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On the dynamics of wave-power devices

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The performance of a class of wave-power devices is studied theoretically by generalizing the known theory of ship dynamics with which there is good agreement with experiment. The extensions to the existing theory introduce the new features of asymmetry and articulation common to many proposed wave energy convertors.

Results are presented for two different devices and a comparison is made between them. The performance calculations correlate very well with the available experimental evidence and moreover it would appear that the two types of device considered in this paper are comparable in their potential operating efficiencies if appropriate scales are chosen.

The Salter duck and a two-pontoon system, semi-elliptical in cross section and hinged at its centre, constrained to move only in the mode in which energy is absorbed, appear to be equivalent. Both structures are designed such that when forced to move in their absorbing mode they generate waves in one preferred direction, the pontoon design relying on the use of a shallow horizontal breakwater in the rear of the moving structure whereas Salter has used a shorter deeper structure which looks almost cylindrical.

When optimally loaded with a simple velocity-proportional damping applied externally, the performance of each system looks almost identical, with the pontoon being slightly better at the low frequency end. This parity is interesting and more general systems where the structures are free to move in many degrees of freedom must be examined. The reasons for this are apparent from calculated sea efficiencies of the devices. For the duck with a fixed centre and reactive loading, a 10 m diameter device may achieve an excellent performance over typical sea states, but if simple (velocity proportional) loading is used the diameter must increase to 15 m to give a comparable performance. If, however, the structure is free to move in other modes of motion with the same simple loading, then to achieve similar performance characteristics the structure has to be 30 m in diameter.

Similar effects are found for the pontoon. Whereas an 80 m long two-pontoon system with a fixed rear section and simple loading will match to the sea conditions very well, a freely floating system with the same loading would need to be 120 m long to achieve a reasonable performance.

List of symbols

A	incident-wave amplitude
B	restoring constant
D	external damping
F	force
I	inertia
K	wavenumber
M	added mass
N	added damping
P	asymptotic amplitude of the radiation potential
R	external spring constant
V	vector of the time dependent body velocity
g	gravitational acceleration
r	polar radius
s	surface profile
t	time
$\boldsymbol{v}$	vector of the time independent body velocity
x, y	plane coordinates
x_0, y_0	centre of rotation
Φ	time dependent velocity potential
Ω	normalized angular frequency
θ	angle
ϕ	time independent velocity potential
Ψ	stream function
ρ	density of water
ω	angular frequency

Subscripts

i	ith mode of motion
ij	the effect in the ith mode due to motion in the jth mode
d	diffraction
n	normal component

Superscripts

ext	external
W	wave induced

1. Introduction

With the recent concern about energy requirements, there has been an increasing interest in renewable energy resources. One of the least investigated is ocean surface waves which offer a potentially large source of energy for which the United Kingdom is well placed. Several studies are in progress (Salter 1974; Glendenning & Count 1976) and recently some effort has been made to develop theoretical models of wave-power devices to assist in understanding the physical processes involved and to predict and optimize some of the full scale characteristics of such structures.

Theoretical studies of the interaction of ships with waves have been developed over a number of years, largely based on work by Ursell (1949) and Newman (1962, 1977). The method demands the solutions of the linearized hydrodynamic equations from which the pressure forces acting on the body surface can be calculated and used to solve the equations of motion of the body in a number of degrees of freedom simultaneously. The technique has been validated experimentally (Takagi 1974; Vugts 1970) with remarkably good agreement, deviations occurring only in cases where the body is forced into oscillations whose amplitudes are significant compared with its own characteristic dimension.

In the past, interest has centred on the dynamics of ship structures and most studies to date have been limited to symmetrical bodies. However, wave-power devices are typically asymmetric and have further significant differences in that they are required to absorb energy from the fluid, which a ship is not. This energy absorbing characteristic can easily be accommodated by inserting dissipative terms into the equations of motion and has little impact on the hydrodynamic problem.

The asymmetry of the structures, however, can be introduced either by the geometrical configuration (Salter 1974) in which case it still looks like a single-hulled ship, albeit asymmetric, or by allowing articulated, asymmetric motions of adjacent elements. An example of the latter is the Cockerell pontoon which has been described by Wooley & Platts (1975). These articulated structures are in fact a number of simple single-hulled structures which are mechanically linked and it will be shown that they can be treated by the same general method used for individual structures, by including a number of extra degrees of freedom to be solved for in the equations of motion. Evans (1976) has noted that symmetrical structures which are mechanically connected to their surroundings in more than one mode of motion can be considered as an efficient wave-power absorber. These, uniquely, have been treated by existing analytic models and the purpose of this paper is to describe the extension of the existing techniques to solve the hydrodynamic equations for all categories of wave-power devices.

The dynamics of a partly or wholly immersed structure may be described by a simultaneous set of equations of motion, in k degrees of freedom,

$$\sum_{j=1}^{k} \{I_{ij}\ddot{X}_j + B_{ij}X_j\} = F_i^{\mathrm{W}}(t) + F_i^{\mathrm{ext}}(X_j, \dot{X}_j, \ddot{X}_j) \quad i = 1, 2, \ldots N, \tag{1}$$

where X_j is the displacement in the jth mode, I_{ij} is the inertial matrix, B_{ij} is the spring matrix due to buoyancy, F_i^{ext} is the external force applied in the ith mode and $F_i^{\mathrm{W}}(t)$ is the corresponding hydrodynamic wave-induced force. The problem is simplified by considering only linear forms of F_i^{ext} $(X_j, \dot{X}_j, \ddot{X}_j)$, linearizing the equations and solving for each frequency component separately. At each frequency the hydrodynamic force is shown to be the sum of a constant and a term proportional to the amplitude of motion (Newman 1962) and calculation of these quantities involves the solution of the full linearized fluid equations.

The hydrodynamic pressure forces are calculated by extending the method derived by Ursell. This is entirely linear and two dimensional and involves mapping the body section to that of a semi-circle and constructing a set of known solutions which are added together so that the velocity boundary conditions on the body are satisfied. The extensions to asymmetric bodies introduce new terms into both the mapping function and the set of standard solutions, usually referred to as multipoles.

After defining the equations of motion and the linearized hydrodynamic equations, the natural partitioning of the potential is described, as suggested by Newman (1962). This demonstrates that in order to obtain all of the forces necessary to solve the equations of motion a potential solution must be found for a forced motion in each independent degree of freedom. The section is completed by explaining exactly how extra modes of motion are formulated for articulated wave-power devices with specific reference to the Cockerell pontoon.

