Hydrodynamic Optimization of the Active surface of a Heaving Point Absorber WEC

M. Alves and A. Sarmento

Wave Energy Centre-WavEC
Av. Manuel da Maia, 36, r/c Dto, 1000-201, Lisbon, Portugal
E-mail: marco@wave-energy-centre.org

Abstract

This paper reports an optimization study to characterize the active surface (surface responsible for the radiation capabilities of the device, i.e., the generation of waves) of an axisymmetric heaving point absorber wave energy converter (*WEC*). For this purpose several hypothesis were considered. Firstly, it was assumed the condition for maximum energy absorption with an axisymmetric point absorber. Additionally, a condition to maximize the device radiation capabilities, which consists of the definition of an active surface depth close to the amplitude of the vertical displacement, was also assumed. Following, a final assumption based on a neglectable near body radiation potential at large distances from the wave generator was also taken into account. Finally, based on all the previous assumptions, it was possible to characterize the *WEC* active surface through an expression that relates the non-dimensional active surface radius and depth with the relative excursion of the device. This expression allows to understand promptly the size of an axisymmetric heaving point absorber required to optimize the energy absorption according to the most relevant local wave climate.

To support some hypothesis and simplifications (based on the physical phenomenon) taken into account in this work, a *BEM* numerical code (*AQUADYN*) was applied for additional confirmation [1]. The code is a three-dimensional radiation-diffraction panel model based on the classic linear water wave theory and potential flow.

Keywords: Active surface, heave, excitation, Froude-Krylov, *Kramers-Kronig, Haskind*, diffraction, radiation, added mass, damping.

Nomenclature

A	=wave amplitude
a	=active surface radius
d	=active surface diameter
D	=hydrodynamic damping coefficient
F	=force
g	=gravity acceleration
h	=water depth
L	=cylinder length
M	=added mass coefficient
m	=device mass or added mass component
n	=normal direction
P	=absorbed power
S	=surface of floatation
u	=vertical velocity
V	=volume
Z	=impedance
z	=active surface draft
β	=wave source distance coefficient
Γ	=mean body wet surface
δ	=additional active surface depth
η	=free surface elevation
θ	=radiated wave direction
λ	=wave length
ξ	=heave displacement
ρ	=water density
ϕ	=potential
φ	=potential for unitary velocity
ω	=angular frequency
Ω	=envelope submerged volume surface

Subscripts

abs	=absorbed
opt	=optimum
d	=diffraction
ext	=excitation
nf	=near field
ff	=far field
FK	=Froude Krylov
r	=radiation
z	=vertical direction
ω	=frequency dependence
0	=incidence
∞	=infinity

Superscripts

^	=complex amplitude
$*$	=dimensionless
$-$	=mean value

Introduction

One of the most important issues regarding the design of point absorbers is the area of the floating surface

(=active surface) and the depth of the active surface. Both these parameters restrain the overall dimension of the device and, in some way, the type of the *PTO*, too. However, the geometrical optimization of a point absorber should not be seen only from a hydrodynamic perspective, but also from the economic point of view, as the interception of both aspects might provide the most suitable solution. Nevertheless, this paper only reports a methodology to define, from the hydrodynamic standpoint, the active surface area and depth of an axisymmetric heaving point absorber *WEC*, and thus its global dimension. The additional economic analysis is a topic for a future work.

To characterize the active surface of a point absorber *WEC* (in terms of area and depth) for maximum power absorption, some assumptions are required. Firstly, the depth of the active surface must be close to the vertical displacement amplitude of the device, to ensure good radiations capabilities [2, 3]. Secondly, the small body approximation and a simplification based on a negligible near body radiation potential (at large distances from the wave generator) should also be introduced [4]. Following, according to the previous assumptions, it seems to be possible to relate the depth and size of the active surface with the device vertical excursion. For this purpose the excitation force is described as a function of the added mass coefficient. Furthermore, the *Haskind* and *Kramers-Kronig* relations allow to redefine the excitation force as a function of the infinite added mass coefficient. On the other hand, using the well known condition for maximum power absorption by a point absorber, and neglecting the near body radiation potential, the excitation force can also be described through the device geometrical parameters (area and depth of the active surface) [4]. Therefore, from the both excitation force descriptions, results an expression that relates the geometrical parameters with the infinite added mass coefficient. Moreover, it also seems to be possible assessing the infinite added mass coefficient through the diameter and depth of the active surface (inside of the expectable range of active surface depths). Then, finally, an expression which relates the non-dimensional active surface radius and depth with the relative excursion is achieved. This expression allows to understand promptly the size of an axisymmetric heaving point absorber required to optimize the energy absorption for the main frequency of the local wave spectrum.

