Project Name: Laser Raman Spectrometer
Project Number: 600045
Engineer: M.Brown
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Title: VORTEX SHEDDING

DESCRIPTION: **The positioning system for the LRS probe head has a canitlever arm that extends approximately 6-8 inches out from the center axis. Given 2-3 knot currents what current induced vibration will there be. Will this vibration effect the functional requirements of incremental positioning within a +/- .002 inch tolerance.**

PROPERTIES & ASSUMPTIONS:

Technical refernce: "Flow Induced Vibration" 2nd Ed. Robert D. Blevins

Length of probe head $L := 380 \cdot mm$ $L = 14.961\,in$

Diameter of Probe Head cylinder $D_1 := 150 \cdot mm$ $D_1 = 5.906\,in$

Strouhal Number: $S := .20$
Dimensionless constant between vortex shedding frequency and free stream velocity.

Density Aluminum $\rho_{alum} := .098 \cdot \frac{lb}{in^3}$

$\rho_{alum} = 2.713 \times 10^3 \frac{kg}{m^3}$

Ocean curren Knots to m/se $kt := .514 \cdot \frac{m}{s}$

Free stream Velocity $V := 3 \cdot kt$ $V = 5.059 \frac{ft}{s}$

Density Sea water $\rho_{sw} := 1026 \cdot \frac{kg}{m^3}$ $\rho_{sw} = 64.051 \frac{lb}{ft^3}$

Sea water Viscosity $\nu := 1.4 \cdot 10^{-3} \cdot N \cdot \frac{sec}{m^2}$

Density Titanium $\rho_{titan} := .160 \cdot \frac{lb}{in^3}$

CALCULATION:

SCENARIO 1: SHEDDING ACROSS PROBE HEAD HOUSING

Vortex shedding frequenc $f_s := \frac{S \cdot V}{D_1}$ $f_s = 2.056\,Hz$

Coeffcient of Drag $C_D := 1.2$

Reynolds Number $Re := \frac{V \cdot D_1 \cdot \rho_{sw}}{\nu}$ $Re = 1.695 \times 10^5$

Drag force per unit length for Probe Head housing $F_D := \frac{\rho_{sw} \cdot V^2 \cdot D_1 \cdot C_D}{2} \cdot L$ $F_D = 83.434\,N$ $F_D = 18.757\,lbf$

Does shedding frequency approach the fundamental frequency of the housing?

Vortex shedding frequency (from above) $f_s = 2.056\,Hz$

Elastic Modulus Titanium $E := 16.5 \cdot 10^6 \cdot \frac{lbf}{in^2}$

Hosing Diameter outside $D_1 = 0.15\,m$ $D_1 = 5.906\,in$

Hosing Diameter inside $D_2 := .08 \cdot m$ $D_2 = 3.15\,in$

Area of housing $A := \frac{\pi}{4} \cdot \left(D_1{}^2 - D_2{}^2\right)$ $A = 0.013\,m^2$

Moment of Inertia of housing $I := \frac{\pi}{64} \cdot \left(D_1{}^4\right)$ $I = 2.485 \times 10^{-5}\,m^4$

Spring Constant (Cylinder) Mark's page 3-62 $k := \frac{E \cdot A}{L}$ $k = 3.786 \times 10^9 \frac{kg}{s^2}$

Weight of housing $W := A \cdot L \cdot \rho_{titan}$ $W = 21.281\,kg$

Mass of housing $m := W$ $m = 21.281\,kg$ $g = 9.807 \frac{m}{s^2}$

Undamped natural frequency in radians/sec $\omega_n := \sqrt{\frac{k}{m}}$ $\omega_n = 1.334 \times 10^4 \frac{rad}{sec}$

Fundamental frequency in cycles per sec Hz $f_n := \frac{\omega_n}{2 \cdot \pi}$ $f_n = 2.123 \times 10^3\,Hz$

Check to see if mast fundamental is within +/- 20% of fs

Fundamental frequency $f_n = 2.123 \times 10^3\,Hz$

Shedding frequency $f_s = 2.056\,Hz$

$$\frac{f_n - f_s}{f_s} \cdot 100 = 1.031 \times 10^5$$

SCENARIO 2: SHEDDING ACROSS CANTILEVER ARM
Analysis for one large 6.0 inch O.D. slide tube