In § 3 the potential problem for any forced oscillation of a structure is formulated and solutions with general application to asymmetric wave-power devices are identified. The computation is discussed and areas where numerical calculations can be simplified to avoid complexity in the programming are stressed. The hydrodynamic forces calculated are then inserted into the equations of motion and § 4 highlights a solution suggested by Evans (1976) which considers a device constrained to move in one mode of motion. From this simple analysis it is concluded that, in two dimensions, a good wave absorber is one that, when forced to move, generates waves in one direction only, and that optimum operating conditions are achieved when the device is tuned to resonate with an applied external damping force exactly equal to the internal (fluid) damping. This is a familiar result in transmission line theory.

The last part of the paper describes how the monochromatic responses can be extended to some sort of meaningful parameter applicable to operation at sea. Results for the Salter duck and Cockerell pontoon are discussed and compared and it is shown that although apparently quite different in their geometrical configuration, they are hydrodynamically very similar in their operating modes of motion.

2. Formulation of the problem

2.1. *The linearized hydrodynamic equations in two dimensions*

Water may be treated to a very good approximation as an incompressible inviscid fluid. Mathematically, this means that solutions can conveniently take the form of a velocity potential $\Phi(x, y; t)$ which satisfies Laplace's equation,

$$\nabla^2 \Phi = 0, \tag{2}$$

and from which the fluid velocities $V(x, y; t)$ are calculated as

$$V(x, y; t) = -\nabla \Phi.$$

The free surface boundary condition equating the velocity of the fluid to that of the surface displacement is linearized and is

$$\frac{\partial \Phi}{\partial y} - \frac{1}{g}\frac{\partial^2 \Phi}{\partial t^2} = 0 \quad \text{on} \quad y = 0, \tag{3}$$

and the continuity condition applied on the mean wetted surface of the body is

$$\partial \Phi / \partial n = V_n(x, y; t), \tag{4}$$

where n denotes the normal direction and $V_n(x, y; t)$ is the normal velocity of the immersed body boundary. Further necessary conditions are that at large distances from the body, simple waves shall exist and the potential decays to zero as y increases (Stoker 1957). Having solved for Φ, the dynamic pressure is given by Bernoulli's theorem (Stoker 1957) as

$$p = -\rho\, \partial \Phi / \partial t, \tag{5}$$

and by evaluating this around the body the nett forces acting on it can be calculated.

2.2. *The form of Φ*

As a consequence of the linearization the time dependence of the solution may be factored out for steady monochromatic solutions as

$$\Phi(x, y; t) = \phi(x, y)\, \mathrm{e}^{-\mathrm{i}\omega t},$$

where ω is the angular frequency of the wave. The spatial part, $\phi(x, y)$ may be separated into any number of superposable solutions satisfying equation (2).

It is convenient to distinguish three such potentials, say ϕ_0, ϕ_d and ϕ_r. The potential ϕ_0 will be that due to an incident plane wave alone in the absence of a body, ϕ_d will be the perturbation on the incident wave due to a body at rest and is known as the diffraction potential and ϕ_r is the potential due to the forced movement of the body in nominally calm water and is known as the radiation potential.

The overall solution is then

$$\phi = \phi_0 + \phi_\mathrm{d} + \phi_\mathrm{r}.$$

This particular choice of separate potentials is particularly convenient since Newman (1962) has demonstrated that the pressure forces acting on the body due to ϕ_d can be determined from knowledge of ϕ_0 and ϕ_r alone. He demonstrated that the exciting forces due to potentials ϕ_0 and ϕ_d in the ith mode is simply

$$\rho g A P_i^{+}\, \mathrm{e}^{-\mathrm{i}\omega t},$$

where A is the incident-wave amplitude and P_i^{+} is the (complex) amplitude of the upstream radiation potential for unit motion in the ith mode alone.

Therefore the total force due to the wave is the sum of the exciting force given above and that due to the potential ϕ_r. The latter is linearly proportional to the motion of the body and is found as the superposition of the force due to motion not only in the ith mode but those induced by motion in all of the other degrees of freedom. Each force is generally resolved into two orthogonal components, one in phase with the acceleration of the forced motion and the other with the velocity, and the terms are appropriately normalized and referred to as the added mass and damping of the system. The total force, $F_i^{\mathrm{W}}(t)$, is then

$$F_i^{\mathrm{W}}(t) = \rho g A P_i^{+} \,\mathrm{e}^{-\mathrm{j}\omega t} - \sum_{j=1}^{k} \{M_{ij}\ddot{X}_j + N_{ij}\dot{X}_j\},$$

where M_{ij} and N_{ij} are the added mass and damping matrices.

2.3. *The number of independent modes of motion*

For a rigid structure in three dimensions there are six independent degrees of freedom, three translational and three rotational modes, whereas in two dimensions there are only three, two translational and one rotational (commonly referred to

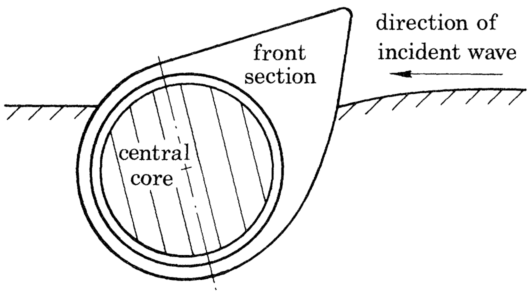

FIGURE 1. The geometry of the Salter duck. An example of a four-degree-of-freedom system which does useful work through relative motion of the front section about the central core.

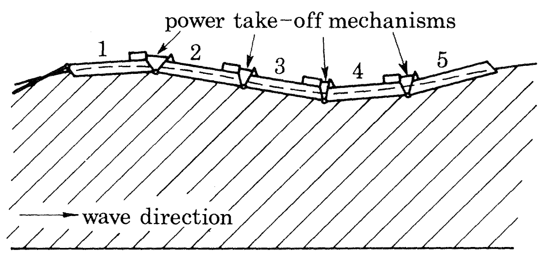

FIGURE 2. The Cockerell wave contouring raft. A five-pontoon system which would have four extra degrees of freedom, giving seven in all.

as heave, sway and roll). However, for articulated structures there will be extra modes involving relative motions between different elements of the structure.

The simplest example, and one often overlooked, is that of the Salter device shown schematically in figure 1 and this has four independent modes of motion which are heave, sway and roll of the complete structure, as if rigid, together with a differential motion of the front section rolling about the central core. In this case the hydrodynamic solutions for the relative roll motion are identical to those of the rolling of the complete structure but there will be different inertia and damping terms between the two modes. However, with an articulated raft the hydrodynamic solutions are different for each mode of motion because the velocity profile changes in each case.