Active Surface Characterization

To improve the radiation capabilities of a *WEC* its active surface might be located as close as possible to the free surface, as *fig. 1* shows [2]. According to this principle the depth of the active surface should be equal to the modulus of the difference between the complex amplitude of the vertical displacement and the amplitude of the incident wave, i.e.,

$$z \cong \left|\hat{\xi} - \hat{\eta}\right| = \left|\hat{\xi} - A\right| . \tag{1}$$

In *Eq. 1*, it was assumed a constant hydrostatic coefficient during the device vertical, in order to avoid significant non-linear effects (slamming).

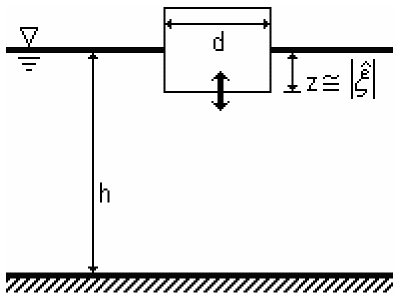

Figure 1. Schematic representation of a geometry which maximizes the radiation capabilities of a partially submerged heaving *WEC*.

Since the optimal energy absorption is achieved when the excitation force is in phase with the device velocity, and thus, in phase quadrature with the displacement, *Eq. 1* might be replaced by

$$z \cong \left|\hat{\xi}\right| . \tag{2}$$

Decomposition of the Excitation Force

In this subsection, the excitation force, acting on the active surface, which maximizes the *WEC* radiation capabilities, will be evaluated. Therefore, taking into account the *Green*'s theorem, and after some mathematical manipulation [4], the excitation force on a floating body can be obtained through,

$$\hat{F}_{ext} = i\omega\rho \int_{\Gamma} \left(\hat{\phi}_0 n_z\right) d\Gamma - i\omega\rho \int_{\Gamma} \left(\varphi_r \frac{\partial \hat{\phi}_0}{\partial n_z}\right) d\Gamma , \tag{3}$$

in which the first term, resulting from the incident wave pressure integration over the wet body surface, is called *Froude-Krylov* force. The second one, named diffraction force, results from the integration, over the same surface, of the pressure inflicted by the water displaced due to the body motions.

Diffraction Force

According to *Eq. 3*, it is possible to relate the diffraction force and the hydrodynamic impedance, Z, through

$$Z = \frac{\hat{F}_r}{\hat{u}} = -i\omega\rho \int_{\Gamma} \varphi_r n_z d\Gamma , \tag{4}$$

where $\hat{F}_r$ is the complex amplitude of the radiation force given by

$$\hat{F}_r = -i\omega\rho \int_{\Gamma} \hat{\phi}_r n_z d\Gamma = -i\omega\rho \int_{\Gamma} \varphi_r \hat{u} n_z d\Gamma , \tag{5}$$

and $\hat{u}$ is the complex amplitude of the vertical body velocity. Thus, for the configuration which maximizes the radiation capability of the device (*see fig. 1*) the diffraction force assumes the form

$$\hat{F}_d = \hat{u}_{0_z} e^{-k|z|} Z. \qquad (6)$$

Usually, the hydrodynamic impedance is decomposed into a reactive term, related to the added mass coefficient, *M*, and a dissipative term, related to the hydrodynamic damping coefficient, *D*, i.e.,

$$Z = i\omega M + D. \qquad (7)$$

Introducing *Eq. 7* into *Eq. 6*, the complex amplitude of the diffraction force becomes

$$\hat{F}_d = -\rho S|z|A\omega^2 e^{-k|z|}\left(M^* - iD^*\right), \qquad (8)$$

where the hydrodynamic coefficients of added mass and damping appear in the dimensionless form. Both are given respectively by

$$M^* = \frac{M}{\rho S|z|} \qquad (9)$$

and

$$D^* = \frac{D}{\omega\rho S|z|}. \qquad (10)$$

Finally, the diffraction force can be rewritten by

$$\hat{F}_d = -\rho S|z|A\omega^2 \left|\chi^*\right| e^{i\beta} e^{-k|z|}, \qquad (11)$$

in which β represents the phase shift between the incident wave and the excitation force acting on the active surface and $\chi^* = \left|\chi^*\right| e^{i\beta} = M^* - iD^*$.

