar pattern. Figure Chapter 3 - -15 shows the pattern that would result from such a series of measurements for a line array of hydrophones. As in the case of the projector arrays, these pictures, called beam patterns, share many of the same features and terminology associated with projector arrays—main lobes, side lobes, axis, shading, and beam widths are all defined the same way. In fact, if you were to measure the beam patterns of an array of transducers operating as a projector array and then as a hydrophone array (recall that they can act as both), the patterns would be identical. This is called the *principle of reciprocity* because the hydrophone arrays follow exactly the same rules that projector arrays do.

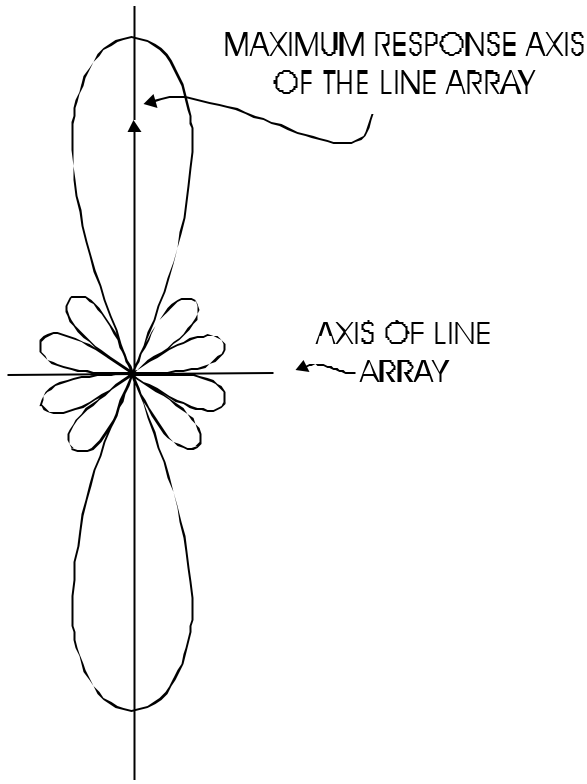


**Figure Chapter 3 - -15: Beam Pattern for a Line Array of Hydrophones**

In short, the important points to remember about the beam patterns of both projector and hydrophone arrays are the following:

- The larger an array is in a dimension, the narrower its main lobe will be in the plane perpendicular to that dimension.
- The width of the main lobe is measured at the half power point.
- Side lobes are undesirable, but unavoidable.
- Side lobes can be reduced through techniques such as shading at the cost of widening the main lobe.
- The beam pattern of a transducer array acting as projectors is the same as that of it acting as hydrophones.

# Beam Steering

The previous section described how an array of hydrophones can be used to receive sound preferentially from specific perpendicular directions. This section describes how an array can be altered to receive preferentially from any of a number of directions. This technique is called *beam steering*. Effectively, it allows the angle of the axis of the beam pattern, pictured in Figure Chapter 3 - -15, to be changed.

Consider, once again, a line array receiving sound from an arbitrary angle θ off the perpendicular (see Figure Chapter 3 - -16). Recall that the sound from this angle arrived at the elements of the hydrophone array out of phase, causing the signals at the different hydrophones to add destructively. The reason for this is that the sound waves must travel different distances to reach each hydrophone. In Figure Chapter 3 - -16, sound waves first strike the hydrophone labeled 3. They must travel the distance labeled *A* before reaching hydrophone 2, and distance *B* before reaching hydrophone 1. These distances can be computed using the spacing between the hydrophones (both assumed in this case to equal *d*, although this is by no means a requirement):

