## 18.5 Implementing the derivative term

In the PID controllers that we have looked at so far, we have seen that the controller acts on a reference signal error given by, $e(t) = r(t) - y_m(t)$, where $r(t)$ is the reference input and $y_m(t)$ is the measured output variable.

If we look at real control applications, we find that the feedback signal $y_f(t)$ is the sum of the measured output $y_m(t)$ and a measurement noise component $y_n(t)$, shown as a dashed line in Figure 18.14. The error signal, $e(t)$, can then be written as follows, $e(t) = r(t) - y_f(t) = r(t) - (y_m(t) + y_n(t)) = e_f(t) - y_n(t)$. What we have found from this expression is that the error signal contains a corrupting noise signal. This noise component has important implications for the use of the derivative term in PID controllers.

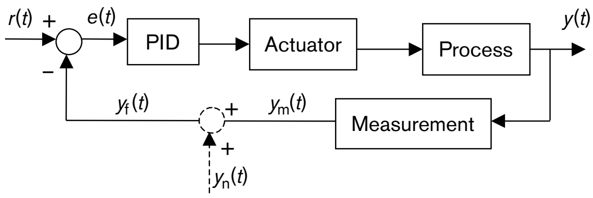


**Figure 18.14** Block diagram of PID control applied to system with measurement noise.

We consider a time-constant form of the PID controller:

$$U(s) = K_p[1 + (1/\tau_i s) + \tau_d s]\, E(s)$$

The differential term (without the gain $K_p$) is given by

$$U_d(s) = \tau_d s E(s) = G_d(s)E(s)$$

If we consider the Bode magnitude plot of $G_d(s)$ in Figure 18.15, where we have let $\tau_d = 1.0$, we find that the plot shows that the derivative term produces amplification of the

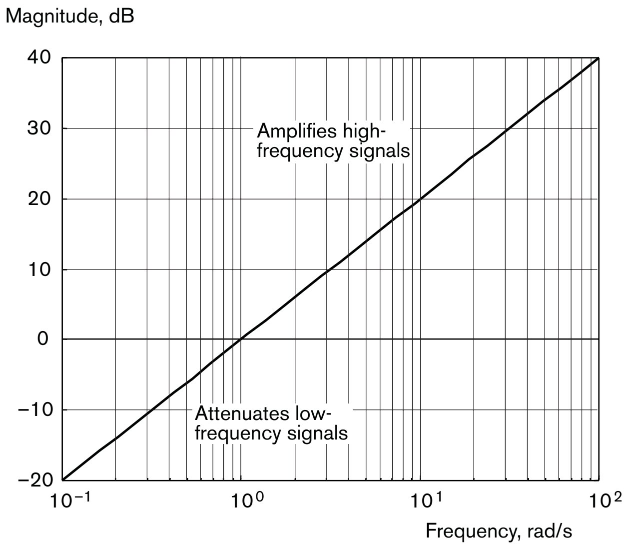


**Figure 18.15** Magnitude plot of derivative term.

high-frequency signals. These high-frequency signals often come from the measurement noise within the system. We have also learnt in previous chapters that the differential term has no effect on steady (constant) signals. We can also see this from the Bode plot. In the low-frequency range we find that the differential term has very small gain values. It is this near-zero gain which attenuates low-frequency signals.

The consequence of the possibility of measurement noise being present within the system is that we do not, in practical applications, apply the derivative directly to the measured output of the process. Instead, we introduce a *low-pass* filter in the D-term. A low-pass filter has the effect of attenuating high-frequency signals.

We incorporate a low-pass filter of the form $G_f(s) = 1/(\tau_f s + 1)$ into the derivative term $U_d(s)$ as follows:

$$U_d(s) = G_f(s)(\tau_d s)E(s) = \frac{\tau_d s}{\tau_f s+1}E(s) = G_{md}(s)E(s)$$

where $G_{md}(s)$ represents the modified derivative transfer function. We note that the high-frequency gain of $G_{md}(s)$ is given by

$$K_{high} = \lim_{\omega\to\infty} |G_{md}(j\omega)| = \frac{\tau_d}{\tau_f}$$

If we parametrise the filter time constant in terms $\tau_f$ in terms of $\tau_d$ by the formula $\tau_f = \tau_d/N$, we obtain a high-frequency gain given by $K_{high} = \tau_d/\tau_f = N$. The final modified derivative term is then given as

$$G_{md}(s) = \frac{\tau_d s}{(\tau_d / N)s+1}$$

In commercial PID controller devices, the value of $N$ is selected to be in the range $5 \le N \le 20$ depending on manufacturer. However, the important fact is that we have created a new modified D-term with limited high-frequency amplification of noisy signals.