If the floater radius, *a*, is much smaller than the wave length $(ka << 1)$, the phase shift β is small and, consequently, $\left|\chi^*\right| \cong M^*$. Therefore, in this case, as the impedance is largely dominated by the reactive term, i.e., $|\omega M| >> D$, the simplified *Eq. 11* assumes the form

$$\hat{F}_d \cong -\rho S|z|\omega^2 A e^{-k|z|} M^*. \qquad (12)$$

Fig. 2 shows, for several relative depths of the active surface, $|z|/a$, the non-dimensionalised added mass and damping coefficients, as well as the modulus $\left|\chi^*\right|$ *vs.* the dimensionless radius *ka* [4]. It is possible to verify that the simplification $\left|\chi^*\right| \cong M^*$, taken into account in *Eq. 12*, is a valid approximation as no significant differences between $\left|\chi^*\right|$ and M^* are visible. This means that, as referred above, in a point absorber the impedance reactive term dominates over the dissipative or resistive term.

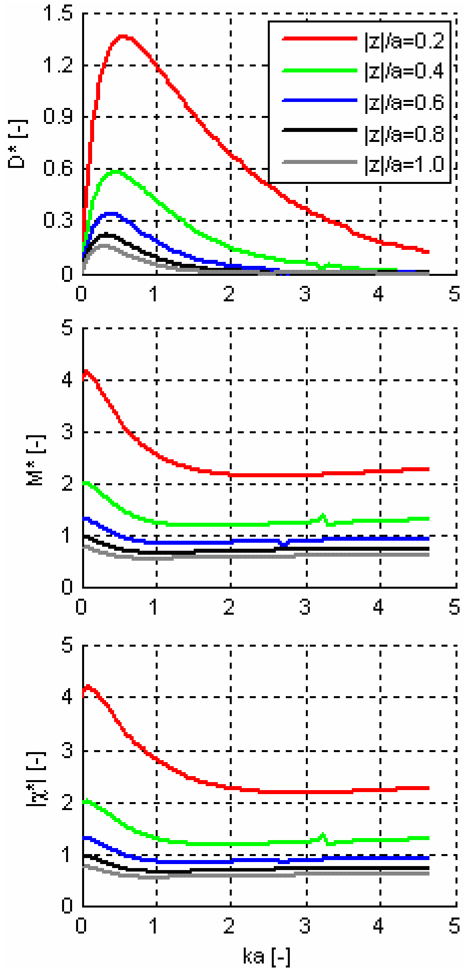

Figure 2. Coefficients D^*, M^* and $|\chi^*|$ vs. *ka*, for several non-dimensionalised depths $|z|/a$.

Even if the simplification introduced in *Eq. 12* is, generally, a valid assumption, it is important to have in mind that, in the case of a tiny depth of the active surface, the simplification assumed could introduce errors superiors to *10%* in the diffraction force, as *fig. 3* shows for $|z|/a = 0.2$ (red curve). However, it is not expectable a point absorber *WEC* designed for $|z|/a \leq 0.2$, as it implies too low velocities (situation which may involve negative added mass coefficients). Thus, the assumption considered is reinforced.

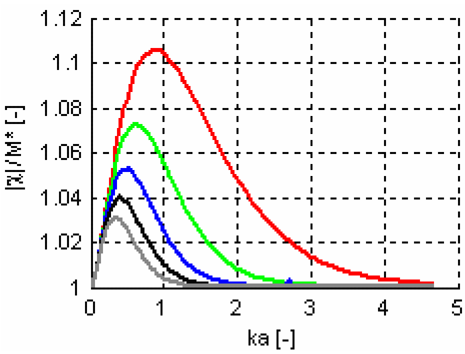

Figure 3. Evaluation of the magnitude of the reactive term in the total hydrodynamic impedance, through the variation of the parameter $|\chi|/M^*$ *vs. ka*.