$$A = d \times \sin\theta \qquad (3.7)$$

$$B = 2d \times \sin\theta$$

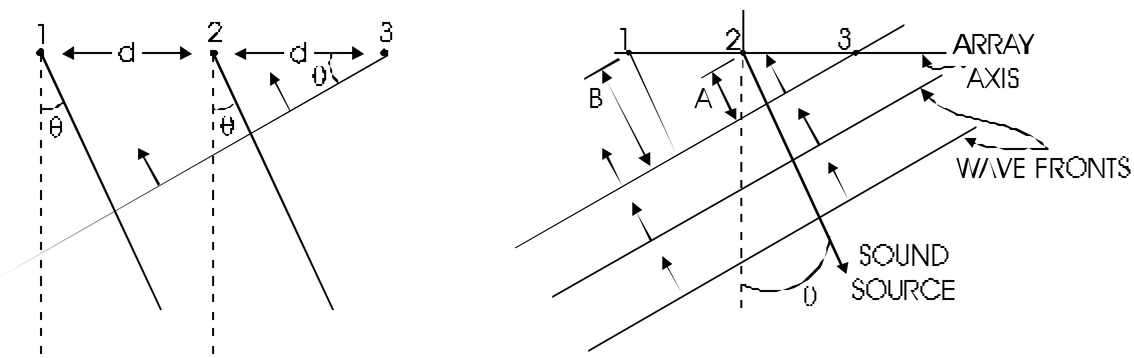


**Figure Chapter 3 - -16: Wavefronts Striking a Hydrophone Array from a Source at Angle** q

The extra times required for the wave front to reach each hydrophone are given by the distances divided by the local sound speed *c*:

$$T_2 \text{ (time to hydrophone 2)} = A / c = (d \sin\theta) / c \qquad (3.8)$$

$$T_1 \text{ (time to hydrophone 1)} = B / c = (2d \sin\theta) / c$$

Knowing these time differences you can cause the hydrophone array to have maximum sensitivity at angle θ by summing the individual hydrophone readings slightly offset in time such that the wave fronts constructively interfere. In this example, you would sum the reading of hydrophone 3 with the reading of hydrophone 2 delayed by $T_2$ and the reading of hydrophone 1 delayed by time $T_1$. This is called introducing a *time delay*. It causes the main lobe of the beam pattern to shift such that its axis is at angle θ from the perpendicular (see Figure Chapter 3 - -17). By applying time delays, or alternatively equivalent *phase delays* to hydrophone readings and summing them (a concept covered in the description of SEA BEAM 2100 processing), an array can be “steered” to maximize its sensitivity to any angle θ.

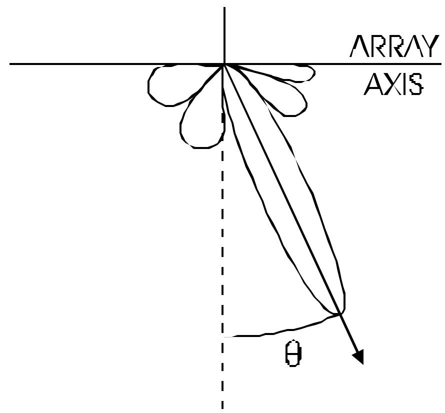


**Figure Chapter 3 - -17: Main Lobe Shifted to Angle** q **by Introducing a Time Delay**

Note that in steering a hydrophone array to be sensitive to a particular angle, nothing about the array itself is changed—only the interpretation of the data it records is altered. By changing the data processing, the same array can be steered to observe any of a large range of angles. In fact, using the same recorded data from the elements of the hydrophone array, different data processing can be used to examine the sounds coming from different angles *simultaneously*. In this way, a hydrophone array can be used to examine the echoes from a single ping at many different locations.

The product of this process—the series of beam-steered traces $B_{\theta 1}(t)$, $B_{\theta 2}(t)$, . . . . $B_{\theta M}(t)$—can be treated as a series of independent measurements of signals from different angles between $\theta_1$ and $\theta_M$. Effectively, beam steering is used to create a series of “virtual” hydrophone arrays, each sensitive to a different angle (see Figure Chapter 3 - -18).