Length of cantilever arm $L_1 := 152 \cdot mm$ $L_1 = 5.984\,in$

Diameter of slide tube $D_O := 6.0 \cdot in$ $D_I := 5.50 \cdot in$

Moment of Inertia of TUBE $I_{t1} := \frac{\pi}{64} \cdot \left(D_O{}^4 - D_I{}^4\right)$ $I_{t1} = 7.783 \times 10^{-6}\,m^4$

Elastic Modulus Aluminum $E_{alum} := 10 \cdot 10^6 \cdot \frac{lbf}{in^2}$

Spring Constant (cantilever) Mark's page 3-62 $k_1 := \frac{3 \cdot E_{alum} \cdot I_{t1}}{L_1{}^3}$ $k_1 = 4.584 \times 10^8\,\frac{kg}{s^2}$

Undamped natural frequency in radians/sec (Tube) $\omega_{tube} := \sqrt{\frac{k_1}{m}}$ $\omega_{tube} = 4.641 \times 10^3\,\frac{rad}{sec}$

Fundamental frequency in cycles per sec Hz (Tube) $f_{tube} := \frac{\omega_{tube}}{2 \cdot \pi}$ $f_{tube} = 738.69\,Hz$

Vortex shedding frequency $f_{s1} := \frac{S \cdot V}{D_O}$ $f_{s1} = 2.024\,Hz$

Check to see if mast fundamental is within +/- 20% of fs

Fundamental frequency (TUBE) $f_{tube} = 738.69\,Hz$

Shedding frequency (TUBE) $f_{s1} = 2.024\,Hz$

$$\frac{f_{tube} - f_{s1}}{f_{s1}} \cdot 100 = 3.64 \times 10^4$$

SCENARIO 3: SHEDDING ACROSS CANTILEVER ARM
Analysis for multiple small tube array

Length of cantilever arm $L_1 := 152 \cdot mm$ $L_1 = 5.984\,in$

Diameter of slide tube Outside & Inside $D_{out} := 1.0 \cdot in$ $D_{out} = 0.025\,m$ $D_{in} := .875 \cdot in$

Moment of Inertia of TUBE $I_{t2} := \frac{\pi}{64} \cdot \left(D_{out}^{4} - D_{in}^{4}\right)$ $I_{t2} = 8.455 \times 10^{-9}\,m^{4}$

Elastic Modulus Aluminum $E_{alum} := 10 \cdot 10^{6} \cdot \frac{lbf}{in^{2}}$

Spring Constant (cantilever) Mark's page 3-62 $k_1 := \frac{3 \cdot E_{alum} \cdot I_{t2}}{L_1^{3}}$ $k_1 = 4.98 \times 10^{5}\,\frac{kg}{s^{2}}$

Mass of one tube per unit length $m_t := .0006 \cdot \frac{kg}{mm}$ $m_t = 0.6\,\frac{kg}{m}$ $mm = 1 \times 10^{-3}\,m$

Undamped natural frequency in radians/sec (Tube) $\omega_{tube} := \sqrt{\frac{k_1}{m}}$ $\omega_{tube} = 152.975\,\frac{rad}{sec}$ $\omega_{tube} = 152.975\,Hz$

Fundamental frequency in cycles per sec Hz (Tube) $f_{tube} := \frac{\omega_{tube}}{2 \cdot \pi}$ $f_{tube} = 24.347\,Hz$

Vortex shedding frequency small tube $f_{s1} := \frac{S \cdot V}{D_{out}}$ $f_{s1} = 12.142\,Hz$

Tube damping factor "Flow induced vibration" pp158 $\zeta := .005$

Constant "C" and exponent "a" from imperical data. see Table on page 160 "Flow induced vibration" pp158 $C := 4.5$ $a := .5$

Critical Velocity for tube array

$$V_{crit} := C \cdot \left[\frac{m_t \cdot (2 \cdot \pi \cdot \zeta)}{\rho_{sw} \cdot D_{out}^{2}}\right]^{a} \cdot \left(f_{tube} \cdot D_{out}\right)$$

