PRESSURE VESSEL CALCULATIONS

PROJECT: **Precision Underwater Positioner** **DATE:** **2/12/03** **ANALYST:** **M.Brown**

Revisions: (last rev date 8/4/99)

1)Added first ten buckling modes for determination of critical buckling pressure.
2) Corrected maximum bending stress equation for cap
3) Added von Mises calculation and corrected buckling calculation
4) Corrected von Mises equation
5) Added O-ring groove diameter for diametral clearance and tab bending calculations.
6) Corrected index problem with radial moment equations

The following calculations are for determining the margin of safety for a pressure vessel at a given depth. The equations are mostly from Roark's Formulas for Stress and Strain and are referenced when used. The user must input the dimensions, material properties, and design depth. A factor of safety will be included in the calculations to determine the minimum margin of safety. The minimum margin of safety must be greater than zero for the design to be suitable for the depth desired.

The format for this analysis package will be as follows:

A) Input of design parameters including dimensions, material properties, and design depth
B) Calculations for deformation and stresses in cylinders (check for thin wall or thick wall condition)
C) Buckling calculations for cylinders
D) Calculations for deformations and stresses in end plates,
E) Weight calculation
F) Summary of input and list of results including diametral gap, stresses, margins of safety and weight.

Note: The minimum margin of safety will be determined based upon the lowest margin calculated in the sections above. Thus, for a give pressure vessel, the maximum working pressure may be limited by the end plate thickness while for another it may be limited by the length of the cylindrical section. Therefor, if a desired allowable working depth is not reached, the limiting factor needs to be determined in order to effectively iterate the design to meet the requirements. Also, the diametral gap must be checked for the different pressures to assure that it falls within the maximum allowable from the o-ring vendor.

A) Design parameters:

1) Dimensional input: **Enter cylinder dimensions:**

Inside Diameter :	Outside Diameter:	O-ring surface diameter:	Length:	Cylinder Base Thickness:	Wall thickness:
$ID_{cyl} \equiv 8.0 \cdot in$	$OD_{cyl} \equiv 9.0 \cdot in$	$ID_{cyl\_oring} \equiv 7.995 \cdot in$	$L_{cyl} \equiv 24.00 \cdot in$	$T_{cyl\_base} \equiv 1.125 \cdot in$	$T_{cyl\_wall} \equiv \frac{OD_{cyl} - (ID_{cyl})}{2}$
					$T_{cyl\_wall} = 0.5\,in$

Enter end cap and gland dimensions:

Plug Diameter:	Groove Width:	Tab Thickness:	Groove Diameter:	Groove Base Thickness:
$OD_{plug} \equiv 7.995 \cdot in$	$W_{groove} \equiv .187 \cdot in$	$T_{endcap\_tab} \equiv .500 \cdot in$	$ID_{groove} \equiv 7.767 \cdot in$	$T_{oring\_tab} \equiv .191 \cdot in$

End Cap OD:	Connector Diameter:	End Cap Thickness:
$OD_{cap} \equiv 9.0 \cdot in$	$D_{conn} \equiv .750 \cdot in$	$T_{cap} \equiv 1.125 \cdot in$

2) Material properties:

Enter material properties: Material: Titanium 6AL-4V

Modulus of elasticity:	Material density:	Poisson's ratio:	Yield Stress:	Shear stress:	Water density:
$E \equiv 16.5 \cdot 10^3 \cdot ksi$	$\rho_m \equiv .170 \cdot \frac{lbf}{in^3}$	$\nu \equiv 0.31$	$S_y \equiv 120 \cdot ksi$	$S_s \equiv 100 \cdot ksi$	$\rho_{sw} \equiv 64 \cdot \frac{lbf}{ft^3}$

3) Design depth and resulting pressure:

$depth \equiv 4000 \cdot m$ Number of pressure steps: $N_{pressure} \equiv 10$ Pressure: $q \equiv depth \cdot \rho_{sw}$ $q = 5.833 \times 10^3 psi$

4) Factors of safety to be used:

for buckling: $FS_{buckling} \equiv 2$ for stress: $FS \equiv 1.5$ $FS = 1.5$

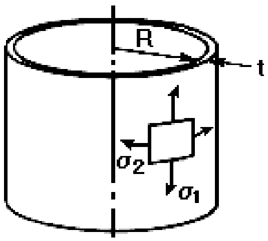
B) Formulas for deformation and stresses in cylinders:

1) Thin walled cylinder calculation:

Reference: Table 28 Formulas for membrane stresses and deformations in thin-walled pressure vessels

Case 1c **Cylindrical shell; uniform internal or external pressure (ends capped)**

Cylindrical shell



Calculated dimensions:

Mean radius: $R_{thin} \equiv \left(\frac{ID_{cyl}}{2} + \frac{T_{cyl\_wall}}{2} \right)$

$R_{thin} = 4.25\,in$

Cylinder Height: $y \equiv L_{cyl}$

$y = 24\,in$

Thin wall stresses:

Conditions:

For the expressions in the case to be valid, the following condition must be true (i.e., condition = 1):

$thinWall \equiv \left(\frac{R_{thin}}{T_{cyl\_wall}} > 10 \right) \quad \frac{R_{thin}}{T_{cyl\_wall}} = 8.5$

$thinWall = 0$

Meridional (axial) stress:

$\sigma_{1thin} \equiv \frac{q \cdot R_{thin}}{2 \cdot T_{cyl\_wall}} \quad \sigma_{1thin} = 24.789\,ksi$

Circumferential (hoop) stress:

$\sigma_{2thin} \equiv \frac{q \cdot R_{thin}}{T_{cyl\_wall}} \quad \sigma_{2thin} = 49.577\,ksi$

Von Mises Stress calculation: $\sigma_{vmthin} \equiv \sqrt{\sigma_{1thin}^{2} - \sigma_{1thin} \cdot \sigma_{2thin} + \sigma_{2thin}^{2}}$

$\sigma_{vmthin} = 42.935\,ksi$

Corresponding margins of safety:

for yield strength: $MS_{cythin} \equiv \left(\frac{S_y}{\sigma_{vmthin} \cdot FS} - 1 \right) \quad MS_{cythin} = 0.863$

Change in height $\Delta y_{thin} \equiv \frac{q \cdot R_{thin} \cdot y}{E \cdot T_{cyl\_wall}} \cdot (0.5 - \nu)$

$\Delta y_{thin} = 0.014\,in$

Radial displacement of circumference: $\Delta R_{max} \equiv \frac{q \cdot R_{thin}^{2}}{E \cdot T_{cyl\_wall}} \cdot \left(1 - \frac{\nu}{2}\right)$

$\Delta R_{max} = 0.011\,in$

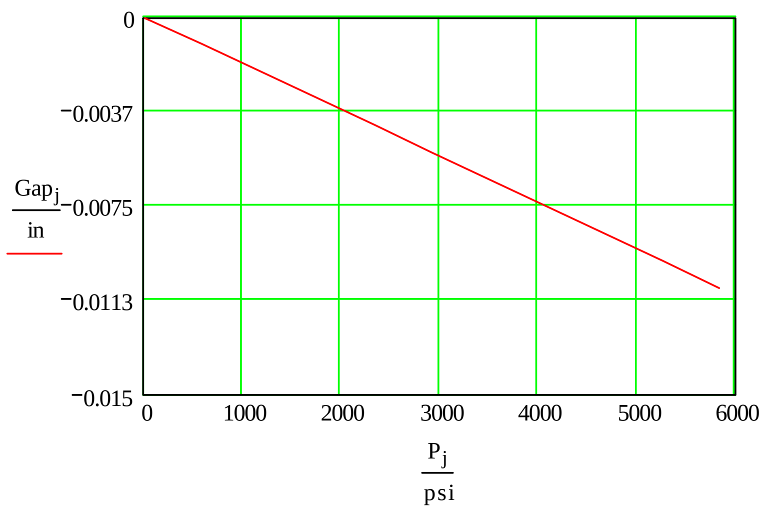
Diametral clearance (extrusion gap):

Extrusion gap (radial):

Pressure step: $P_{step} := \frac{q}{N_{pressure}}$ $P_{step} = 583.26\,psi$ $j := 0 .. 10$

$P_j := j \cdot P_{step}$ $\Delta R_j := \left[\frac{P_j \cdot R_{thin}^{2}}{E \cdot T_{cyl\_wall}} \cdot \left(1 - \frac{\nu}{2}\right)\right]$

$Gap_j := \frac{ID_{cyl\_oring} - OD_{plug}}{2} - \Delta R_j$



$\frac{Gap_j}{in} =$

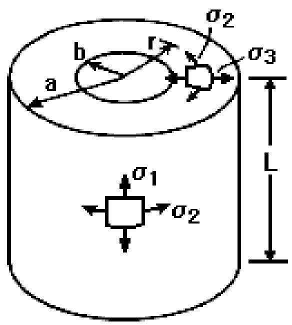
0
$-1.079 \cdot 10^{-3}$
$-2.158 \cdot 10^{-3}$
$-3.237 \cdot 10^{-3}$
$-4.316 \cdot 10^{-3}$
$-5.395 \cdot 10^{-3}$
$-6.474 \cdot 10^{-3}$
$-7.553 \cdot 10^{-3}$
$-8.632 \cdot 10^{-3}$
$-9.711 \cdot 10^{-3}$
-0.011

2) Thick walled cylinder calculations:

Table 32 Formulas for thick-walled vessels under internal and external loading

Case 1d **Cylindrical disk or shell; uniform external pressure in all directions, ends capped**

Cylindrical disk or shell



Calculated dimensions:

$a_{thick} \equiv 0.5 \cdot OD_{cyl}$ $b_{thick} \equiv 0.5 \cdot ID_{cyl}$ $L \equiv L_{cyl}$

$a_{thick} = 4.5\,in$ $b_{thick} = 4\,in$ $L = 24\,in$

Formulas for change in outer and inner radii, and change in length:

Outer radius:

$$\Delta a_{thick} \equiv \frac{-q \cdot a_{thick}}{E} \cdot \frac{a_{thick}^2 \cdot (1 - 2\cdot\nu) + b_{thick}^2 \cdot (1 + \nu)}{a_{thick}^2 - b_{thick}^2}$$

$\Delta a_{thick} = -0.011\,in$

Inner radius:

$$\Delta b_{thick} \equiv \frac{-q \cdot b_{thick}}{E} \cdot \frac{a_{thick}^2 \cdot (2 - \nu)}{a_{thick}^2 - b_{thick}^2}$$

$\Delta b_{thick} = -0.011\,in$

$$\Delta L \equiv \frac{-q \cdot L}{E} \cdot \frac{a_{thick}^2 \cdot (1 - 2\cdot\nu)}{a_{thick}^2 - b_{thick}^2}$$

$\Delta L = -0.01536\,in$

Formulas for normal stresses as a function of radial thickness, r:

radius range variable $step := \frac{a_{thick} - b_{thick}}{10}$ $r := b_{thick}, b_{thick} + step \,..\, a_{thick}$