We can see the effect of the low-pass filter when we look at the Bode magnitude plot of $G_{md}(s)$ in Figure 18.16. This plot shows the magnitude of $G_{md}(s)$ with $N$ set to 5. From our analysis, we know that the high-frequency gain is given by $N$ or, in dB, $20 \log_{10}(N) = 20 \log_{10}(5) = 13.9794 \simeq 14$ dB. We can see from the magnitude plot that the high-frequency gain is indeed 14 dB.

We conclude this section by setting the formula for the modified derivative control term in the full PID controller form to obtain:

$$U(s) = K_p\left(1+\frac{1}{\tau_i s}+\frac{\tau_d s}{(\tau_d / N)s+1}\right)E(s)$$

where $K_p$ is the proportional gain, $\tau_i$ is the integral time constant, $\tau_d$ is the derivative time constant and $N$ is a derivative filter parameter satisfying $5 \le N \le 20$. We should note that different manufacturers often use different forms for the modified D-term and even different notation for the PID controller itself. As an example, the SCADA interface shown in Figure 18.3 earlier in the chapter used the notation, G, TI, TD, TF for the terms $K_p$, $T_i$, $\tau_d$ and $\tau_f$, and the PID formula used by this manufacturer was

$$U(s) = [G_{pid}(s)]E(s) = G\left[1+\frac{1}{TIs}+\frac{TDs}{TFs+1}\right]E(s)$$

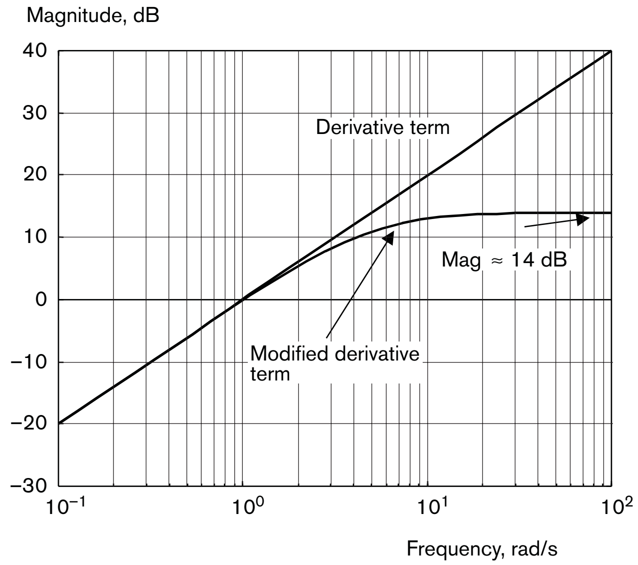


**Figure 18.16** Bode plot for the pure differential term, $G_d(s) = s$, and the modified differential term, $G_{md}(s) = s/(0.2s + 1)$.

## 18.6 Industrial PID controller structures

We have looked at how the individual PID controller terms are implemented for use in industrial applications. We have looked at the specific ways in which the integral and derivative terms are changed to overcome wind-up and noise amplification effects respectively. Again, motivated by the practical problems of industrial control, we now examine the flexibility of the structure of the PID controller itself. This eventually leads to the larger family of PID controllers that are widely available for industrial applications.

### 18.6.1 Proportional kick

We find that the structural innovations introduced to the PID controller are based on direct experience of using these controllers in industrial applications. In this section, we follow an empirical investigation using the Simulink software to show why these structural changes were made; we start with *proportional kick*. We use a simple Simulink simulation for the textbook PI control of a first-order process model. This is shown in Figure 18.17, where a unit step change occurs in the reference signal at $t = 1$.