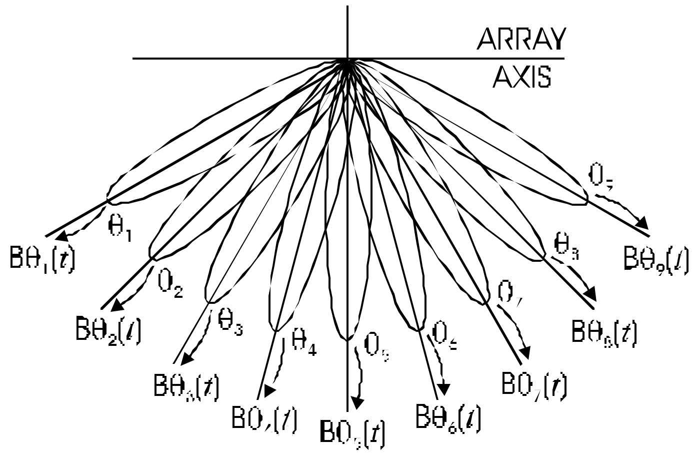


**Figure Chapter 3 - -18: Hydrophone Array Processing Used to Observe Different Beam Patterns Simultaneously**

# The Mills Cross Technique

Recall that each of the beam patterns in Figure Chapter 3 - -8, Figure Chapter 3 - -15, Figure Chapter 3 - -17, and Figure Chapter 3 - -18 are two-dimensional representations of three-dimensional patterns. The true beam pattern in each of these situations can be seen by rotating the figure about the axis of the line array (see Figure Chapter 3 - -6 and Figure Chapter 3 - -7). A projector line array transmits sound preferentially in all directions perpendicular to the axis of the array, ensonifying a strip of the ocean bottom (see Figure Chapter 3 - -19). Similarly, a hydrophone array aligned parallel to the projector array receives echoes from all locations along a similar strip of the ocean floor. If you want to accurately locate echoes on the ocean floor, this is not very useful! The projector array will cause echoes all along the ensonified strip, and the hydrophone array will pick up echoes from a similar strip. There will be no way of telling where the echoes are occurring along these strips.

 

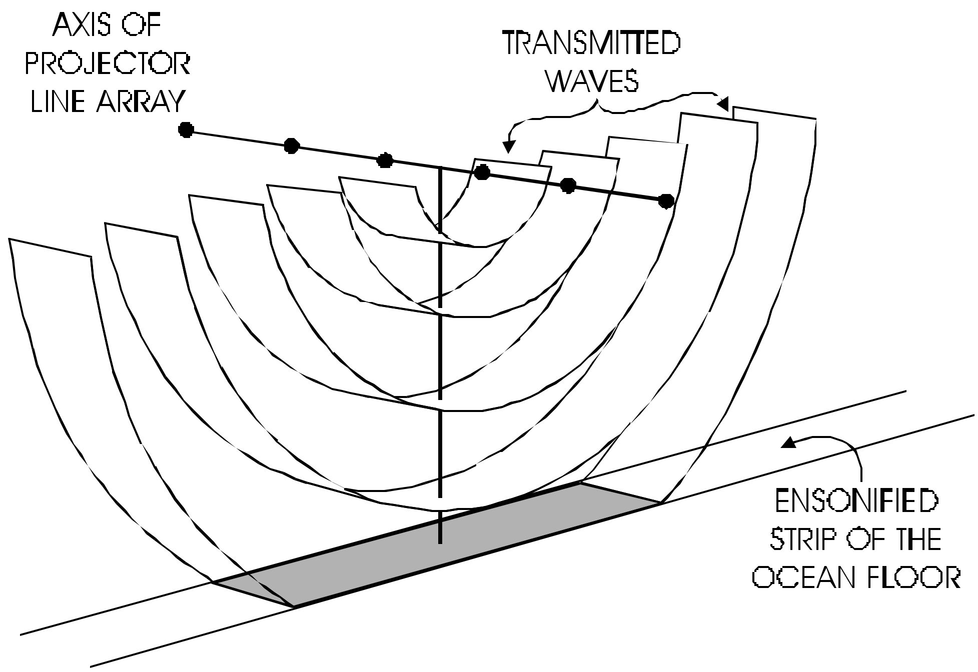


**Figure Chapter 3 - -19: Projector Array Ensonifying a Strip of the Ocean Floor**

If, however, the projector and hydrophone arrays are perpendicular to each other, the strip of the ocean floor ensonified by the projectors will intersect with the strip of the ocean floor observed by the hydrophones. This occurs in only a small area with dimensions that correspond approximately to the projector and hydrophone array beamwidths (see Figure Chapter 3 - -20). While echoes occur along the entire ensonified area, and sound may be received from the entire observed area, the only part of the bottom both ensonified by the projector array and observed by the hydrophone array beam is the area where the two strips overlap. The amplitude trace from the hydrophone array will contain only those echoes from the transmitted ping that occur in this area. The perpendicular arrangement of the projector and hydrophone line arrays is called a *Mills Cross*, named after a pioneering radio astronomy instrument built in New South Wales, Australia.