Longitudinal stress:

$$\sigma_{1thick} \equiv \frac{-q \cdot a_{thick}^2}{a_{thick}^2 - b_{thick}^2}$$

$\sigma_{1thick} = -2.779 \times 10^4\,psi$

Circumferential stress:

$$\sigma_{2thick}(r) \equiv \frac{-q \cdot a_{thick}^2 \cdot \left(b_{thick}^2 + r^2\right)}{r^2 \cdot \left(a_{thick}^2 - b_{thick}^2\right)}$$

$\sigma_{2thick}(b_{thick}) = -5.558 \times 10^4\,psi$

Radial stress:

$$\sigma_{3thick}(r) \equiv \frac{-q \cdot a_{thick}^2 \cdot \left(r^2 - b_{thick}^2\right)}{r^2 \cdot \left(a_{thick}^2 - b_{thick}^2\right)}$$

$\sigma_{3thick}(b_{thick}) = 0\,psi$

Shear stress:

$$\tau_{thick} \equiv \frac{q \cdot a_{thick}^2}{a_{thick}^2 - b_{thick}^2}$$

$\tau_{thick} = 2.779 \times 10^4\,psi$

Von Mises Stress calculation:

$$\sigma a_{vmthick} \equiv \sqrt{\frac{\left(\sigma_{1thick} - \sigma_{2thick}\left(a_{thick}\right)\right)^2 + \left(\sigma_{2thick}\left(a_{thick}\right) - \sigma_{3thick}\left(a_{thick}\right)\right)^2 + \left(\sigma_{3thick}\left(a_{thick}\right) - \sigma_{1thick}\right)^2}{2}} \qquad \sigma a_{vmthick} = 38.032\,\text{ksi}$$

$$\sigma b_{vmthick} \equiv \sqrt{\frac{\left(\sigma_{1thick} - \sigma_{2thick}\left(b_{thick}\right)\right)^2 + \left(\sigma_{2thick}\left(b_{thick}\right) - \sigma_{3thick}\left(b_{thick}\right)\right)^2 + \left(\sigma_{3thick}\left(b_{thick}\right) - \sigma_{1thick}\right)^2}{2}} \qquad \sigma b_{vmthick} = 48.135\,\text{ksi}$$

$$\sigma_{vmthick} \equiv \max\left(\sigma a_{vmthick}, \sigma b_{vmthick}\right) \qquad \sigma_{vmthick} = 4.813 \times 10^4\,\text{psi}$$

Corresponding margins of safety:

for yield strength: $MSyield_{cythick} \equiv \left(\frac{S_y}{\sigma_{vmthick} \cdot FS} - 1\right)$ for shear strength: $MSshear_{cythick} \equiv \left(\frac{0.5 \cdot S_s}{\tau_{thick} \cdot FS} - 1\right)$

$MSyield_{cythick} = 0.662$

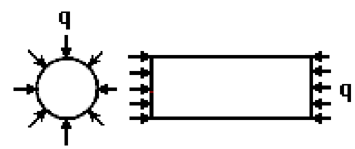
$MSshear_{cythick} = 0.199$

C) Buckling calculation for cylinders:

Table 35 Formulas for elastic stability of plates and shells

Case 20a Thin tube with closed ends under uniform external pressure, lateral and longitudinal; ends held circular

Thin tube with closed ends under uniform external pressure, lateral and longitudinal



Calculated dimensions:

Tube dimensions: radius: $r_{buckling} \equiv \left(\frac{ID_{cyl}}{2} + \frac{T_{cyl\_wall}}{2}\right)$ length: $L = 24\,in$

$r_{buckling} = 4.25\,in$

Conditions: For the expressions in the case to be valid, the following condition must be true (i.e., condition = 1):

$condition := \frac{r_{buckling}}{T_{cyl\_wall}} > 10$ $condition = 0$

Critical pressure:

The critical pressure for the initial buckling of a cylinder is: number of lobes: $n \equiv 2..10$

critical pressure:

$$p_n \equiv \frac{E\cdot\frac{T_{cyl\_wall}}{r_{buckling}}}{1+\frac{1}{2}\cdot\left(\frac{\pi\cdot r_{buckling}}{n\cdot L}\right)^2}\cdot\left[\frac{1}{n^2\cdot\left[1+\left(\frac{n\cdot L}{\pi\cdot r_{buckling}}\right)^2\right]^2}+\frac{n^2\cdot T_{cyl\_wall}{}^2}{12\cdot r_{buckling}{}^2\cdot\left(1-\nu^2\right)}\cdot\left[1+\left(\frac{\pi\cdot r_{buckling}}{n\cdot L}\right)^2\right]^2\right]$$

$p =$

	0
0	0
1	0
2	$1.348\cdot10^4$
3	$2.368\cdot10^4$
4	$4.083\cdot10^4$
5	$6.309\cdot10^4$
6	$9.033\cdot10^4$
7	$1.225\cdot10^5$
8	$1.597\cdot10^5$
9	$2.018\cdot10^5$
10	$2.489\cdot10^5$

psi

$p_{max} \equiv \max(p)$ $p_0 \equiv p_{max}\cdot 10$ $p_1 \equiv p_{max}\cdot 10$

$p_{critical} \equiv 0.8\cdot\min(p)$ $p_{critical} = 1.079\times10^4\,psi$

$q = 5.833\times10^3\,psi$

Margin of safety on buckling:

$MS_{buckling} \equiv \left(\frac{p_{critical}}{q\cdot FS_{buckling}} - 1\right)$ $MS_{buckling} = -0.075$