The Cockerell articulated raft, as shown in figure 2, in its equilibrium position looks very much like a shallow draught ship and the motion of this type of structure can easily be solved by the multipole technique since its conformal transform is approximately that of an ellipse with a large major : minor-axis ratio. Therefore, for heave, sway and rolling motions of the complete structure, there is no theoretical problem, but a difficulty arises because the body is free to deform at each hinge position.

The key to the solution of the problem introduced by the articulated raft is to recognize that the additional degrees of freedom are simply the relative angular motions between adjacent pontoons and can be solved hydrodynamically, exactly as for rigid motions except that the boundary conditions are changed. For example, a structure with n raft sections will have $(n-1)$ extra degrees of freedom, which are the first m rafts rigidly coupled, rotating about the rear $(n-m)$ rafts which are fixed and $1 \leqslant m \leqslant (n-1)$. These extra motions are quite legitimate mathematically since the boundary conditions are applied on the mean wetted surface of the body and the only proviso is that the imposed velocity profile is continuous along that boundary. Therefore if we had a pontoon system of n component parts, it would be necessary to solve the hydrodynamic equations in $(n+2)$ degrees of freedom.

There have been alternative suggestions (Katory 1977) where each element is allowed to move in three degrees of freedom (heave, sway and roll) in the presence of the other pontoons, and all of the hinge constraints have been built into the equations of motion. This method requires $3n$ hydrodynamic solutions against the $(n+2)$ suggested above, making it less economical; and with the smaller set of equations it is easy to work backwards to compute the required hinge forces for design purposes.

It remains to solve for radiation potentials with specified unit motions in each of the degrees of freedom in order to calculate the wave forces. These can then be substituted in equation (1) which can be solved for specified linear external forces.

3. The radiation problem

3.1. *The simplified equations*

The problem is to find a potential $\phi(x, y)$ which decays to zero as y becomes large, satisfies Laplace's equation (2), together with the modified free surface condition,

$$\partial\phi/\partial y + K\phi = 0 \quad \text{on} \quad y = 0, \tag{6}$$

$$\partial\phi/\partial n = \boldsymbol{v}_n(x, y) \quad \text{on the mean body surface,} \tag{7}$$

and

$$\phi \rightarrow P^{\pm} \mathrm{e}^{-Ky} \mathrm{e}^{\pm \mathrm{i}Kx} \quad \text{as} \quad x \rightarrow \pm\infty, \tag{8}$$

where $K = \omega^2/g$, $V_n(x,y;t) = \boldsymbol{v}_n(x,y)\,\mathrm{e}^{-\mathrm{i}\omega t}$ and $P^{\pm}$ are (complex) constants. The positive and negative signs correspond to the upstream and downstream respectively, referred to the direction of propagation of the incident wave.

Since this is a two-dimensional Laplace problem the method of conformal transforms may be used where the exterior region of the body is transformed to the exterior of a circle. The technique has previously been successfully applied to symmetric ship sections, and the following analysis extends the previous work to asymmetric sections.

3.2. *The conformal representation*

Conformal transformations are discussed at length by Kantorovich & Krylov (1958) and in general x and y are related to the new coordinates, r and θ,

$$x = r\sin\theta + \sum_{n=0}^{N}(-1)^n[a_{2n}r^{-2n}\cos 2n\theta + a_{2n+1}r^{-(2n+1)}\sin(2n+1)\theta],$$

and
$$y = r\cos\theta + \sum_{n=0}^{N}(-1)^n[a_{2n}r^{-2n}\sin 2n\theta - a_{2n+1}r^{-(2n+1)}\cos(2n+1)\theta],$$

where the body surface is mapped to the geodesic $r = r_0$, θ ranges from $-\frac{1}{2}\pi$ to $\frac{1}{2}\pi$, $r \geqslant r_0$ and the a_n are the mapping coefficients. The free surface condition (6) now becomes

$$\partial\phi/\partial\theta + K\phi\left[\pm r + \sum_{n=0}^{N}\{2na_{2n}r^{-2n} \pm (2n+1)\,a_{2n+1}r^{-(2n+1)}\}\right] = 0 \qquad (9)$$

on $\theta = \pm\frac{1}{2}\pi$ where $y = 0$, and

$$\partial\phi/\partial y + K\phi = 0 = \partial\phi/\partial\theta + K\phi\,\partial y/\partial\theta.$$

3.3. *The form of the solution*

In order to calculate a complete set of functions representing only local motions, the property that a conformal transformation leaves the Laplace equation invariant is used. Therefore, any solution must satisfy equation (2) in the polar coordinates, (r,θ), namely

$$r\partial/\partial r(r\,\partial\phi/\partial r) + \partial^2\phi/\partial\theta^2 = 0.$$

Solutions of this equation which decay to zero for large r, take the form of $r^{-n}\cos n\theta$ (or $r^{-n}\sin n\theta$), where n is a positive integer greater than 1. Further, from § 3.2, if equation (9) is to be satisfied then a little algebra will demonstrate that the combinations, usually called multipoles,

$$\phi_{2m+3} = S_{2m+1} + K\left[\frac{S_{2m}}{2m} + \sum_{n=0}^{N}(-1)^n\left\{\frac{2na_{2n}C_{2m+2n+1}}{(2m+2n+1)} + (2n+1)\frac{a_{2n+1}S_{2m+2n+2}}{(2m+2n+2)}\right\}\right],$$

and

$$\phi_{2m+4} = C_{2m} + K\left[\frac{C_{2m-1}}{(2m-1)} - \sum_{n=0}^{N}(-1)^n\left\{\frac{2na_{2n}S_{2m+2n}}{(2m+2n)} - (2n+1)\frac{a_{2n+1}C_{2m+2n+1}}{(2m+2n+1)}\right\}\right],$$

are adequate where $m = 1, 2, \ldots$,

$$C_n = r^{-n} \cos n\theta,$$

and

$$S_n = r^{-n} \sin n\theta.$$

These functions, ϕ_{2m+3} and ϕ_{2m+4}, alone are insufficient to satisfy the radiation condition (8), and to complete the solution, a function, or set of functions, satisfying (2), (6) and (8) must be added to the above multipoles.

