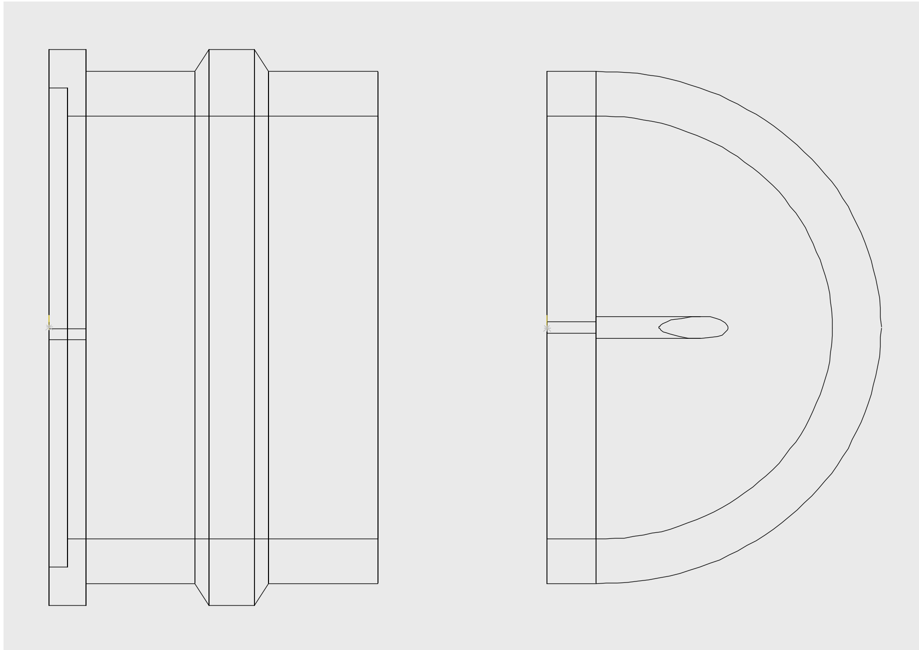
SPHERICAL ENDCAP EXTERNAL PRESSURE



PROJECT: 600045 LASER RAMAN

DATE: 11/2/00

ANALYST: Mark Brown

A) Design parameters:

1) Dimensional input:

Enter cylinder dimensions:

$ksi \equiv 1000 \cdot psi$

Inside Diameter : $ID_{cyl} \equiv 9.5 \cdot in$

Outside Diameter: $OD_{cyl} \equiv 11.5 \cdot in$

Length: $L_{cyl} \equiv 11 \cdot in$

Cylinder Base Thickness: $BaseThick_{cyl} \equiv 2.0 \cdot in$

Enter Spherical end cap dimensions:

Spherical Thckness: $t_{sphere} \equiv 1 \cdot in$

Radius : $R_1 \equiv 5.75$

Radius Mid-Plane: $R_2 \equiv 5.25 \cdot in$

2) Material properties:

Enter material properties:

Material: Aluminum alloy: 7075-T6

Modulus of elasticity: $E \equiv 10 \cdot 10^3 \cdot ksi$

Poisson's ratio: $\nu \equiv 0.33$

Material density: $\rho_m := 0.098 \cdot \frac{lbf}{in^3}$

Water density: $\rho_{sw} \equiv 64 \cdot \frac{lbf}{ft^3}$

Yield Stress: $S_y \equiv 73 \cdot ksi$

Shear stress: $S_s \equiv 48 \cdot ksi$

3) Design depth and resulting pressure:

Operating depth: $d_o \equiv 4000 \cdot m$

Margin of uncertainty $MU \equiv 1.10$

Enter design depth: $depth \equiv d_o \cdot MU$ Number of pressure steps: $N_{pressure} \equiv 10$

Pressure: $q \equiv depth \cdot \rho_{sw}$ $q = 6.416 \times 10^3 psi$

4) Factors of safety to be used:

for buckling: $FS_{bk} := 1.25$

for stress: $FS := 1.25$

A) Formulas for deformation and stresses in Spherical shells:

Change in Radius due to Pressure:

$$\Delta R := \frac{q \cdot R_2^{\,2}}{2 \cdot E \cdot t_{sphere}} \cdot (1 - \nu)$$

$$\Delta R = 5.924 \times 10^{-3} in$$

Circumferential (Hoop) Stress:

$$\sigma_{cir} := \frac{q \cdot R_2}{2 \cdot t_{sphere}}$$

$$\sigma_{cir} = 1.684 \times 10^4 psi$$

Hydrostatic Buckling Pressure:

$$P \equiv \frac{2 \cdot E \cdot t_{sphere}^{\,2}}{R_2^{\,2} \cdot \sqrt{3(1 - \nu^2)}}$$

$$P = 4.438 \times 10^5 psi$$

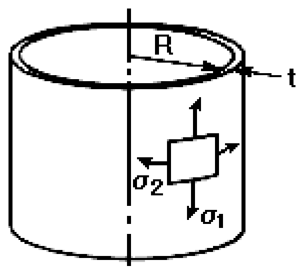
B) Formulas for deformation and stresses in cylinders:

1) Thin walled cylinder calculation:

Reference: Table 28 Formulas for membrane stresses and deformations in thin-walled pressure vessels

Case 1c **Cylindrical shell; uniform internal or external pressure (ends capped)**

Cylindrical shell



Calculated dimensions:

Wall thickness: $t := \frac{OD_{cyl} - (ID_{cyl})}{2}$ $t = 1\,in$

Mean radius: $R := \frac{ID_{cyl}}{2} + \frac{t}{2}$ $R = 5.25\,in$

Cylinder Height: $y := L_{cyl}$ $y = 11\,in$

Thin wall stresses:

Conditions: For the expressions in the case to be valid, the following condition must be true (i.e., condition = 1):

$$condition := \frac{R}{t} > \frac{R}{t} = 5.25 \qquad condition = 0$$

Meridional (axial) stress: $\sigma_1 := \frac{q \cdot R}{2 \cdot t}$ $\sigma_1 = 16.842\,ksi$

Circumferential (hoop) stress: $\sigma_2 := \frac{q \cdot R}{t}$ $\sigma_2 = 33.683\,ksi$

Maximum principal stress: $S_t := (\sigma_1 \;\; \sigma_2)$

$$\sigma b1_{max} := max(S \cdot \sigma b1_{max} = 33.683\,ksi$$

Change in height: $\Delta y := \frac{q \cdot R \cdot y}{E \cdot t} \cdot (0.5 - \nu)$ $\Delta y = 6.299 \times 10^{-3}\,in$

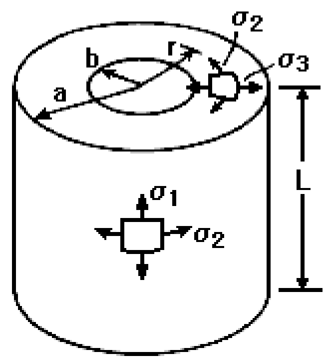
Radial displacement of circumference: $\Delta R_{max} := \frac{q \cdot R^2}{E \cdot t} \cdot \left(1 - \frac{\nu}{2}\right.$ $\Delta R_{max} = 0.015\,in$

2) Thick walled cylinder calculations:

Table 32 Formulas for thick-walled vessels under internal and external loading

Case 1d **Cylindrical disk or shell; uniform external pressure in all directions, ends capped**

Cylindrical disk or shell



Calculated dimensions:

$a := .5 \cdot OD_{cyl}$

$b := .5 \cdot ID_{cyl}$

$a = 5.75\,in$ $\quad L := L_{cyl}$

$b = 4.75\,in$ $\quad L = 11\,in$

Formulas for change in outer and inner radii, and change in length:

Outer radius: $$\Delta a := \frac{-q \cdot a}{E} \cdot \frac{a^2 \cdot (1 - 2 \cdot \nu) + b^2 \cdot (1 + \nu)}{a^2 - b^2} \qquad \Delta a = -0.014\,in$$