Case 10a Solid circular plate simply supported; uniformly distributed pressure from r_o to a

Solid circular plate

Uniformly distributed pressure from r_o to a

(Roark Table 24, Case 10a:)

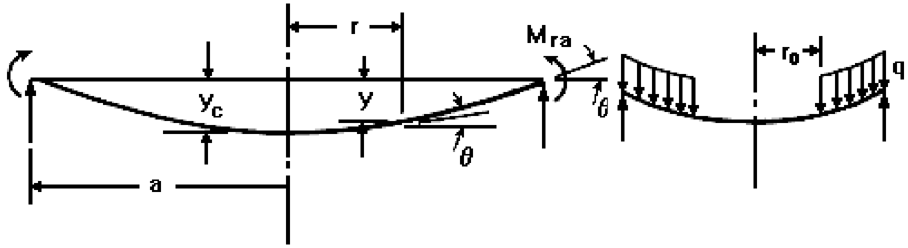


PLATE CALCULATIONS: Check stress in bottom:

Cylinder Base Thickness:

$t_{base} \equiv T_{cyl\_base}$

$a_{base} \equiv 0.5 \cdot ID_{cyl}$

If assumed simply supported:

$t_{base} = 0.029\,m$

$a_{base} = 4\,in$

$r_{base0} \equiv 0$

Plate Constant: $D_{base} \equiv \dfrac{E \cdot t_{base}^{3}}{12 \cdot \left(1 - \nu^{2}\right)}$ $D_{base} = 2.166 \times 10^{6}\,lbf \cdot in$ $G_{17} \equiv \dfrac{(3 + \nu)}{16}$

$M_{cbase} \equiv \dfrac{q \cdot a_{base}^{2} \cdot (3 + \nu)}{16}$ $M_{cbase} = 1.931 \times 10^{4}\,lbf \cdot \dfrac{in}{in}$ $\sigma_{base} \equiv 6 \cdot \dfrac{M_{cbase}}{t_{base}^{2}}$ $\sigma_{base} = 91.524\,ksi$

$Q_{base} \equiv \dfrac{-q \cdot a_{base}}{2}$ $Q_{base} = -1.167 \times 10^{4}\,\dfrac{lbf}{in}$

$ycenter_{base} \equiv \dfrac{-q \cdot a_{base}^{4}}{64 \cdot D_{base}} \cdot \dfrac{(5 + \nu)}{(1 + \nu)}$ $ycenter_{base} = -0.044\,in$

Margin: $MS_{base} \equiv \left(\dfrac{S_y}{\sigma_{base} \cdot FS} - 1\right)$ $MS_{base} = -0.126$

check small deflection assumption (deflection < t/2) $\dfrac{ycenter_{base}}{t_{base}} = -0.039$

$S_y = 1.2 \times 10^{5}\,psi$

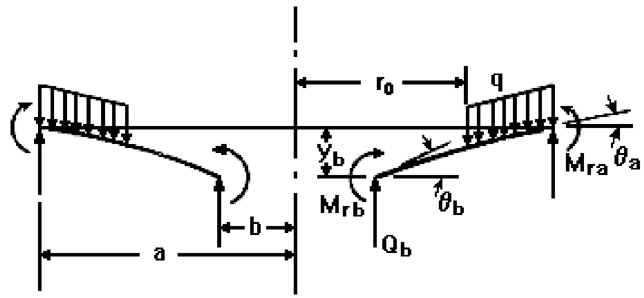
Table 24 Formulas for shear, moment and deflection of flat circular plates of constant thickness

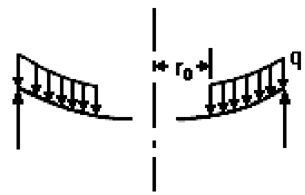
This file corresponds to Cases 2a - 2d in *Roark's Formulas for Stress and Strain.*

Annular plate with uniformly distributed pressure q over the portion from r_o to a; outer edge simply supported

Annular plate with a uniformly distributed pressure q over the portion from r_o to a

Outer edge simply supported, inner edge free





$$D_{endcap} \equiv \frac{E \cdot T_{cap}^{3}}{12 \cdot \left(1 - \nu^{2}\right)}$$

Plate dimensions:

thickness: $t_{endcap} \equiv T_{cap}$ outer radius: $a_{endcap} \equiv \frac{OD_{cap}}{2}$ inner radius: $b_{endcap} \equiv \frac{D_{conn}}{2}$

Shear modulus: $G \equiv \frac{E}{2 \cdot (1 + \nu)}$ $G = 6.298 \times 10^{6} \text{psi}$

$b_{endcap} = 0.375 \, \text{in}$

$M_{rbendcap} \equiv 0 \cdot \text{in} \cdot \text{lbf}$ $Q_{bendcap} \equiv 0 \cdot \frac{\text{lbf}}{\text{in}}$ $y_{aendcap} \equiv 0 \cdot \text{in}$ $M_{raendcap} \equiv 0 \cdot \text{in} \cdot \text{lbf}$ $r_{0endcap} \equiv b_{endcap}$

$$C_{endcap4} \equiv \frac{1}{2}\cdot\left[(1+\nu)\cdot\frac{b_{endcap}}{a_{endcap}} + (1-\nu)\cdot\frac{a_{endcap}}{b_{endcap}}\right]$$

$$C_{endcap1} \equiv \frac{1+\nu}{2}\cdot\frac{b_{endcap}}{a_{endcap}}\cdot\ln\left(\frac{a_{endcap}}{b_{endcap}}\right) + \frac{1-\nu}{4}\cdot\left(\frac{a_{endcap}}{b_{endcap}} - \frac{b_{endcap}}{a_{endcap}}\right)$$