To find the additional solutions it is convenient to revert back to the Cartesian coordinates (x, y) where the so-called dipole functions are constructed, which satisfy (2) and (6) (Ursell 1949), and are

$$\phi_1 = \pi \mathrm{e}^{-Ky} \cos Kx,$$

$$\phi_2 = -\int_0^\infty \mathrm{e}^{\mp kx} \frac{[k \cos ky - K \sin ky]}{(K^2+k^2)} \mathrm{d}k \pm \pi \mathrm{e}^{-Ky} \sin Kx,$$

$$\phi_3 = \pi \mathrm{e}^{-Ky} \sin Kx,$$

and

$$\phi_4 = \pm \int_0^\infty k \mathrm{e}^{\mp kx} \frac{[k \cos ky - K \sin ky]}{(K^2+k^2)} \mathrm{d}k \pm \pi \mathrm{e}^{-ky} \cos Kx,$$

where the upper and lower signs correspond to positive and negative values of x respectively. If any linear combination, $A\phi_1 + B\phi_2 + C\phi_3 + D\phi_4$ is added to a corresponding component $A\phi_2 - B\phi_1 - C\phi_4 + D\phi_3$ which is $\frac{1}{2}\pi$ out of phase, say,

$$[A\phi_1 + B\phi_2 + C\phi_3 + D\phi_4] \cos \omega t + [A\phi_2 - B\phi_1 - C\phi_4 + D\phi_3] \sin \omega t,$$

then this combination of ϕ_1, ϕ_2, ϕ_3 and ϕ_4 will satisfy (8) recast in its real form as opposed to the complex form

$$\Phi \to |P^\pm| \, \mathrm{e}^{-Ky} \cos (Kx \mp \omega t + \epsilon^\pm) \quad \text{as} \quad x \to \pm \infty,$$

where

$$|P^\pm| = \pi [(A \pm D)^2 + (B \pm C)^2]^{\frac{1}{2}},$$

$$\epsilon^\pm = \arctan [-(B \pm C)/(A \pm D)],$$

and A, B, C and D are constants.

The complete solution then is a superposition of the multipole and dipole functions and may be written as

$$\Phi(x, y; t) = [A\phi_1 + B\phi_2 + C\phi_3 + D\phi_4] \cos \omega t + [A\phi_2 - B\phi_1 - C\phi_4 + D\phi_3] \sin \omega t$$
$$+ \sum_{m=1}^{\infty} E_{m+4} \phi_{m+4} \cos \omega t + \sum_{m=1}^{\infty} F_{m+4} \phi_{m+4} \sin \omega t,$$

where the constants A, B, C, D, $\{E_{m+4}\}$ and $\{F_{m+4}\}$ can be determined from the boundary condition (7) on the body surface $r = r_0$. It has been found that this can be achieved by truncating the infinite sum and fitting the unknown constants in a least squares sense.

3.4. *The computation*

For computational convenience it is interesting to note that to evaluate $\partial\phi/\partial n$ around the body profile is a very tedious process, whereas in terms of the stream function, $\psi(x,y)$, the normal velocity is $\partial\psi/\partial s$ where $s(x,y)$ is the surface profile. The advantage of this is that in terms of s, the normal velocity V_n can be expressed as $\cos\omega t\,\partial f/\partial s$, where f is some known spatial function, $\{f(x,y) \equiv x$ for heave, $-y$ for sway and $\frac{1}{2}[(x-x_0)^2+(y-y_0)^2]$ for rolling about the point $(x_0, y_0)\}$. Therefore the equation

$$\partial\psi/\partial s = \cos\omega t\,\partial f/\partial s$$

can be integrated to give

$$\Psi(x,y;t) = \cos\omega t f(x,y)+g(t) \quad \text{on the body surface,}$$

where $g(t)$ is an arbitrary function of time. In order to eliminate this unknown function of time we can simply consider the equation

$$\Psi(x,y;t)-\Psi(x_1,y_1;t) = \{f(x,y)-f(x_1,y_1)\}\cos\omega t,$$

where (x_1, y_1) are the coordinates of an arbitrary reference point on the boundary.

Therefore, after carrying out the computations, the equations of motion can be solved in detail. An interesting solution has been suggested by Evans (1976) and is described in the following section.

4. A particular solution for one degree of freedom

Suppose that a wave-power device is constrained to move in one mode of motion only and the added mass and damping are respectively frequency dependent functions $M(\omega)$ and $N(\omega)$. If the displacement in this mode is X, then the equation of motion (1) reduces to

$$[I+M(\omega)]\ddot{X}+N(\omega)\dot{X}+BX = \rho gAP^{+}\mathrm{e}^{-\mathrm{i}\omega t}+F^{\mathrm{ext}}(X,\dot{X},\ddot{X}),$$

where I is the body inertia, B is the spring term and $\rho gAP^{+}\mathrm{e}^{-\mathrm{i}\omega t}$ is the remaining part of the hydrodynamic force given in § 2.3. If the external force is

$$F^{\mathrm{ext}} = -D\dot{X}-RX,$$

then the above equation can easily be solved to give

$$X = \rho gAP^{+}\mathrm{e}^{-\mathrm{i}\omega t}/\{[R+B-\omega^2(I+M(\omega))]-\mathrm{i}\omega[D+N(\omega)]\},$$

and noting that the incident power is (Stoker 1957)

$$\rho g^2A^2/4\omega,$$

whereas the absorbed power is

$$\tfrac{1}{2}D\omega^2|X|^2,$$

the efficiency of the device may be calculated as

$$2D\rho\omega^3|P^{+}|^2/\{[R+B-\omega^2(I+M(\omega))]^2+\omega^2[D+N(\omega)]^2\}. \tag{10}$$