In the case of a submerged *WEC*, similar to the *AWS* (*Archimedes Wave Swing*), with the active surface up side down, as *fig. 4* illustrates, *Eq. 12* is still applicable. However, in this situation, the device mass, *m*, should be understood as the mass between the top of the *WEC* and the undisturbed water free surface.

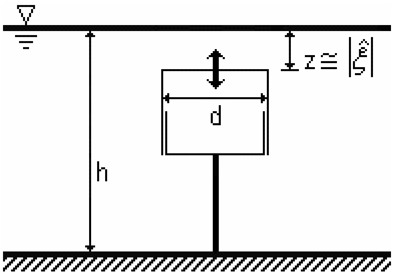

Figure 4. Schematic representation of the geometric condition which maximizes the radiation capabilities of a *WEC* with a work principle similar to the *AWS*.

Froude-Krylov Force

The *Froude-Krylov* force results from the integration of the dynamic pressure induced by the incident wave over the body wet surface. According to *Eq. 3* the complex amplitude of the *Froude-Krylov* force is given by

$$\hat{F}_{FK} = i\omega\rho \int_{\Gamma} \left(\hat{\phi}_0 n_z\right) d\Gamma. \tag{13}$$

Through the *Gauss* theorem, *Eq. 13* becomes

$$\begin{aligned}\hat{F}_{FK} &= i\omega\rho \int_{\Omega} \hat{\phi}_0 n_z d\Omega - i\omega\rho \int_{S} \hat{\phi}_0 n_z dS = \\ &i\omega\rho \int_{V} \nabla \hat{\phi}_0 n_z d\Omega + \rho g \int_{S} \hat{\eta}_0 n_z dS.\end{aligned} \tag{14}$$

where *S* is the floating area and Ω the envelope surface of the submerged volume *V*, i.e., the sum of *S* and Γ (mean body wet surface). Introducing the incident wave potential into *Eq. 14*, the complex amplitude of the *Froude-Krylov* force, taking into account the condition which improves the device radiation capabilities (*see fig 1*), becomes

$$\hat{F}_{FK} = i\omega\rho \hat{u}_{0_z} \frac{S|z|}{k}\left(1 - e^{-k|z|}\right) + \rho g S|A|. \tag{15}$$

Considering only the first and second terms of the series decomposition of the exponential function, $e^{-k|z|}$ (which is a valid assumption as $k|z|<1$), the *Froude-Krylov* complex amplitude turns into

$$\hat{F}_{FK} = A\rho g S - A\omega^2 m e^{-k|z|}, \tag{16}$$

in which the first term represents the buoyancy variation due to the incident wave and the second one results from the dynamic pressure action on the active surface due to the incident wave. The decay of the dynamic pressure for progressive increment of the active surface depth, $|z|$, is translated by $e^{-k|z|}$.

For a submerged body with the active surface up side down, as shown in *fig 4*, the *Froude-Krylov* complex amplitude, bearing in mind the optimization of the device radiation capabilities, is given by

$$\hat{F}_{FK} = i\omega\rho \int_{S} \hat{\phi}_0 n_z dS = -\rho g A S e^{-k|z|}, \tag{17}$$

where *S* is the device active surface. The series decomposition of the exponential function allows to rewrite the complex amplitude of the *Froude-Krylov* force by

$$\hat{F}_{FK} = -A\rho g S + A\omega^2 m e^{-k|z|}, \tag{18}$$

in which, *m* represents the mass of the water column above the top of the device. The first term in *Eq. 18* corresponds to the weight variation of the water column above the device top and the second one results from the dynamic pressure action on the active surface, induced by the incident wave. As in the case of a partially submerged buoy, the function $e^{-k|z|}$ gives the decay of the dynamic pressure for progressive increments of the active surface depth.