$$C_{endcap7} \equiv \frac{1}{2}\cdot\left(1-\nu^2\right)\cdot\left(\frac{a_{endcap}}{b_{endcap}} - \frac{b_{endcap}}{a_{endcap}}\right)$$

$$L_{endcap14} \equiv \frac{1}{16}\cdot\left[1 - \left(\frac{r_{0endcap}}{a_{endcap}}\right)^4 - 4\cdot\left(\frac{r_{0endcap}}{a_{endcap}}\right)^2\cdot\ln\left(\frac{a_{endcap}}{r_{0endcap}}\right)\right]$$

$$L_{endcap11} \equiv \frac{1}{64}\cdot\left[1 + 4\cdot\left(\frac{r_{0endcap}}{a_{endcap}}\right)^2 - 5\cdot\left(\frac{r_{0endcap}}{a_{endcap}}\right)^4 - 4\cdot\left(\frac{r_{0endcap}}{a_{endcap}}\right)^2\cdot\left[2 + \left(\frac{r_{0endcap}}{a_{endcap}}\right)^2\right]\cdot\ln\left(\frac{a_{endcap}}{r_{0endcap}}\right)\right]$$

$$L_{endcap17} \equiv \frac{1}{4}\cdot\left[1 - \left(\frac{1-\nu}{4}\right)\cdot\left[1 - \left(\frac{r_{0endcap}}{a_{endcap}}\right)^4\right] - \left(\frac{r_{0endcap}}{a_{endcap}}\right)^2\cdot\left[1 + (1+\nu)\cdot\ln\left(\frac{a_{endcap}}{r_{0endcap}}\right)\right]\right]$$

$$y_{endcapb} \equiv \frac{-q\cdot a_{endcap}^{\,4}}{D_{endcap}}\cdot\left(\frac{C_{endcap1}\cdot L_{endcap17}}{C_{endcap7}} - L_{endcap11}\right) \qquad y_{endcapb} = -0.074\,\text{in}$$

$$Q_a \equiv \frac{-q}{2\cdot a_{endcap}}\cdot\left(a_{endcap}^{\,2} - r_{0endcap}^{\,2}\right)$$

$$Q_a = -1.303\times 10^4\,\frac{\text{lbf}}{\text{in}}$$

$$\theta_{endcapb} \equiv \frac{q\cdot a_{endcap}^{\,3}}{D_{endcap}\cdot C_{endcap7}}\cdot L_{endcap17} \qquad \theta_{endcapb} = 0.521\,\text{deg}$$

$$\theta_{endcapa} \equiv \frac{q\cdot a_{endcap}^{\,3}}{D_{endcap}}\cdot\left(\frac{C_{endcap4}\cdot L_{endcap17}}{C_{endcap7}} - L_{endcap14}\right) \qquad \theta_{endcapa} = 1.366\,\text{deg}$$

$$M_{endcapt} \equiv \frac{\theta_{endcapb}\cdot D_{endcap}\cdot\left(1-\nu^2\right)}{b_{endcap}} \qquad M_{endcapt} = 4.745\times 10^4\,\text{lbf}$$

$$\sigma_{endcap} \equiv \frac{6\cdot M_{endcapt}}{t_{endcap}^{\,2}} \qquad \sigma_{endcap} = 2.25\times 10^5\,\text{psi}$$

check small deflection assumption
(deflection < t/2)

$$\frac{y_{endcapb}}{t_{endcap}} = -0.066$$

Margin: $MS_{endcap} \equiv \left(\frac{S_y}{\sigma_{endcap}\cdot FS} - 1\right)$ $\qquad MS_{endcap} = -0.644$

Stresses in tabs:

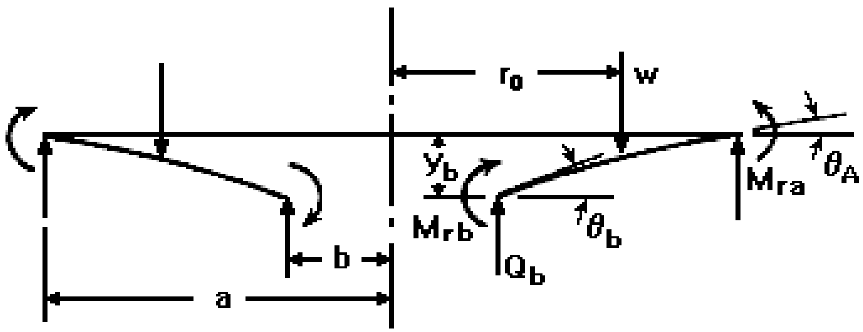
For a conservative approach, assume the outer edge of the tabs are free and the inner edge fixed

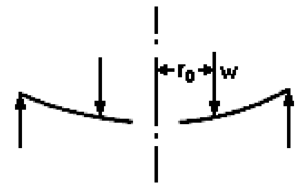
A) Top tab:

Table 24 Formulas for shear, moment and deflection of flat circular plates of constant thickness

Annular plate with uniform annular line load w at radius r_o; outer edge free

Case 1a Outer edge simply supported, inner edge free





Loading on tab: $W_t \equiv q \cdot \pi \cdot \frac{ID_{cyl\_oring}^2}{4}$ $W_t = 2.928 \times 10^5 \, lbf$

Applied unit load: $w_t \equiv \frac{W_t}{\pi \cdot ID_{cyl}}$ $w_t = 1.165 \times 10^4 \frac{lbf}{in}$