Energy conservation demands (Newman 1962)

$$2K(\omega) = \rho\omega[|P^+|^2 + |P^-|^2], \tag{11}$$

where $P^{\pm}$ are the upstream and downstream amplitudes of the radiation potential, and substitution of this expression for the efficiency gives an expression which maximizes when

$$R = \omega^2(I + M(\omega)) - B \quad \text{and} \quad D = N(\omega),$$

as

$$|P^+|^2/[|P^+|^2 + |P^-|^2].$$

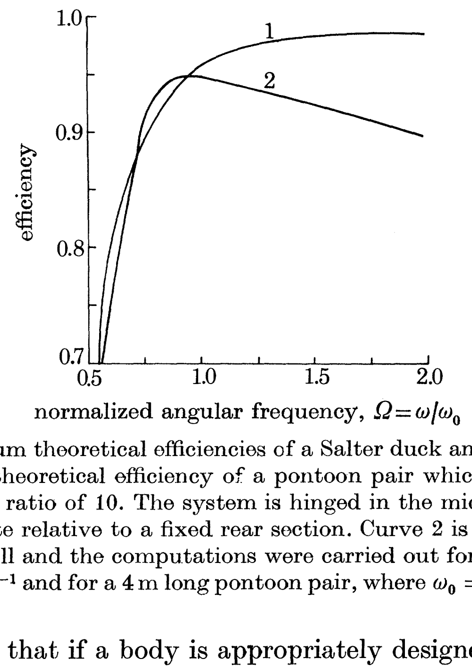

FIGURE 3. The maximum theoretical efficiencies of a Salter duck and a pontoon pair. Curve 1 is the maximum theoretical efficiency of a pontoon pair which is semi-elliptical with a major : minor-axis ratio of 10. The system is hinged in the middle and the front section is allowed to rotate relative to a fixed rear section. Curve 2 is the equivalent for a duck allowed only to roll and the computations were carried out for a 2 m diameter duck, for which $\omega_0 = 2\,\mathrm{rad\,s^{-1}}$ and for a 4 m long pontoon pair, where $\omega_0 = 3.4\,\mathrm{rad\,s^{-1}}$.

This result shows that if a body is appropriately designed so that when forced to move it generates waves in one direction only, (i.e. $|P^-|/|P^+| \ll 1$) and if this is maintained over a large frequency range, by tuning the damping, N, and the spring, R, a very broad bandwidth response may be obtained. This appears to be what the Salter device has achieved in practice by choosing a circular rear section about a fixed roll centre, but there is no reason to suppose that the shape of the front section is unique, and this can be demonstrated by the results shown in figure 3.

However, in a real sea situation where a spectrum of waves is incident upon a device, then only one damping and inertial force can be externally applied and therefore to calculate realistic performance characteristics the added mass and damping must be determined (figure 4) and an ideal shape would be one which has a broad band response with constant damping and spring. This is achieved by the Salter duck in small-scale experiments. The device used was 0.1 m in diameter (figure 1) and all of the experimental values have been appropriately scaled up for a duck of 2 m diameter at which scale it was convenient to carry out the computations. Figure 5 compares the experimental values of efficiency against those computed with a dry inertia of $4.4\,\mathrm{t\,m^2\,m^{-1}}$, an external damping constant of $6.7\,\mathrm{kN\,m\,rad^{-1}\,s\,m^{-1}}$ and a centre of gravity 0.75 m from the pivot point making an angle of 23° with the horizontal, about which a mass of $2.81\,\mathrm{t\,m^{-1}}$ acts. The values of the dry inertia and external damping are presented in figure 4 for comparison with the hydrodynamic values.

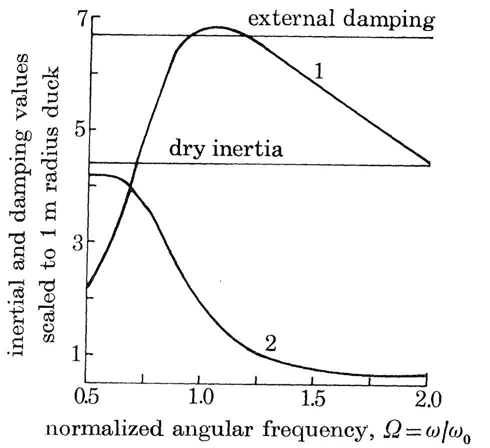

FIGURE 4.

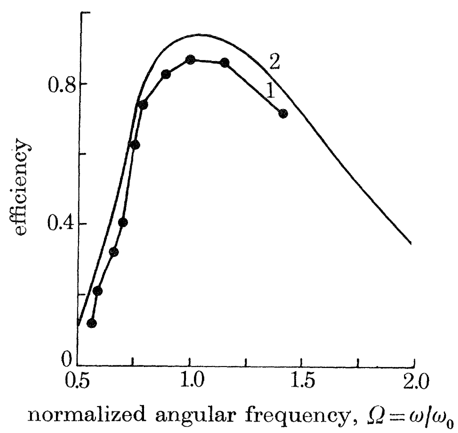

FIGURE 5.

FIGURE 4. The added inertia and damping curves of a Salter duck in the roll mode of motion. Curve 1 is the added damping of a 2 m diameter duck for which $\omega_0 = 2\,\mathrm{rad\,s^{-1}}$ and the units presented are $\mathrm{kN\,m\,rad^{-1}\,s\,m^{-1}}$. Curve 2 is the added mass curve for the same device and is measured in $\mathrm{t\,m^2\,m^{-1}}$. Both curves are computed for a unit width of device and the experimental values of dry inertia and external damping used by Salter in this small-scale experiment are presented appropriately scaled to the dimensions used in the computations.

FIGURE 5. A comparison between the calculated and measured efficiencies of a Salter duck. Curve 1 is an experimental curve of Salter's (September 1976) appropriately scaled to a 2 m diameter duck and this is compared with the theoretical curve, 2, which used the experimental values presented in figure 4.

5. The design and loading of devices

5.1. *The optimum geometry and motion*

The characteristics of linearly loaded wave-power devices, constrained to move in a single mode of motion, can be explained by considering equations (10) and (11) which give an expression for the efficiency as

$$\left\{\frac{|P^+|^2}{|P^+|^2+|P^-|^2}\right\}\frac{4\omega^2 DN(\omega)}{\{[R+B-\omega^2(I+M(\omega))]^2+\omega^2[D+N(\omega)]^2\}}, \tag{12}$$

in which the first term represents the ultimate limit due to hydrodynamic effects, and the second term is a measure of the dynamic tuning available.

In § 4 it was shown that with suitable choice of D and R the second term can be maximized to unity. This implies that the geometry and motion of an efficient wave-power device must be conceived such that when force to oscillate in the particular chosen mode, waves are generated in one preferred direction. As stated this does not imply that there is a unique shape and motion for such structures. Figure 3 presents two apparently quite different configurations possessing this property while constrained to a rolling motion. Moreover, the added mass and

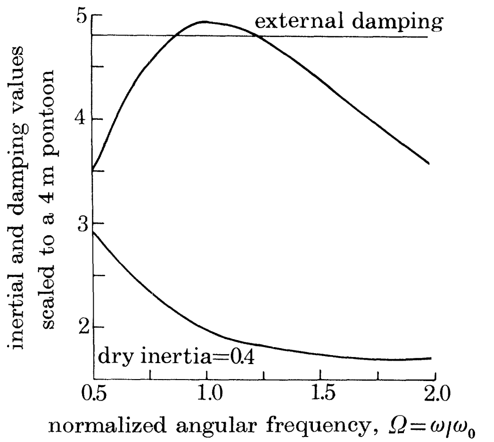

Figure 6.