Entire Excitation Force

The resulting excitation force acting on a body is obtained through the addition of both the *Froude-Krylov* and diffraction components. Then, according to *Eq. 16* and *11*, the excitation force on a partially submerged body, as *fig. 2* represents, can be written by

$$\begin{aligned}\hat{F}_{ext} &= \hat{F}_{FK} + \hat{F}_d = \\ &A\left(\rho g S - \omega^2 m e^{-k|z|} - \omega^2 M e^{-k|z|}\right).\end{aligned} \tag{19}$$

Introducing in *Eq. 19* the dimensionless added mass coefficient M^*, given by *Eq. 9*, the complex amplitude of the excitation force becomes

$$\hat{F}_{ext} = A\rho g S\left(1 - \left(1 + M^*\right)k|z|e^{-k|z|}\right), \tag{20}$$

where, due to the simplicity of the geometry under analysis, the displaced water mass, *m*, is given by

$$m = \rho S|z|. \tag{21}$$

Finally, considering, once more, only the two first terms of the series decomposition of the exponential function, $e^{-k|z|}$, the complex amplitude of the excitation force, given by *Eq. 20*, turns into

$$\begin{aligned}\hat{F}_{ext} &\cong A\rho g S\left(1 + k|z|M^*\right)e^{-k|z|} \\ &\cong A\rho g S e^{-k|z|\left(1+M^*\right)}.\end{aligned} \tag{22}$$

In the case of a submerged body, as represented schematically by *fig. 4*, the excitation force is, similarly, obtained by adding the *Froude-Krylov*, *Eq.*

18, and diffraction, *Eq. 11*, components. Thus, the resulting excitation force is given by

$$\hat{F}_{ext} = \hat{F}_{FK} + \hat{F}_d = A\left(-\rho g S + \omega^2 m e^{-k|z|} - \omega^2 M e^{-k|z|}\right). \tag{23}$$

Introducing in *Eq. 23* the dimensionless added mass coefficient M^*, given by *Eq. 9*, and taking, as before, only the two first terms of the series decomposition of the exponential function, the complex amplitude of the excitation force becomes

$$\hat{F}_{ext} \cong A\rho g S\left(1 - k|z|M^*\right)e^{-k|z|} \cong A\rho g S e^{-k|z|\left(1-M^*\right)}. \tag{24}$$

As referred before, *m*, represents the mass of the water column above the top of the device, which is also given by *Eq. 21*.

Eq. 22 and *24* allow to verify that the excitation force acting on the submerged body is higher than the force over the emerged body with equal area and depth of the active surface. This happens because the diffraction and *Froude-Krylov* components of the excitation force are in phase instead of phase opposition, as it happens in the emerged body case. Therefore, according to the *Haskind* relation, it is expectable to notice higher radiation capabilities in a submerged body, assuming similar circumstances (i.e. same bathymetry, wave climate as well as the same area and depth of the active surface). In other words the result seems to indicate that, under these similar circumstances, to get the same level of wave energy absorption with both devices, one submerged and the other partially emerged, it will be necessary to increase the velocity or, instead, the diameter of the last one.

The excitation force acting on both types of heaving axisymmetric point absorbers under consideration can be even more simplified if we analyse the added mass coefficient behaviour. Furthermore, through *Eq. 22* or *24* and the *Haskind* relation, the damping coefficient could be written by

$$D = \frac{1}{2}\omega k \rho g S^2\left(1 \pm k|z|M^*\right)^2 e^{-2k|z|}, \tag{25}$$

in which the addition or subtraction of the non-dimensionalised added mass coefficient depends on the device type, namely, if it is totally or partially submerged, as discussed above. Following, according to *Eq. 25*, the *Kramers-Kronig* relation allows to write the frequency dependent part of the non-dimensionalised added mass coefficient by

$$M^* = m^*_\omega = m^*_\infty - k^2 S D^*\left(k|z|m^*_\infty \pm 1\right)e^{-2k|z|} - \frac{\omega k S}{\pi|z|}e^{-2k|z|\left(1 \pm m^*_\infty\right)}\int_0^\infty \frac{1}{\omega^2 - y^2}dy. \tag{26}$$

The integral in the third term of *Eq. 26*, resulting from the *Kramers-Kronig* relations, should be seen as a *Cauchy* principal value. Then, as this principal value integral is zero, the frequency dependent component of the non-dimensionalised added mass coefficient assumes the simplified form

$$m^*_\omega = m^*_\infty - k^2 S D^*\left(k|z|m^*_\infty \pm 1\right)e^{-2k|z|}. \tag{27}$$