$D_t \equiv \frac{E \cdot t_t^3}{12 \cdot (1 - \nu^2)}$ $D_t = 1.901 \times 10^5 \, lbf \cdot in$

Plate dimensions:

thickness: $t_t \equiv T_{endcap\_tab}$ $t_t = 0.5 \, in$

outer radius: $a_t \equiv \frac{OD_{cap}}{2}$ $a_t = 4.5 \, in$

inner radius: $b_t \equiv \frac{OD_{plug}}{2}$ $b_t = 3.998 \, in$

Radial location of applied load: $r_{t0} \equiv b_t$

$M_{trb} \equiv 0 \cdot lbf$ $Q_{tb} \equiv 0 \cdot \frac{lbf}{in}$

$$C_{t1} \equiv \frac{1 + \nu}{2} \cdot \frac{b_t}{a_t} \cdot \ln\left(\frac{a_t}{b_t}\right) + \frac{1 - \nu}{4} \cdot \left(\frac{a_t}{b_t} - \frac{b_t}{a_t}\right)$$

$y_{ta} \equiv 0 \cdot in$ $M_{tra} \equiv 0 \cdot lbf \cdot in$

$$C_{t7} \equiv \frac{1}{2} \cdot (1 - \nu^2) \cdot \left(\frac{a_t}{b_t} - \frac{b_t}{a_t}\right)$$

$$C_{t4} \equiv \frac{1}{2}\cdot\left[(1+\nu)\cdot\frac{b_t}{a_t} + (1-\nu)\cdot\frac{a_t}{b_t}\right]$$

$$L_{t3} \equiv \frac{r_{t0}}{4\cdot a_t}\cdot\left[\left[\left(\frac{r_{t0}}{a_t}\right)^2 + 1\right]\cdot\ln\left(\frac{a_t}{r_{t0}}\right) + \left(\frac{r_{t0}}{a_t}\right)^2 - 1\right] \qquad L_{t6} \equiv \frac{r_{t0}}{4\cdot a_t}\cdot\left[\left(\frac{r_{t0}}{a_t}\right)^2 - 1 + 2\cdot\ln\left(\frac{a_t}{r_{t0}}\right)\right] \quad L_{t9} \equiv \frac{r_{t0}}{a_t}\cdot\left[\frac{1+\nu}{2}\cdot\ln\left(\frac{a_t}{r_{t0}}\right) + \frac{1-\nu}{4}\cdot\left[1-\left(\frac{r_{t0}}{a_t}\right)^2\right]\right]$$

$$L_{t6} \equiv \frac{1}{4}\cdot\frac{r_{t0}}{a_t}\cdot\left(\frac{r_{t0}^2}{a_t^2} - 1 + 2\cdot\ln\left(\frac{a_t}{r_{t0}}\right)\right)$$

$$y_{bt} \equiv \frac{-w_t\cdot a_t^3}{D_t}\cdot\left(\frac{C_{t1}\cdot L_{t9}}{C_{t7}} - L_{t3}\right) \quad Q_{at} \equiv -w_t\cdot\frac{r_{t0}}{a_t} \qquad \theta_{bt} \equiv \frac{w_t\cdot a_t^2}{D_t\cdot C_{t7}}\cdot L_{t9} \qquad \theta_{at} \equiv \frac{w_t\cdot a_t^2}{D_t}\cdot\left(\frac{C_{t4}\cdot L_{t9}}{C_{t7}} - L_{t6}\right)$$

$y_{bt} = -0.577\,\text{in}$ $\qquad Q_{at} = -1.035\times10^4\,\frac{\text{lbf}}{\text{in}}$ $\qquad \theta_{bt} = 67.067\,\text{deg}$ $\qquad \theta_{at} = 64.66\,\text{deg}$

$$M_t \equiv \frac{\theta_{bt}\cdot D_t\cdot\left(1-\nu^2\right)}{r_{t0}} + \nu\cdot M_{trb} \qquad \sigma_t \equiv \frac{6\cdot M_t}{t_t^2}$$

$\sigma_t = 1.208\times10^6\,\text{psi}$

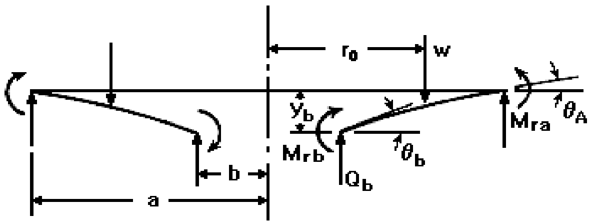
check small deflection assumption
(deflection < t/2) $\qquad \frac{y_{bt}}{t_t} = -1.155$

Margin: $MS_t \equiv \left(\frac{S_y}{\sigma_t\cdot FS} - 1\right)$ $\qquad MS_t = -0.934$

B) O-ring groove tab:

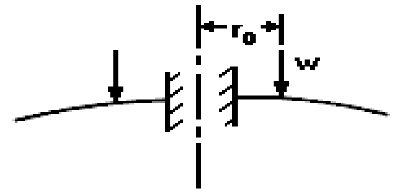
Table 24 Formulas for shear, moment and deflection of flat circular plates of constant thickness

Annular plate with uniform annular line load w at radius r_o; outer edge free



Case 1l:

Outer edge free, inner edge fixed



For o-ring tab: $t_{oring} \equiv T_{oring\_tab}$

$$D_{oring} \equiv \frac{E \cdot t_{oring}^{3}}{12 \cdot (1-\nu^2)} \qquad D_{oring} = 1.06 \times 10^{4}\,\mathrm{lbf \cdot in}$$