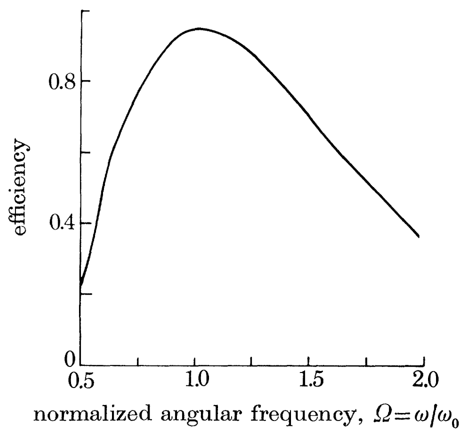

Figure 7.

Figure 6. The added inertia and damping curves for a semi-elliptical pontoon pair. Curve 1 is the added damping of a 4 m (ω_0 = 3.4 rad s^{-1}) semi-elliptical pontoon pair where one section rotates relative to an equal length, fixed section. The units of damping, for a unit width of structure are kN m rad^{-1} s m^{-1} and curve 2 is similarly the added inertia curve in t m^2 m^{-1}. Superimposed is the design value of external damping used in figure 7, but the dry inertia is off the scale since its value is 0.4 t m^2 m^{-1}.

Figure 7. The calculated efficiency of a pontoon pair. This shows the efficiency of the pontoon pair derived from figure 6, with the external values as shown in the same figure.

damping of the two equal pontoons has been calculated (figure 6) by assuming that the complete structure is semi-elliptical with a major : minor-axis ratio of 10; and the efficiency of the system is plotted over a range of frequencies, in figure 7, for a system in which only one pontoon is allowed to rotate relative to the other which is fixed horizontally in space. The configuration was 4 m long in total and the moving section had a dry inertia of 0.4 t m^2 m^{-1}, a damping constant of 4.8 kN m rad^{-1} s m^{-1} and a centre of gravity which was on the horizontal through the hinge. The values of the inertia and damping constant are presented in figure 6 for comparison with the equivalent hydrodynamic quantities as for the duck. It is interesting to note that the curves for the Salter device and pontoon system look almost identical. To explain this effect, equation (12) must be examined in more detail.

5.2. The hydrodynamic effects

If a geometry, which, constrained to move in some specified motion, is designed such that

$$|P^+|^2/|P^+|^2+|P^-|^2,$$

is close to unity over a reasonable frequency range, then the performance characteristics are dominated by the tuning term of equation (12). Clearly the best performance will be obtained at the resonant frequency where the mass and spring terms cancel, reducing the expression to

$$4DN(\omega)/[D+N(\omega)]^2. \tag{13}$$

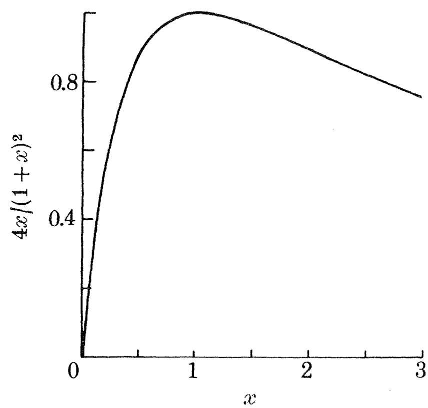

FIGURE 8. The variation of $4x/(1+x)^2$ against x.

This is plotted as a function of $D/N(\omega)$ in figure 8 and it is easily seen that the maximum is 1, at $D/N(\omega) = 1$, with a sharp fall off for $D/N(\omega) < 1$ and a relatively slow decay for $D/N(\omega) > 1$. Since the damping factor D can only be tuned by a given constant amount, this implies that it is better to set it too high rather than too low. For frequencies around resonance,

$$R+B-\omega^2(I+M(\omega)) \leqslant \omega[D+N(\omega)],$$

and equation (13) remains a close approximation to the second term of equation (12).

It would therefore appear that the form of the added damping, $N(\omega)$, is very important in determining the bandwidth of any wave-power structure. Physically, $N(\omega)$ is always positive and vanishes for both low and high frequencies, and therefore by Rolle's theorem it must have a maximum value at some frequency, ω_0, say. If the results from equation (13) are coupled with this fact then a sensible design would be to choose an external damping close to the maximum value and adjust the spring and mass terms so that the body resonates at ω_0. This is exactly what Salter has achieved experimentally, (figure 4) and has been used to determine the optimal characteristics of the pontoon system considered. This procedure led to a 'natural' frequency of ω_0 and all frequencies have been normalized to this, for each particular device, producing a non-dimensional frequency Ω, against which all of the response characteristics are plotted.

By using the above method for fixing the external damping and inertia of the device the efficiency will be a broad or narrow-band response accordingly as the added damping characteristic is broad or narrow respectively. Figures 4 and 6 demonstrate that the duck and pontoon system both have similarly shaped added-damping curves and therefore almost identical efficiencies over the same, normalized frequency range. To convert the frequencies to their actual values a device dimension must be selected and all quantities scaled accordingly. This gives the remarkable result that, subject to the constraint of one mode of motion, for any Salter duck there is an equivalent pontoon system and their differences lie only in the ease of engineering design, cost and feasibility, together with liability to practical problems such as slamming or capsizing, and not in their performance characteristics.