If we look at the process output plot as shown in Figure 18.18, we find, as might be expected, a first-order-like step rise to the steady output value of unity. We know that the integral term in the PI controller will ensure no steady state offset, and that is what we find in the output plot. But now if we look at the control signal plot in Figure 18.18, we see a surprising spike in the signal. This is *proportional kick*. The term *kick* refers to the effect the control signal will have at the input to the actuator. For example, this might be

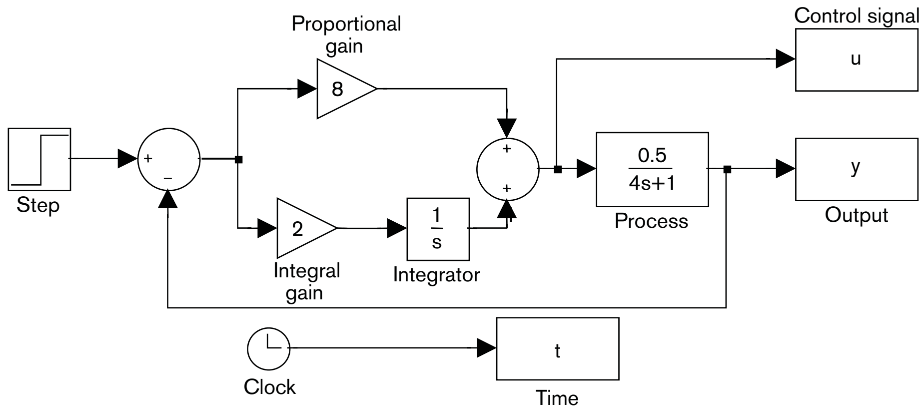


**Figure 18.17** Standard textbook PI control of a first-order process model.

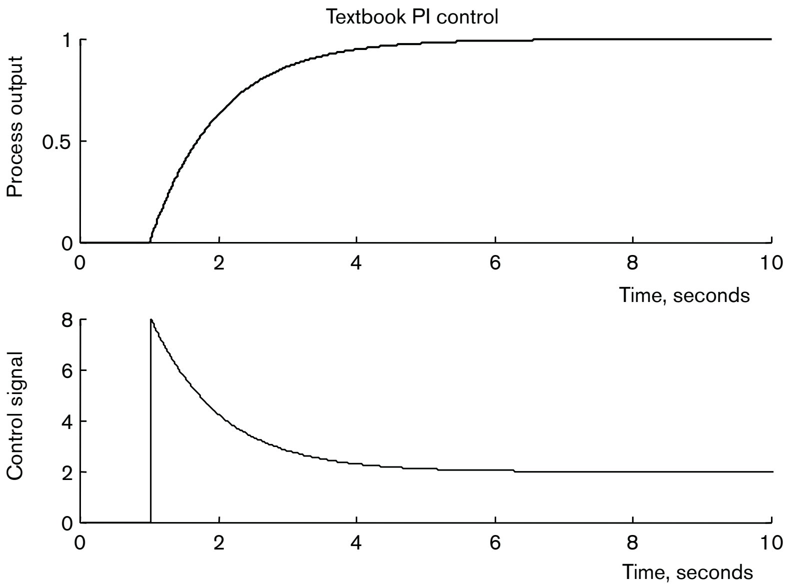


**Figure 18.18** Output and control signals in the standard PI control of a first-order process model.

a kick in the current controlling a valve actuator and, as can be imagined, the effect of the sudden increase in current could be quite damaging.

To remove the proportional kick, the remedy is to move the proportional term into the feedback path so that it does not directly operate on reference changes (Figure 18.19).

This move restructures the form of the PI controller. The formula for this modified controller is now

$$U(s) = -K_p Y(s) + \frac{K_i}{s} E(s)$$

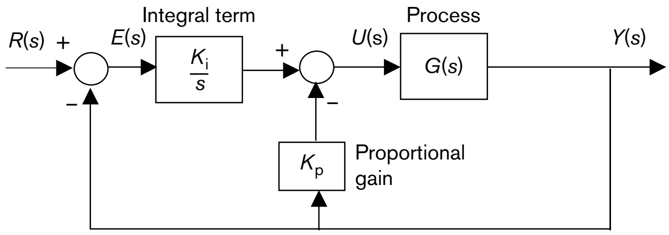


**Figure 18.19** Restructuring the PI controller.

Using the terminology of process control, the system output is often called the process variable and denoted 'PV', the reference signal is called the set point, denoted 'SP', and the error, $e(t) = r(t) - y(t)$, is termed set point error and denoted 'E'. The restructured PI controller is then referred to as 'P on process variable and I on set point error'. To see that this new structure does in fact remove the proportional kick, we use the Simulink simulation of Figure 18.20.