$V_{crit} = 0.47\,\frac{m}{s}$ $V_{crit} = 1.541\,\frac{ft}{sec}$

M B A R I

FREE VIBRATION OF A VISCOUSLY DAMPED STRUCTURE

Mode factor from wake oscillator model (FIV pp72) $\gamma := 1.0$

Mass per unit length $m_{titan} := .078 \cdot \frac{kg}{mm}$ $m_{titan} = 78 \frac{kg}{m}$

Added Mass (Table 2-2) FIV pp25 $m_{added} := \rho_{titan} \cdot \pi \cdot a^2 \cdot b$ $b \equiv L$ $m_{added} = 3.478\,kg$

Added mass per unit length $m_{addm} := \frac{m_{added}}{L}$ $m_{addm} = 9.154 \frac{kg}{m}$

Reduced damping

$$\delta_r := \frac{2 \cdot \left(m_{titan} + m_{addm}\right) \cdot (2 \cdot \pi \cdot \zeta)}{\rho_{sw} \cdot D_1^{\,2}}$$

$\delta_r = 0.237$

Dimensionless Displacement Amplitude at resonance Blevins 1977, (pp. 71)

$$A_{yn} := \frac{.07 \cdot \gamma}{\left(1.9 + \delta_r\right) \cdot S^2} \cdot \sqrt{.3 + \frac{0.72}{\left(1.9 + \delta_r\right) \cdot S}} \cdot D_1$$

$A_{yn} = 0.173\,m$

$A_{yn} = 6.812\,in$

Amplitude (Meter & Inches) from Avian software package

$A_y := 4.9 \cdot 10^{-8} \cdot in$ $A_y = 1.245 \times 10^{-9}\,m$

$A_x := \frac{A_y}{D_1}$ $A_x = 8.297 \times 10^{-9}$

RESULTS

Natural frequency and shedding frequency are not in the synchronization range (fig. 3-10) Significant amplitudes do not exist. Amplitude check at given flow velocity of 60 in/sec shows an amplitude of 2.7 x 10e8 inches.

DAMPING IN A FLUID FLOW

Natural frequency	$f_x := f_{s1}$	$f_x = 12.142\,\text{Hz}$
Diameter of geometry	$D_x := D_O$	$D_x = 0.152\,\text{m}$
Mass of geometry	$m_x := m_t$	$m_x = 0.6\,\frac{\text{kg}}{\text{m}}$
Free stream velocity	$U := V$	$U = 1.542\,\frac{\text{m}}{\text{s}}$

Flow induced damping $\zeta_{x\_drag} := \frac{1}{4 \cdot \pi} \cdot \frac{U}{f_x \cdot D_x} \cdot \frac{\rho_{sw} \cdot D_x^2}{m_x} \cdot C_D$

$\zeta_{x\_drag} = 3.161$

SCENARIO 4: FORCE ON HOUSING DUE TO CURRENT

Pressure on housing due to flow	$P_1 := \frac{1}{2} \cdot \rho_{sw} \cdot V^2$	$P_1 = 1.22 \times 10^3\,\text{Pa}$	$P_1 = 0.177\,\text{psi}$
Radius of cylinder	$r := \frac{D_1}{2}$	$r = 0.075\,\text{m}$	$\alpha := 180$
length of cylinder arc	$L_c := \frac{r \cdot \alpha \cdot \pi}{180}$	$L_c = 0.236\,\text{m}$	$L_c = 9.276\,\text{in}$
Area of cylinder face	$A_f := L \cdot L_c$	$A_f = 0.09\,\text{m}^2$	$A_f = 138.78\,\text{in}^2$
Force on housing	$F := P_1 \cdot A_f$	$F = 109.215\,\text{N}$	$F = 24.552\,\text{lbf}$

Radius of housing	$r = 0.075\,\text{m}$	$a := r$
Length of housing	$L = 0.38\,\text{m}$	$b := L$

Added mass $m_{added} := \rho_{sw} \cdot \pi \cdot a^2 \cdot b$

$m_{added} = 6.89\,\text{kg}$