Inner radius: $$\Delta b := \frac{-q \cdot b}{E} \cdot \frac{a^2 \cdot (2 - \nu)}{a^2 - b^2} \qquad \Delta b = -0.016\,in$$

$$\Delta L := \frac{-q \cdot L}{E} \cdot \frac{a^2 \cdot (1 - 2 \cdot \nu)}{a^2 - b^2} \qquad \Delta L = -7.55566 \times 10^{-3}\,in$$

Formulas for normal stresses as a function of radial thickness, r:

radius range variable $$step := \frac{a - b}{10} \quad r := b, b + step \,..\, a$$

Longitudinal stress: $$\sigma_1 := \frac{-q \cdot a^2}{a^2 - b^2} \qquad \sigma_1 = -2.02 \times 10^4\,psi$$

Circumferential stress: $$\sigma_2(r) := \frac{-q \cdot a^2 \cdot \left(b^2 + r^2\right)}{r^2 \cdot \left(a^2 - b^2\right)} \qquad \sigma_2(b) = -4.04 \times 10^4\,psi$$

Radial stress: $$\sigma_3(r) := \frac{-q \cdot a^2 \cdot \left(r^2 - b^2\right)}{r^2 \cdot \left(a^2 - b^2\right)} \qquad \sigma_3(a) = -6.416 \times 10^3\,psi$$

Maximum principal stress:

$$S_t := \left(\left|\sigma_1\right| \quad \left|\sigma_2(b)\right| \quad \left|\sigma_3(a)\right| \right)$$

$$\sigma b2_{max} := \max\left(S_t\right) \qquad \sigma b2_{max} = 40.405\,ksi$$

Maximum shear stress (at inner radius, r=b):

$$\tau b2_{max} := \frac{q \cdot a^2}{a^2 - b^2} \qquad \tau b2_{max} = 20.202\,\text{ksi}$$

Von Mises Stress calculation:

$$\sigma_{vm} := \left[\frac{\left[\left(\sigma_1 - \sigma_2(b)\right)^2 + \left(\sigma_2(b) - \sigma_3(a)\right)^2 + \left(\sigma_3(a) - \sigma_1\right)^2\right]}{2}\right]^{.5} \qquad \sigma_{vm} = 29.609\,\text{ksi}$$

Calculate maximum principal stress based upon wall thickness criteria:

$$s_{max} := \begin{cases} \sigma b1_{max} & \text{if condition} \\ \sigma b2_{max} & \text{otherwise} \end{cases} \qquad s_{max} = 40.405\,\text{ksi}$$

Corresponding margins of safety:

for yield strength: $$MS_{cy} := \frac{S_y}{s_{max} \cdot FS} - 1 \qquad MS_{cy} = 0.445$$

for shear strength: $$MS_{cs} := \frac{S_s}{\tau b2_{max} \cdot FS} - 1 \qquad MS_{cs} = 0.901$$

Check stress in bottom: (Roark Table 24, Case 10:

Cylinder Base Thickness: If assumed simply supported:

$$t_b := BaseThick_{cyl} \qquad t_b = 2\,\text{in} \qquad a_b := .5 \cdot ID_{cyl} \qquad a_b = 4.75\,\text{in}$$

Constants: Shear modulus:

$$G \equiv \frac{E}{2 \cdot (1 + \nu)} \quad G = 3.759 \times 10^6\,\text{psi} \quad D_b := \frac{E \cdot t_b^3}{12 \cdot \left(1 - \nu^2\right)} \quad D_b = 7.481 \times 10^6\,\text{lbf} \cdot \text{in}$$

$$Gb_{11} := \frac{1}{64} \quad Gb_{14} := \frac{1}{16} \quad Gb_{17} := \frac{(3 + \nu)}{16}$$

$$Mb_b := q \cdot a_b^2 \cdot \frac{(3 + \nu)}{16} \qquad Mb_b = 3.013 \times 10^4\,\text{lbf} \cdot \frac{\text{in}}{\text{in}}$$

$$\sigma b_b := 6 \cdot \frac{Mb_b}{t_b^2} \qquad \sigma b_b = 45.192\,\text{ksi}$$