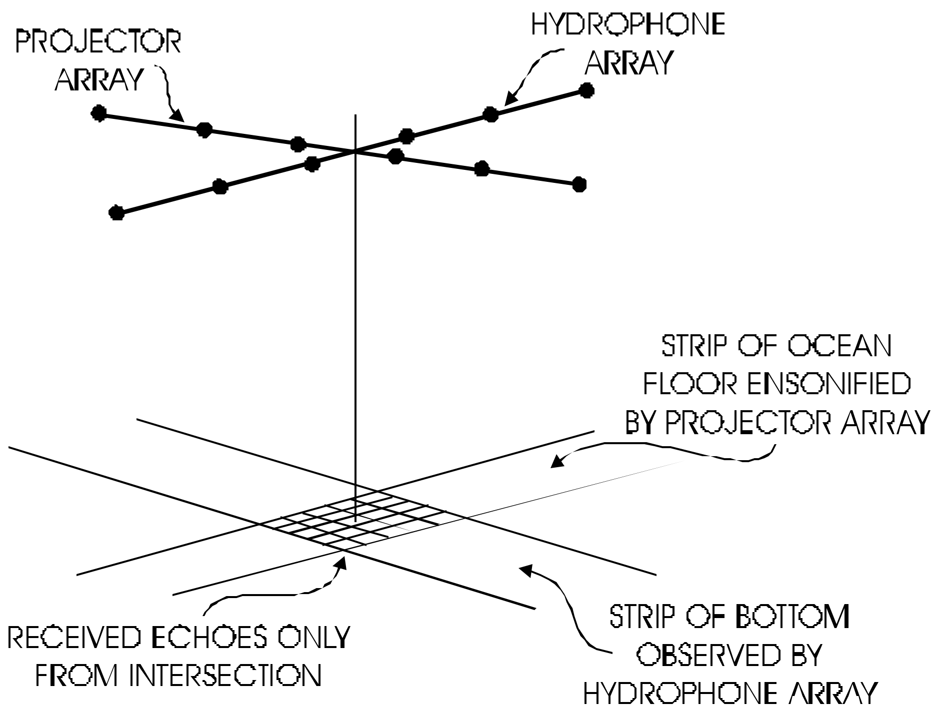


**Figure Chapter 3 - -20: Projector and Hydrophone Arrays Arranged in a "Mills Cross"**

Recall that beam steering can be used to observe echoes generated from multiple angles with the hydrophone array. These different angles observe parallel strips of the bottom, which in the Mills Cross arrangement, will intersect with the ensonified area in a series of small patches (see Figure Chapter 3 - -21). In this way, the multiple steered beams can be used to receive the echoes from discrete locations all along the ensonified area, allowing the sonar to determine ranges to a strip of locations with each ping. This technique is used by all modern multibeam sonar systems.

 