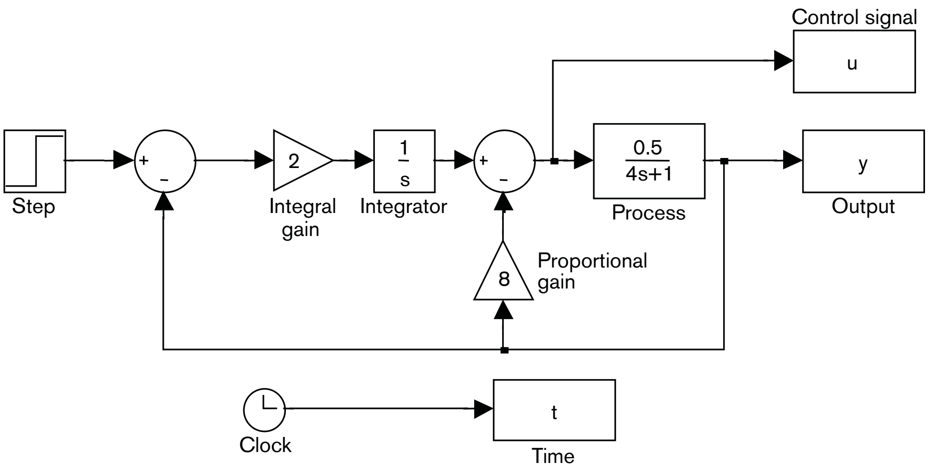


**Figure 18.20** The P on process variable, I on set point error controller used on a first-order process model.

The step response signals are shown in Figure 18.21. Comparing Figures 18.18 and 18.21, we immediately see that the spike due to proportional kick no longer appears in the control signal of the 'P on process variable and I on set point error' controller. But if we look at the output plot from the new controller, we see that the speed of response is much slower.

We can experiment with new tuning for the I-term since we want the steady state value to be reached quicker. If we increase the I-gain to 6, the output and control signals of Figure 18.22 are obtained. Here we can see that the output response has a similar performance to the original textbook PI controller, yet the control signal is much smoother and although an overshoot has appeared the signal is devoid of any *kick* effect.

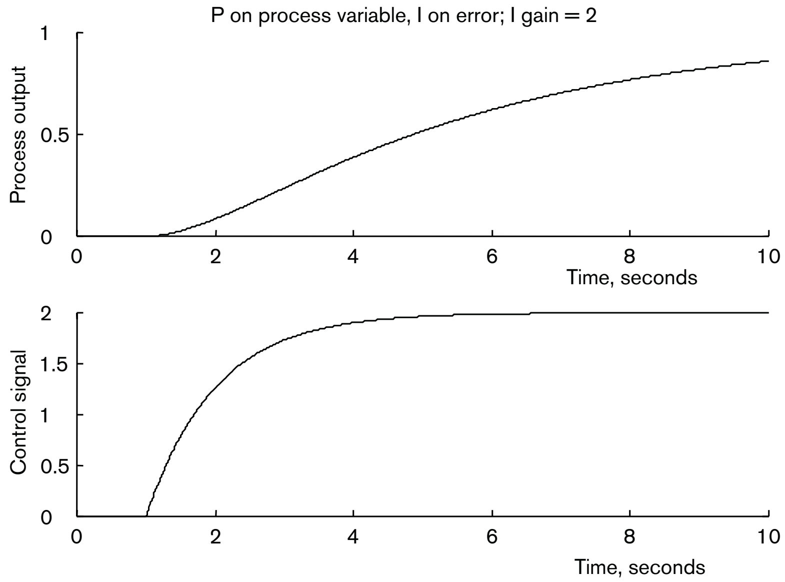


**Figure 18.21** Output and control signals in the restructured PI control of a first-order process model. Controller removes proportional kick.