Margin: $$MS_{bb} := \frac{S_y}{\sigma b_b \cdot FS} - 1 \qquad MS_{bb} = 0.292$$

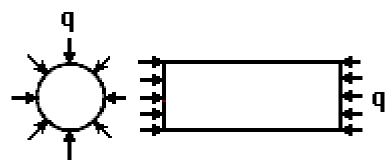
C) Buckling calculation for cylinders:

Table 35 Formulas for elastic stability of plates and shells

Case 20a

Thin tube with closed ends under uniform external pressure, lateral and longitudinal; ends held circular

Thin tube with closed ends under uniform external pressure, lateral and longitudinal



Calculated dimensions:

Tube dimensions: $r := R \qquad t = 1\,\text{in}$

$r = 5.25\,\text{in}$

(1/3 of hemispherical end cap added per R. Johnson, NRAD)

$L_b := L$

Conditions: For the expressions in the case to be valid, the following condition must be true (i.e., condition = 1):

$$\text{condition} := \frac{r}{t} > 10 \qquad \frac{r}{t} = 5.25 \qquad \text{condition} = 0$$

Critical pressure:

The critical pressure for the initial buckling of a cylinder is:

number of lobes: $n := 1 .. 10 \qquad p_0 := 100 \cdot \text{ksi}$

critical pressure:

$$p_n := \frac{E \cdot \frac{t}{r}}{1 + \frac{1}{2} \cdot \left(\frac{\pi \cdot r}{n \cdot L}\right)^2} \cdot \left[\frac{1}{n^2 \cdot \left[1 + \left(\frac{n \cdot L}{\pi \cdot r}\right)^2\right]^2} + \frac{n^2 \cdot t^2}{12 \cdot r^2 \cdot \left(1 - \nu^2\right)} \cdot \left[1 + \left(\frac{\pi \cdot r}{n \cdot L}\right)^2\right]^2 \right]$$

$p_3 = 88.281\,\text{ksi}$

$p_{min} := \min(p) \quad p_{min} = 88.281\,\text{ksi}$

$p =$

	0
0	100
1	461.687
2	97.365
3	88.281
4	127.363
5	184.171
6	254.849
7	338.703
8	435.566
9	545.39
10	668.155

ksi

$p_{max} := .8 \cdot p_{min} \qquad p_{max} = 70.625\,\text{ksi}$

Margin of safety on buckling:

$$MS_{bk} := \frac{p_{max}}{q \cdot FS_{bk}} - 1 \qquad MS_{bk} = 7.806$$

F) SUMMARY SHEET FOR PRESSURE VESSEL CALCULATIONS

Dimensional input:

Cylinder dimensions:

Inside Diameter : $ID_{cyl} = 9.5\,in$ Outside Diameter: $OD_{cyl} = 11.5\,in$

Length: $L_{cyl} = 11\,in$ Cylinder Base Thickness: $BaseThick_{cyl} = 2\,in$

Material Input:

Modulus of elasticity: $E = 1 \times 10^4\,ksi$ Poisson's ratio: $\nu = 0.33$

Shear stress: $S_s = 48\,ksi$ Material density: $\rho_m = 0.098\,\frac{lbf}{in^3}$

Yield Stress: $S_y = 73\,ksi$

Depth/working pressure:

Depth: $depth = 4.4 \times 10^3\,m$ working pressure: $q = 6.416 \times 10^3\,psi$

Stresses in cylinder:

Thin / thick wall assumption

$condition = 0$ (condition =1 for thin wall)

maximum principal stress: $s_{max} = 40.405\,ksi$

von mises stress: $\sigma_{vm} = 29.609\,ksi$

Shear stress: $\tau b2_{max} = 20.202\,ksi$ (only applicable for thick wall cylinder)

Critical buckling pressure: $p_{max} = 70.625\,ksi$

Margins of Safety:

for yield strength: $MS_{cy} = 0.445$

for shear strength: $MS_{cs} = 0.901$