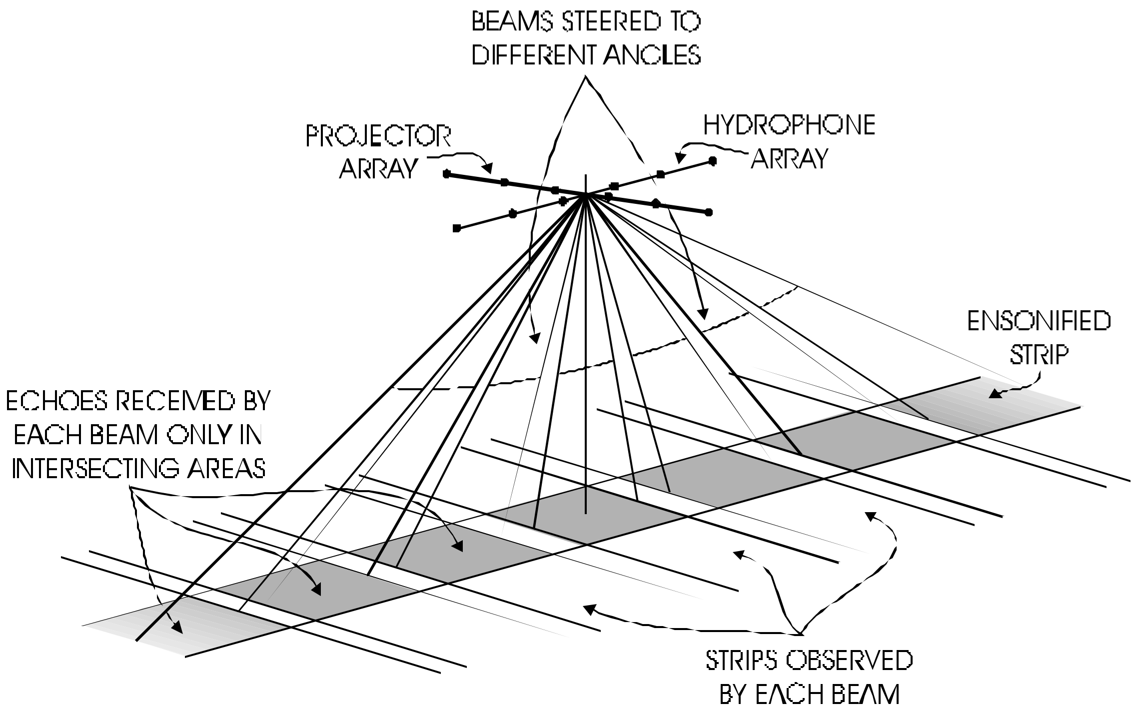


**Figure Chapter 3 - -21: Mills Cross with Multiple Steered Beams**

# The Mills Cross Applied in the SEA BEAM 2100 System

The SEA BEAM 2100 multibeam sonar utilizes the Mills Cross technique to produce high-resolution maps of the ocean floor. This section describes the projector and hydrophone array configurations used by the 2100, the beams formed by those systems, and how beam steering is accomplished.

# The SEA BEAM 2100 Projector and Hydrophone Arrays

The SEA BEAM 2100 12 kHz projector system consists of a line of 10 to 14 projector elements mounted along the keel of its survey vessel. Because it is far longer in the *alongtrack* direction (parallel to the ship's keel and its direction of motion) than in the *acrosstrack* or *athwartship* direction (perpendicular to the ship's keel), the complete projector array is similar to a line array. It projects a beam that is narrow in the alongtrack direction (roughly 2 degrees), and wide in the acrosstrack direction, illuminating a wide swath of the bottom. Dolph-Chebyshev shading techniques are used to even the illumination in the acrosstrack direction, reducing side lobe levels to between -25 and -30 dB, depending on the number of projector elements. A cross-section of the beam pattern of the SEA BEAM 2100 projector array at 12 kHz is depicted in Figure Chapter 3 - -22. This pattern has been tailored such that more energy is directed towards the wide acrosstrack directions where return signals usually arrive with lower signal-to-noise levels. Less energy is directed near the vertical where backscattering strengths are usually high and travel distances are shorter.