5.3. *Reactive loading*

The previous section has discussed the performance of a system in the absence of external springs and with an applied damping, proportional to velocity. In this situation the performance was dominated by the damping terms since the inertial and spring terms cancelled for frequencies at or near the resonant point. They do, however, degrade the performance slightly when the device is working off resonance since their effect is to increase the value of the denominator. To refine and increase the performance either side of resonance, reactive loading can be applied which acts as negative springs and inertias, and the first term of the denominator in equation (12) would be

$$\{B - R - \omega^2(I - I' + M(\omega))\},$$

where R and I' are the magnitudes of the negative spring and inertia respectively. If R and I' are chosen to minimize this expression over a required frequency range, then the device will work very close to resonance at all frequencies over this range. Results are given in figure 9 for the Salter duck and compared against his experimental points. The appropriately scaled experimental values for a 2 m diameter duck were similar to those specified in § 4 (figure 5) except that the damping constant was $5.5\,\mathrm{kN\,m\,rad^{-1}\,s\,m^{-1}}$, negative inertia, I', was $1.1\,\mathrm{t\,m^2\,m^{-1}}$ and the negative spring, R, was $9.6\,\mathrm{kN\,m\,rad^{-1}\,m^{-1}}$. This can be dramatically improved upon if a damping constant of $6.7\,\mathrm{kN\,m\,rad^{-1}\,s\,m^{-1}}$, a negative inertia of $5.4\,\mathrm{t\,m^2\,m^{-1}}$ and a negative spring of $19\,\mathrm{kN\,m\,rad^{-1}}$ are used but the practical difficulties in achieving a loading characteristic such as this would be formidable. It is interesting to note that since the curves of added inertia, the $M(\omega)$ in figures 4 and 6, are similar for both the duck and pontoon, this refinement is equally valid for both systems.

The only major difference for these two systems is in the relative values of their external inertias. With the pontoon the value is lower by about a factor of 4 or 5 than the added inertia (figure 6) whereas with the duck the opposite is true. This means that the pontoon is very insensitive to the external inertia, a fact that was discovered experimentally in the C.E.G.B. experiments (Count *et al.* 1976), but the duck performance varies significantly with the external inertia and this is demonstrated in figure 10 where efficiency curves for dry inertias of 2, 3, 4.5 and 6 t m^2 m^{-1} are plotted. This could be an important fact in the engineering design although such effects must be examined more closely when the structure is not necessarily constrained to one mode of motion, but is free to move in all degrees of freedom.

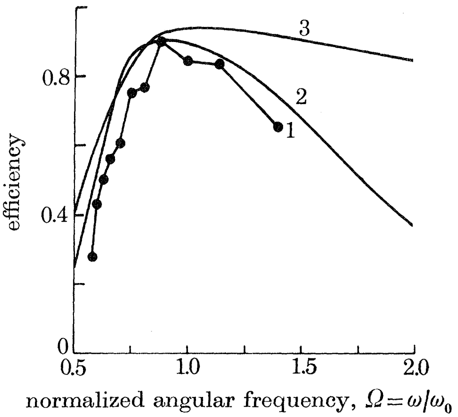

FIGURE 9.

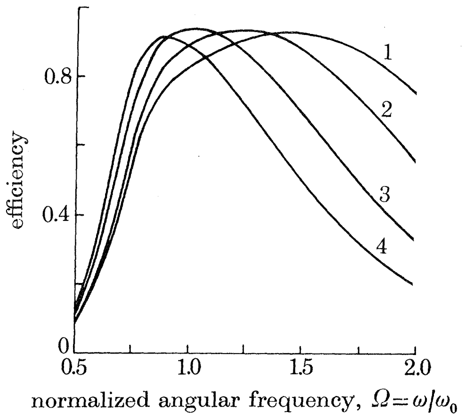

FIGURE 10.

FIGURE 9. The efficiency of a Salter duck with reactive loading. Curve 1 shows Salter's experimental results (September 1976), appropriately scaled to a 2 m diameter duck ($\omega_0 = 2$ rad s^{-1}). Curves 2 and 3 are theoretical computations with the specified parameters of § 5.3, curve 2 being the equivalent experimental values and curve 3 is an optimal performance derived entirely from theoretical considerations.

FIGURE 10. The variation of efficiency of a Salter duck with inertia. Curves 1, 2, 3 and 4 illustrates the efficiency of a 2 m duck ($\omega_0 = 2$ rad s^{-1}) with dry inertias of 2, 3, 4.5 and 6 t m^2 m^{-1} respectively. In all cases the damping is 6.7 kN m rad^{-1} s m^{-1} and the mass and centre of gravity are as specified in § 4.

5.4. *A multi-degree-of-freedom system*

In a practical system any wave-power structure will undergo a complicated set of motions which may or may not have an adverse effect on the performance. Recently suggestions have been made to construct large modules on which smaller structures are connected so that phase cancellation of the wave forces along the length of the large module will provide a stability that would only allow the smaller parts to move in a single degree of freedom. It is likely that the motion of such configurations will lie between a multi-degree-of-freedom system excited by a plane wave and that of a single-degree-of-freedom system.

The physics involved in both situations is basically the same although the optimization procedures are more complex. In one mode of motion, a design was chosen so that a balance was achieved where the waves induced by its motion cancelled in magnitude and phase with those scattered by the device due to the incident wave. This minimized the nett energy radiating away from the structure and hence maximized the power transfer from the fluid. The situation is exactly analogous if more degrees of freedom are introduced. Optimally, springs, inertias and damping must be applied so that the total wave induced by the various motions cancels with the scattered wave field. There will be limitations on the way that this can be achieved since energy may only be extracted in a limited number of modes and inertial values must not be so large that the structure will sink!

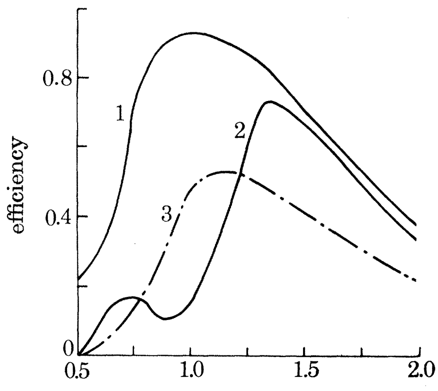

FIGURE 11. The change in performance when devices are allowed to move in all degrees of freedom. Curve 1 is the same as curve 2 of figure 5 and curves 2 and 3 represent non-optimized performances of a freely floating duck and pontoon pair with the external parameters specified in § 5.4.

This type of problem will need to be examined more closely since, although both the duck and pontoon systems are attractive in a single degree of freedom, the optimization may be difficult to achieve physically since they may simply not be the best geometries in this rather more general system. The first step in this direction must be to optimize given structures subject to real engineering constraints and it will be interesting to see if the duck and pontoon systems maintain their parity.

As an example of the caution that must be applied on the interpretation of existing results to full scale, experimental values of damping and inertia are used for the duck in a system with four degrees of freedom. The efficiencies calculated are compared (figure 11) with the fixed-centre results of figure 5, and the difference is significant although at present it must be noted that no optimization has been attempted. Results have been computed by using the same parameters as specified in § 4, for the extraction mode of motion (§ 2.3) together with a translational mass of 4.7 t m^{-1} and a total rolling inertia of 6.0 t $m^2 m^{-1}$. A similar curve is presented in figure 11 for the pontoon pair where, as above, the parameters specified in § 5.1 are used for the relative motion, together with a translational mass of 0.628 t m^{-1} and the total rolling inertia of 1.25 t $m^2 m^{-1}$.