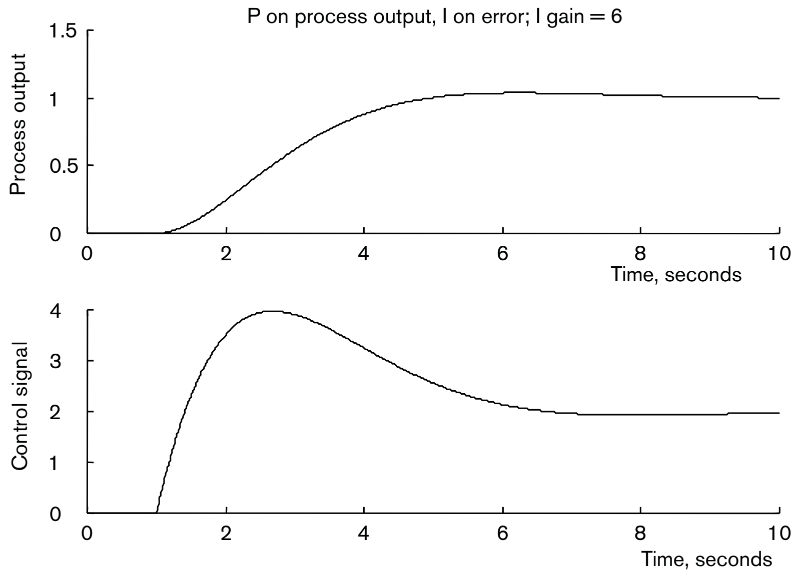


**Figure 18.22** Output and control signals from the re-tuned controller which removes proportional kick.

## 18.6.2 Derivative kick

We use derivative control to enhance the closed-loop stability of a loop and to shape the response by tuning the damping in systems with approximately second-order dynamics. We have already seen how it is necessary to modify the formula of the D-term to avoid problems with measurement noise amplification. Now we look at practical implications of the position of the D-term in the usual textbook structure of the PID controller. In Figure 18.23 we have a Simulink model of PID control of a simple first-order process where the controller is in the usual forward path position. We have used the modified D-term which incorporates noise filtering. If we write a general transfer function for the PID controller of Figure 18.23, we would have:

$$U(s) = G_{\mathrm{pid}}(s)E(s) = \left(K_{\mathrm{p}} + \frac{K_{\mathrm{i}}}{s} + \frac{K_{\mathrm{d}}s}{\tau_{\mathrm{f}}s+1}\right)E(s)$$

where $K_{\mathrm{p}} = 2.6$, $K_{\mathrm{i}} = 0.763$, $K_{\mathrm{d}} = 0.224$ and $\tau_{\mathrm{f}} = 0.112$.

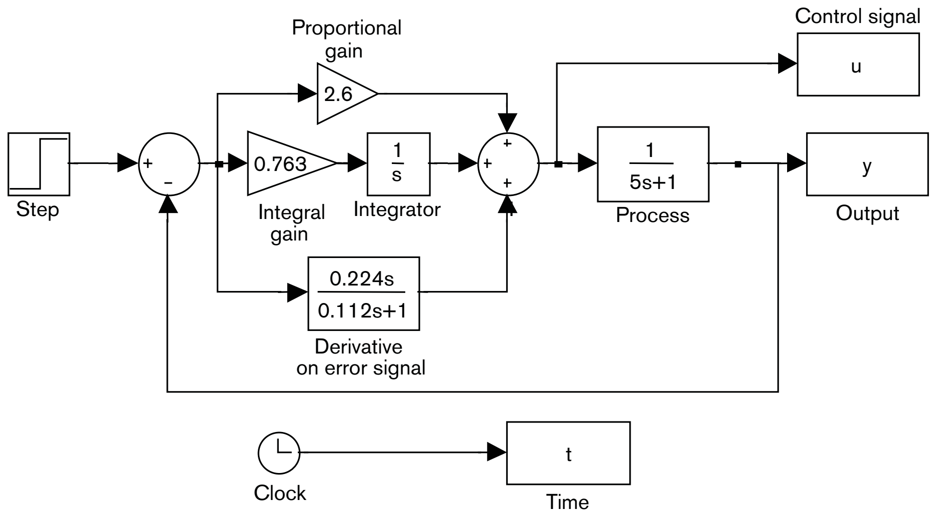


**Figure 18.23** A Simulink model of standard PID-on-error applied to a first-order process.

We run the simulation to look at the process variable output signal and the control signal. The signal plots are shown in Figure 18.24, where a step change in reference has occurred at $t = 1$.