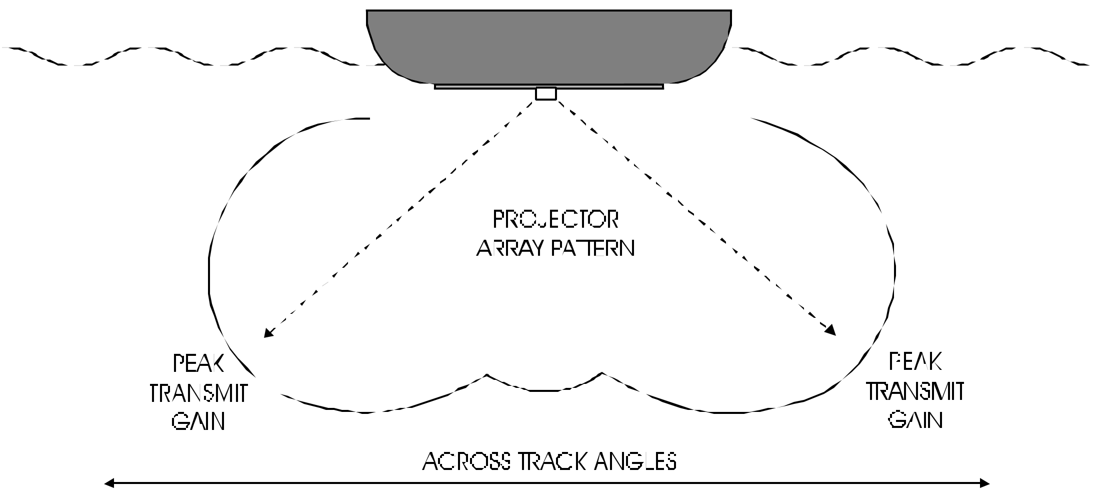


**Figure Chapter 3 - -22: Projector Array Pattern**

The SEA BEAM 2100 hydrophone array is mounted in the athwartship direction, forming the Mills Cross. It consists of between 48 and 80 hydrophones, depending on the installation, and can either be mounted in a "flat" configuration if the survey vessel has a flat bottom, or in a "V" configuration if the survey ship has v-shaped bottom (see Figure Chapter 3 - -23). In the flat configuration, the received echoes of the entire array are processed together, while in the "V" configuration the two sides of the array are processed separately. The reasons for this are related to the high-speed data processing shortcuts used in beam steering.

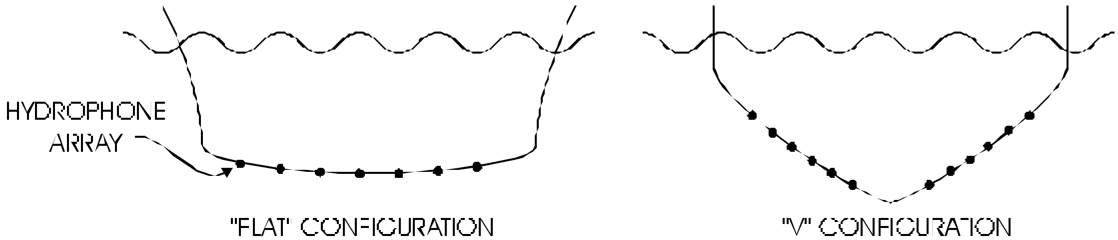


**Figure Chapter 3 - -23: "Flat" and "V" Configurations for Hydrophone Arrays**

## SEA BEAM 2100 Beam Steering

The data from the hydrophones is used to form a maximum of 151 steered beams, which have beam widths in the athwartship direction of approximately 2 degrees, and spacing of roughly 1 degree between their axes. Beam steering is accomplished through digital signal processing.

Each element *i* of the *N* elements of the hydrophone array ( *N* for the SEA BEAM 2100 is between 48 and 80) records a signal trace $S_i(t)$. It is important to understand the relationship between the measurement $S(t)$ and the amplitude $A(t)$ of a signal. The amplitude $A(t)$ measures the change in signal strength. It does not measure the oscillations of a wave that are observed by instantaneous measurements. Consider the signal segment pictured in Figure Chapter 3 - -24.