6. Extension to full scale

In order to make a preliminary assessment of the overall picture of wave power it is necessary to extrapolate the ideas given in the previous sections to predict relevant performances of a full-scale device in a typical Atlantic sea. Any such extrapolation is bound to be difficult since it is by no means clear what is a typical sea state and the linearity assumptions made throughout are probably invalid during certain times, when the spectral components of the sea reinforce to produce

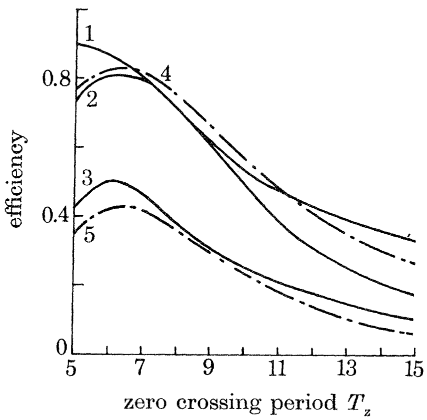

Figure 12. The sea efficiencies of the duck and pontoon pair. Curve 1 shows the sea performance of a 10 m diameter, fixed centre duck with optimum reactive loading, whereas curve 2 shows a 15 m diameter device with a fixed centre and simple loading. Curve 3 demonstrates the change in performance of the simple fixed centre, simple loading characteristics are applied to a freely floating system which is 30 m in diameter. Curves 4 and 5 illustrate the same behaviour of a pontoon system which is 80 m long constrained to one mode of motion and 120 m long while freely floating.

large forces and hence large displacements. However, it will be useful to attempt such extrapolations if only to examine the general trends and compare different types of systems subject to the same restricting assumptions.

Together with linearity it will be assumed that at full scale all wave-power devices will work in deep water where the fluid equations applicable to infinite depth may be applied. In this case the wavelength, and period, T, of the waves are related (Stoker 1957) by

$$\lambda = gT^2/2\pi,$$

and all dimensionless quantities are functions of the device dimension (L): wavelength ratio, which can be reinterpreted in terms of frequency and L through the simple relation given above.

Therefore in order to scale any efficiency curve $\eta(f)$, obtained by considering a device of dimension L_1, to be applicable to one of dimension L_2, it is only necessary to multiply the frequency range by $(L_1/L_2)^{\frac{1}{2}}$. To calculate the sea efficiency for the device of dimension L_2, the efficiency at each frequency must be multiplied by the power contained in the corresponding component of wave spectrum, summed over the entire frequency range and normalized to the total power of the spectrum. For the calculations in this Paper it will be assumed that the Pierson–Moskowitz (1964) energy spectrum is a reasonable representation of Atlantic wave climate. This has the form (Glendenning & Count 1976)

$$\epsilon(f) = Gf^{-5}\,\mathrm{e}^{-Qf^{-4}},$$

where

$$G = H_{\mathrm{s}}^2/4\pi T_{\mathrm{z}}^4, \quad Q = 1/\pi T_{\mathrm{z}}^4,$$

and H_{s} and T_{z} are the significant wave height and zero crossing period respectively, which are the usual published parameters defining wave climates (Draper 1966). To form a power spectrum from this it can be simply noted that the energy content of each component, $\epsilon(f)\,\mathrm{d}f$, propagates at a speed $g/4\pi f$, and the overall efficiency of the system is

$$\int_0^\infty \epsilon(f)\,\eta(f)\frac{\mathrm{d}f}{f}\bigg/\int_0^\infty \epsilon(f)\frac{\mathrm{d}f}{f},$$

where the denominator is a measure of the total power available. As expected, the performance is not dependent on the scale of the sea, as categorized by H_{z}, but is only a function of T_{z} which supposedly contains all the spectral information. Sea states typically lie in the region, $6\,s < T_{\mathrm{z}} < 12\,s$ and efficiencies are given in figure 12 for a number of different devices with appropriate dynamic loading.

7. Conclusions

The analysis presented has been applied to versions of the Salter device and a pontoon system which may not be the final or even currently preferred configurations, but serve to demonstrate the main findings. The Salter duck appears to be equivalent to a two-pontoon system, semi-elliptical in cross section and hinged at its centre, constrained to move only in the mode in which energy is absorbed. Both structures are designed such that, when forced to move in their absorbing mode, they generate waves in one preferred direction, the pontoon design relying on the use of a shallow horizontal breakwater in the rear of the moving structure, whereas Salter has used a shorter deeper structure which looks almost cylindrical.

When optimally loaded with a simple damping proportional to velocity applied externally, the performance of each system looks almost identical, with the pontoon being slightly better at the low-frequency end. This is because the added damping decays more slowly at low frequencies than that of the duck (figures 4 and 6) and therefore the external damping is closer to optimum. There is a fine adjustment owing to the fact that the added inertia of the duck increases more rapidly than that of the pontoon at this low-frequency end and there is a greater tendency for cancellation of the inertia and spring terms for the duck, but this effect is small.

More general systems where the structures are free to move in many degrees of freedom must be examined. The reasons for this are apparent from calculated sea efficiencies of the devices. For the duck with a fixed centre and reacting loading, a 10 m diameter device may achieve an excellent performance over typical sea states, but if simple (velocity-proportional) loading is used the diameter must increase to 15 m to give a comparable performance. If, however, the structure is free to move in other modes of motion with the same simple loading, then to achieve similar characteristics the structure has to be 30 m in diameter.

Similar effects are found for the pontoon, for, whereas an 80 m long two-pontoon system with a fixed rear section and simple loading will match to the sea conditions very well, a freely floating system with the same loading would need to be 120 m long to achieve a reasonable performance.

It should be noted that in the freely floating systems no attempt has been made to optimize the performance, and the loading characteristics of the one-degree-of-freedom system have simply been carried over to the more complex situation. This does, however, serve to demonstrate that there are enormous differences in the ultimate design of any wave-power structure according to the method of power extraction and the degree of optimization that can be achieved in the freely moving situation. Until more detailed parametric studies have been carried out, no definite design parameters can be given. However, since it is unlikely that reactive loading will be achieved in a practical situation, then for the wave spectrum assumed here, the duck would be between 15 m and 30 m in diameter and the entire pontoon system would be between 80 and 120 m in length at full scale.

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