The process output response looks satisfactory, the I-term ensures there is no steady state error and the rise time is about 4 time units. A look at the control signal in Figure 18.24 shows a remarkably different interpretation of the apparently benign output response. We see a sharp spike on the control signal at the time of the step reference change. This is a *derivative kick* effect. In practice this control signal could be driving an actuator device like a motor or a valve, and the kick would create serious problems for any electronic circuitry used in the device. Derivative kick is very similar in origin to proportional kick, explained in the previous section. The remedy for derivative kick also follows that used for proportional kick, so that we move the derivative term into the feedback path and restructure the PID controller (Figure 18.25).

Using the process control terminology, we call this new controller a PI-D controller, with 'P and I on set point error and D on process variable'. The formula for the PI-D controller is given as

$$U(s) = \left(K_{\mathrm{p}} + \frac{K_{\mathrm{i}}}{s}\right)E(s) - \frac{K_{\mathrm{d}}s}{\tau_{\mathrm{f}}s+1}Y(s)$$

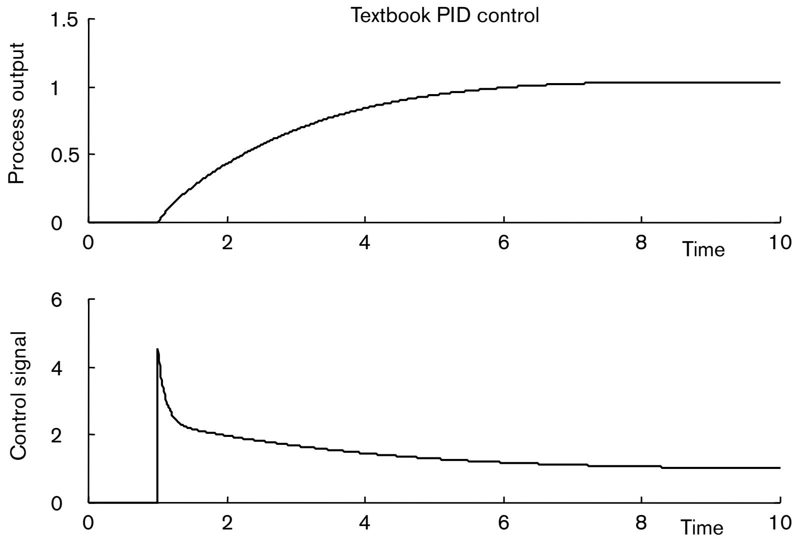


**Figure 18.24** Output and control signals in the standard PID control of a first-order process model.

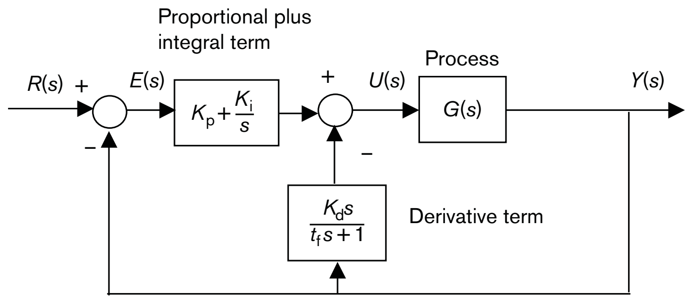


**Figure 18.25** PID structure: PI on error, D on process variable.

The modified Simulink simulation is shown in Figure 18.26, and the associated output and control signal traces are shown in Figure 18.27.

Comparing the control signals of Figures 18.24 and 18.27, we immediately see that the spike due to derivative kick is much reduced in the new PI-D controller response. In the standard PID control signal the spike reached a peak of 5 units and was very narrow and sharp, while in the PI-D control signal the peak is 2.7 units and appears as a gentle exponential decay. The difference in the two process output signals is hardly noticeable. The restructuring of the standard PID controller into the PI-D controller to avoid derivative kick is a second example of how industrial engineers have modified the PID control method to solve practical implementation problems. This structural flexibility is discussed in more detail in the next section.