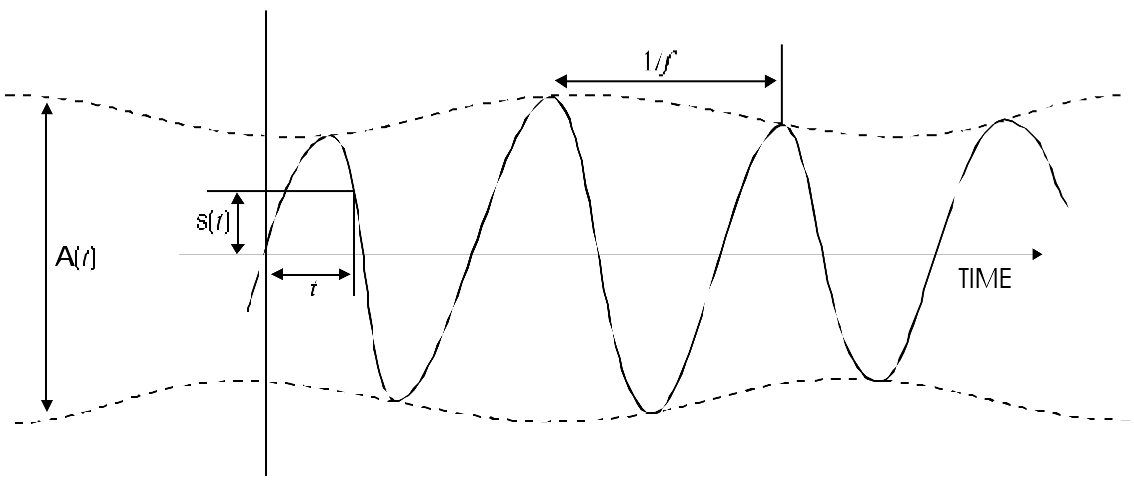


**Figure Chapter 3 - -24: Instantaneous Measurements $S(t)$ of a Signal with Amplitude $A(t)$**

The amplitude $A(t)$, which changes with time, is represented by the dashed lines. Within the envelope of the changing amplitude, the signal oscillates with its frequency *f*.

$S(t)$ is an instantaneous measurement of this signal made at time *t*. Knowing the frequency of the signal and its amplitude $A(t)$ at time *t*, the instantaneous signal measurement is given by:

$$S(t) = A(t) \cos(2\pi f t) \tag{3.9}$$

This equation can be rewritten in terms of the phase $\phi(t)$ of the signal at time *t*:

$$S(t) = A(t) \cos (\phi(t)) \quad (3.10)$$

where:

$$\phi(t) = 2\pi f t \quad (3.11)$$

or equivalently:

$$S(t) = A(t) e^{j\Phi(t)} \quad (3.12)$$

The phase $\phi(t)$ is simply a measure of where a signal is in its oscillations. An instantaneous measure of a signal will yield $S(t)$, as described above, but what you are really interested in is $A(t)$, the amplitude. If measurements of $S(t)$ and $\phi(t)$ are made simultaneously, $A(t)$ can be extracted using Equation 3.12.

The SEA BEAM 2100 measures both signal strength $S(t)$ and phase $\phi(t)$. Both pieces of information are converted from continuous analog signals to a series of discrete digital measurements through the analog-to-digital converters. These measurements are made at the *sampling rate*, which is between 1 and 3 milliseconds. All of the signal and phase data for all hydrophones from one of these samples is called a *time slice*.

To form a steered beam, the signals from the hydrophones $S_1(t)$ through $S_N(t)$ must be combined with the correct adjustments for the path differences between different elements. Recall from Equation 3.7 that the path difference of a signal from steered beam angle $\theta$ between two elements of an array spaced $d$ apart is given by:

$$d \sin \theta \quad (3.13)$$

If the $N$ hydrophones are numbered $i = 0$ through $i = N - 1$, the path differences required to properly combine the signals of each are given by:

$$id \sin \theta \quad (3.14)$$

For computational purposes, it is easier to speak in terms of the *phase difference* between hydrophone elements rather than the path difference. The phase difference is a measure of the fractional number of wave oscillations that occur in the path difference. The phase difference in one complete oscillation or cycle is $2\pi$. The fractional difference in phase caused by the path difference $id \sin \theta$ is given by:

$$\frac{2p}{l} id \sin q \quad (3.15)$$

The in-phase signal received from an angle $\theta$ by each hydrophone is the measured signal adjusted by the appropriate phase delay. Using Equations 3.9 and 3.15, you can find the signal of hydrophone element $i$ with the appropriate phase delay required to form a steered beam at angle $\theta$, called $B_i(q)$, in terms of the instantaneous measurements $S_i$ and $\phi_i$ within a time slice:

$$B_i(\theta) = A_i e^{j(\Phi + \frac{2\pi}{\lambda} id \sin\theta)} \quad (3.16)$$

$$= A_i e^{j(\Phi_i)} e^{j(\frac{2\pi}{\lambda} id \sin\theta)} \quad (3.17)$$

$$= S_i^{j(\frac{2\pi}{\lambda} id \sin\theta)} \tag{3.18}$$

The combined steered beam at angle $\theta$ for a particular time slice is found by summing the contributions of all hydrophones, with appropriate shading coefficients $s_i$:

$$B(\theta) = \sum_{i=0}^{N-1} s_i S_i e^{j\left(\frac{2\pi}{\lambda} id \sin\theta\right)} \tag{3.19}$$