Loading on tab:

$$W_{oring} \equiv q \cdot \pi \cdot \frac{\left(OD_{plug}^{2} - ID_{groove}^{2}\right)}{4} \qquad W_{oring} = 1.646 \times 10^{4}\,\mathrm{lbf} \qquad w_{oring} \equiv \frac{W_{oring}}{\pi \cdot ID_{groove}}$$

$b_{oring} \equiv ID_{groove}$ $\qquad t_{oring} \equiv T_{oring\_tab}$

$a_{oring} \equiv OD_{plug}$

$r_{oring0} \equiv a_{oring}$

$y_{oringb} \equiv 0 \quad \theta_{oringb} \equiv 0 \qquad M_{oringra} \equiv 0 \qquad Q_{oringa} \equiv 0$

$$C_{oring2} \equiv \frac{1}{4} \cdot \left[1 - \left(\frac{b_{oring}}{a_{oring}}\right)^2 \cdot \left(1 + 2 \cdot \ln\left(\frac{a_{oring}}{b_{oring}}\right)\right)\right]$$

$$C_{oring8} \equiv \frac{1}{2} \cdot \left[1 + \nu + (1-\nu) \cdot \left(\frac{b_{oring}}{a_{oring}}\right)^2\right]$$

$$C_{oring3} \equiv \frac{b_{oring}}{4 \cdot a_{oring}} \cdot \left[\left[\left(\frac{b_{oring}}{a_{oring}}\right)^2 + 1\right] \cdot \ln\left(\frac{a_{oring}}{b_{oring}}\right) + \left(\frac{b_{oring}}{a_{oring}}\right)^2 - 1\right]$$

$$C_{oring9} \equiv \frac{b_{oring}}{a_{oring}} \cdot \left[\frac{1+\nu}{2} \cdot \ln\left(\frac{a_{oring}}{b_{oring}}\right) + \left(\frac{1-\nu}{4}\right) \cdot \left[1 - \left(\frac{b_{oring}}{a_{oring}}\right)^2\right]\right]$$

$$L_3 \equiv \frac{r_{oring0}}{4 \cdot a_{oring}} \cdot \left[\left[\left(\frac{r_{oring0}}{a_{oring}}\right)^2 + 1\right] \cdot \ln\left(\frac{a_{oring}}{r_{oring0}}\right) + \left(\frac{r_{oring0}}{a_{oring}}\right)^2 - 1\right]$$

$$L_{oring6} \equiv \frac{r_{oring0}}{4 \cdot a_{oring}} \cdot \left[\left(\frac{r_{oring0}}{a_{oring}}\right)^2 - 1 + 2 \cdot \ln\left(\frac{a_{oring}}{r_{oring0}}\right)\right]$$

$$L_{oring9} \equiv \frac{r_{oring0}}{a_{oring}} \cdot \left[\frac{1+\nu}{2} \cdot \ln\left(\frac{a_{oring}}{r_{oring0}} \right) + \frac{1-\nu}{4} \cdot \left[1 - \left(\frac{r_{oring0}}{a_{oring}} \right)^2 \right] \right]$$

$$M_{oringrb} \equiv \frac{-w_{oring} \cdot a_{oring}}{C_{oring8}} \left(\frac{r_{oring0} \cdot C_{oring9}}{b_{oring}} - L_{oring9} \right)$$

$$M_{oringrb} = -157.591\,\mathrm{lbf}$$

$$y_{oringmax} \equiv \frac{-w_{oring} \cdot a_{oring}^{4}}{b_{oring} \cdot D_{oring}} \cdot \left(\frac{C_{oring2} \cdot C_{oring9}}{C_{oring8}} - C_{oring3} \right)$$

$$y_{oringmax} = -2.552 \times 10^{-4}\,\mathrm{in}$$

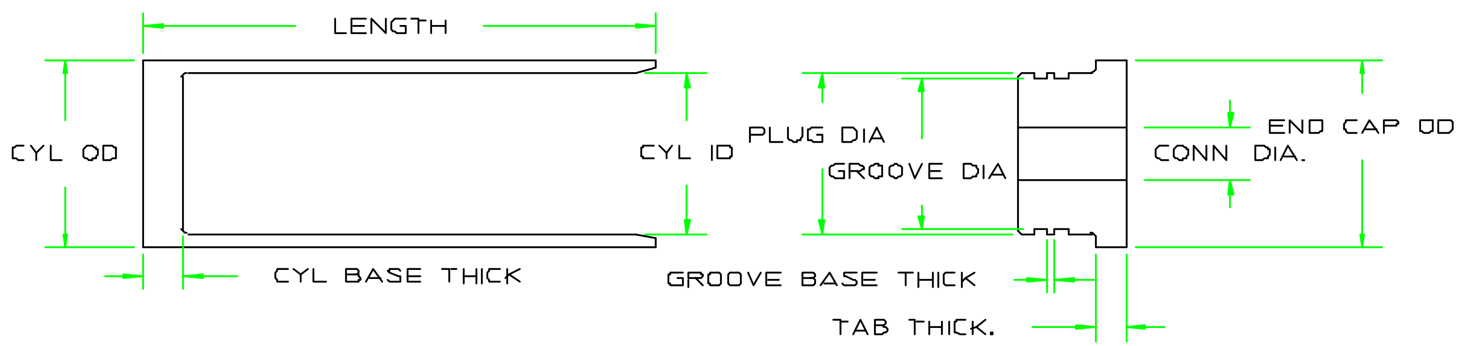
$$\sigma_{oring} \equiv \frac{6 \cdot M_{oringrb}}{t_{oring}^{2}} \qquad \sigma_{oring} = -2.592 \times 10^{4}\,\mathrm{psi}$$

$$Q_{oringb} \equiv \frac{w_{oring} \cdot r_{oring0}}{b_{oring}} \qquad Q_{oringb} = 694.481\,\frac{\mathrm{lbf}}{\mathrm{in}}$$