The calculations required to form $M$ steered beams from the data of $N$ hydrophones within one time slice can be represented using matrix algebra. The phase delay required for hydrophone $i$ at a beam steered to angle $\theta_m$ as $D_{mi}$ is defined as:

$$D_{mi} \equiv e^{j(\frac{2\pi}{\lambda} id \sin\theta m)} \tag{3.20}$$

Then the operation required to find a steered beam becomes:

$$B(\theta_m) = \sum_{i=0}^{N-1} s_i S_i D_{mi} \tag{3.21}$$

The logic required to form $M$ steered beams can then be expressed as a matrix multiplication:

$$\begin{bmatrix} B(\theta_1) \\ B(\theta_2) \\ . \\ . \\ B(\theta_M) \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & . & . & D_{1N} \\ D_{21} & D_{22} & & & D_{2N} \\ . & & & & . \\ . & & & & . \\ D_{M1} & D_{M2} & . & . & D_{MN} \end{bmatrix} \times \begin{bmatrix} S_1 \\ S_2 \\ . \\ . \\ S_N \end{bmatrix}$$

To create $M$ steered beams out of data from $N$ hydrophones requires on the order of $M \times N$ operations. These computations must be completed for each time slice in real time—before data appears from the next time slice. The time between time slices is only a few milliseconds—far too short for the matrix multiplication to be performed. Fortunately, some computation short cuts can be used.

Equation 3.19 is similar to a familiar equation—that of a *Fast Fourier Transform* (FFT[1]):

$$H_k = \sum_{i=0}^{N} h_i e^{j\left(\frac{2\pi ik}{N}\right)} \tag{3.22}$$

where $k$ is an integer.

This is a useful similarity if we make the substitutions:

[1] The subject of FFTs is vast and deep. Interested readers will find more information in *Discrete-Time Signal Processing* by A. V. Oppenheim and R. W. Schafer: Prentice Hall, 1989.

$$H_k \equiv B(\theta_k)$$

$$\frac{2\pi}{N} ik = \frac{2\pi}{\lambda} id \sin\theta_k \tag{3.23}$$

Equation 3.19 can be used to find $B(\theta)$ at any arbitrary θ. However, the FFT can be used only to solve for angles $\theta_k$ where the substitution in Equation 3.23 holds true. Solving this equation for $\theta_k$:

$$\theta_k = \sin^{-1}\left(\frac{\lambda}{d} \cdot \frac{k}{N}\right) \tag{3.24}$$

From Equation 3.24, you can see that the beam steering angles that emerge from the FFT processing will be limited to a discrete set of angles which will depend on $\lambda/d$ and on $N$, the number of hydrophones in the array. These angles can be found using values of $k$, which range from $-N/2$ to $N/2 - 1$. Using a hydrophone spacing $d$ of $\lambda/2$, a 48-hydrophone installation would yield angles of:

| **k** | **beam angle** |
|---|---|
| 0 | 0° |
| 1 | ± 2.4° |
| 2 | ± 4.8° |
| 3 | ± 7.2° |
| 4 | ± 9.6° |
| 5 | ± 12.0° |
| etc. | |

Recall that θ is measured such that 0.0 is perpendicular to the hydrophone array.

For certain sizes of $N$ (preferably powers of 2), special techniques for solving FFTs are available. Fortunately, $N$ is not strictly limited to the number of physical hydrophones in an array. Equation 3.22 and FFT processing work fine if the value of $h_i$ (corresponding to signal $S_i$ and shading $s_i$) is 0. The SEA BEAM 2100 adds a number of "virtual" hydrophones with $S_j$ and $s_j$ equal to 0 in order to trick the FFT into processing data with $N$ a power of 2. This is also done to increase beam density.

The FFT computations, which are performed on high-speed digital signal processors (DSPs), allow the SEA BEAM 2100 beam steering to be accomplished in real time. The FFT processing produces a time series of amplitudes for beams with axes at the beam steering angles. Echoes recorded from these time series would correspond to locations on the sea floor to which each of these angles was pointing. These data must be translated into measurements of ocean depths. The processing required to do this translation is the subject of the next chapter.

 