Margin: $$MS_{oring} \equiv \left(\frac{S_y}{\left|\sigma_{oring}\right| \cdot FS} - 1 \right) \qquad MS_{oring} = 2.087$$

E) WEIGHT CALCULATION:

	cross sectional area:	$ac_x \equiv \frac{\pi \cdot \left(OD_{cyl}^2 - ID_{cyl}^2\right)}{4}$	$ac_x = 13.352\,in^2$
	internal volume:	$vc_i \equiv \pi \cdot \frac{ID_{cyl}^2}{4} \cdot \left(L_{cyl} - T_{cyl\_base}\right)$	$vc_i = 1.15 \times 10^3\,in^3$
	external volume:	$vc_o \equiv \frac{\pi \cdot OD_{cyl}^2}{4} \cdot L_{cyl}$	$vc_o = 1.527 \times 10^3\,in^3$
	Tab:	$v_{tab} \equiv \frac{\pi \cdot OD_{cap}^2}{4} \cdot T_{endcap\_tab}$	$v_{tab} = 31.809\,in^3$
	Plug:	$v_{plug} \equiv \frac{\pi \cdot OD_{plug}^2}{4} \cdot \left(T_{cap} - T_{endcap\_tab}\right)$	$v_{plug} = 31.377\,in^3$
	Cap:	$V_{cap} \equiv v_{plug} + v_{tab}$	$V_{cap} = 63.185\,in^3$
End Cap:	Weight:	$W_{cap} \equiv V_{cap} \cdot \rho_m$	$W_{cap} = 10.742\,lbf$
Cylinder:	weight:	$W_{cyl} \equiv \rho_m \cdot \left(vc_o - vc_i\right)$	
		$W_{cyl} = 64.088\,lbf$	
	Total weight:	$Wp_{dry} \equiv W_{cyl} + W_{cap}$	$Wp_{dry} = 74.83\,lbf$
		$Wp_{wet} \equiv Wp_{dry} - \left(vc_o + v_{tab}\right) \cdot \rho_{sw}$	$Wp_{wet} = 17.103\,lbf$


LENGTH
CYL OD
CYL ID
CYL BASE THICK
PLUG DIA
GROOVE DIA
END CAP OD
CONN DIA.
GROOVE BASE THICK
TAB THICK.

$ksi \equiv 1000 \cdot psi$

SUMMARY

Thin Wall Calcs: Is it thin wall (1=yes)? $thinWall = 0$

Meridional (axial) stress: $\sigma_{1thin} = 24.789\,ksi$

Change in height: $\Delta y_{thin} = 0.014\,in$

Circumferential (hoop) stress: $\sigma_{2thin} = 49.577\,ksi$

Radial displacement of circumference: $\Delta R_{max} = 0.011\,in$

Margin of Safety: $MS_{cythin} = 0.863$

Thick Wall Calcs

Axial Stress: $\sigma_{1thick} = -2.779 \times 10^4\,psi$

Change in OD: $\Delta a_{thick} = -0.011\,in$

Hoop Stress: $\sigma_{2thick}(b_{thick}) = -5.558 \times 10^4\,psi$

Change in ID: $\Delta b_{thick} = -0.011\,in$

Radial Stress: $\sigma_{3thick}(b_{thick}) = 0\,psi$

Change in Length: $\Delta L = -0.01536\,in$

Shear Stress: $\tau_{thick} = 2.779 \times 10^4\,psi$

Von Mises Stress: $\sigma_{vmthick} = 48.135\,ksi$

Margins of Safety: for yield strength:

$MSyield_{cythick} = 0.662$

for shear strength:

$MSshear_{cythick} = 0.199$

Closed End Cylinder Base Calcs:

$\sigma_{base} = 91.524\,ksi$

$Q_{base} = -1.167 \times 10^4 \frac{lbf}{in}$

$ycenter_{base} = -0.044\,in$

Margin: $MS_{base} = -0.126$

Buckling Calcs

$p_{critical} = 10.786\,ksi$

Margin of safety on buckling: $MS_{buckling} = -0.075$

END CAP Calcs

END CAP Circular Plate

$y_{endcapb} = -0.074\,\text{in}$ $\quad Q_a = -1.303 \times 10^4 \dfrac{\text{lbf}}{\text{in}}$ $\quad \sigma_{endcap} = 2.25 \times 10^5\,\text{psi}$

Margin: $MS_{endcap} = -0.644$

END CAP tabs

$\sigma_t = 1.208 \times 10^6\,\text{psi}$ $\quad Q_{at} = -1.035 \times 10^4 \dfrac{\text{lbf}}{\text{in}}$

MARGIN OF SAFETY: $MS_t = -0.934$

ORING tab

$\sigma_{oring} = -2.592 \times 10^4\,\text{psi}$ $\quad y_{oringmax} = -2.552 \times 10^{-4}\,\text{in}$ $\quad Q_{oringb} = 694.481 \dfrac{\text{lbf}}{\text{in}}$

Margin: $MS_{oring} = 2.087$

WEIGHTS

$W_{cyl} = 64.088\,\text{lbf}$ $\quad W_{cap} = 10.742\,\text{lbf}$

$Wp_{dry} = 74.83\,\text{lbf}$ $\quad Wp_{wet} = 17.103\,\text{lbf}$