The last result seems to indicate that the higher radiation capabilities observed for the submerged body relatively to the partially emerged body under identical circumstances (similar bathymetry, wave climate and the same area and depth of the active surface) is a less important aspect because inside the most interesting range of active surface depths $k|z|m^*_\infty << 1$. Furthermore, as in the optimal absorption conditions the hydrodynamic damping coefficient and the excitation force are related by

$$2D|\hat{u}|_{opt} = |F_{ext}| \Leftrightarrow 2\omega|z|D = |F_{ext}|, \tag{28}$$

or, in a non-dimensionalised way, through

$$2k|z|^2 D^* = A\left|F^*_{ext}\right|, \tag{29}$$

the simplified non-dimensional excitation force amplitude, for any kind of heaving *WEC*, can be described by means of

$$\left|\hat{F}^*_{ext}\right| = \frac{\left(1 - k|z|m^*_\infty\right)}{e^{3k|z|} - \frac{k^2 S A}{2|z|}\left(k|z|m^*_\infty \pm 1\right)}e^{2k|z|}. \tag{30}$$

Radiation Potential

The radiation potential, $\hat{\phi}_r$, may be decomposed in two parts which translate, respectively, the radiation phenomenon near the body, $\hat{\phi}_{r_{nf}}$, and faraway from the body, $\hat{\phi}_{r_{ff}}$, i.e.,

$$\hat{\phi}_r = \hat{\phi}_{r_{nf}} + \hat{\phi}_{r_{ff}}. \tag{31}$$

The radiation potential in the body vicinity is linked with the reactive energy which flows between the body and the adjacent water volume. So, it is related to the evanescent agitation and the added mass effect. Besides, the radiation potential in the remote area of the body translates the hydrodynamic damping. Thus, it reflects the energy transported by the waves generated, which, progressively, move away from the body. Hence, for distances higher than the wave length, i.e., faraway from the body (wave source), it is expected to observe a neglectable near body radiation potential, $\hat{\phi}_{r_{nf}}$. Therefore, it is possible to assume, under these circumstances, a radiation potential completely described by the far field component. So, it becomes

$$\hat{\phi}_r \approx \hat{\phi}_{r_{ff}} = C(\theta)e(kz)(kr)^{-1/2}e^{-ikr}, \tag{32}$$

where r is the distance between any point at the free surface and the body vertical axis and C is the far field coefficient, which, for an axisymmetric device, is independent of the radiated wave direction, θ, as this wave is circular. This coefficient translates the progressive reduction of the wave amplitude as it moves away from the source point. For large distances from the origin, i.e. $kr \geq 2\pi$, where the near body potential, $\hat{\phi}_{r_{nf}}$, is insignificant, the variation of $(kr)^{-1/2}$ with the wave front is a less important factor. So, the far field coefficient, C, can be described, as for a plan wave, by the relation

$$C = \frac{|A_r|g}{\omega}(kr)^{1/2}, \tag{33}$$

in which A_r is the radiated wave amplitude. Following, the energy transported per unit of radiated wave front is obtained through

$$P_r = \int_0^{2\pi} J(r,\theta)r d\theta, \tag{34}$$

where J represents the energy radiated per unit of wave front, resulting from the depth integration of the time average energy propagated per unit of vertical area in the longitudinal direction. Then, it is given by

$$J(r,\theta) = \frac{\rho g^2 D(kh)}{4\omega}|A_r|^2, \tag{35}$$

where $D(kh)$ is the depth function. In this analysis it is assumed a high sea bottom depth, i.e., $kh >> 1$, which means that $D(kh) \approx 1$. Therefore, introducing *Eq. 35* into *Eq. 34*, the radiated power assumes the form

$$P_r = \frac{\omega\rho}{4k}2\pi C^2. \tag{36}$$

Besides, knowing the damping coefficient, D, the power radiated due to the heave motion can be obtained through

$$P_r = \frac{1}{2}\omega^2 D|z|^2. \tag{37}$$

Thus, through *Eq. 36* and *37*, it is possible to write the far field coefficient by

$$C = \left(\frac{\omega k D}{\rho\pi}\right)^{1/2}|z|. \tag{38}$$

Introducing *Eq. 33* in *Eq. 38* and considering the *Haskind* relation, the amplitude of the radiated wave, A_r, at any distance, r, from the source point is given by

$$|A_r| = k|z|\frac{|\hat{F}_{ext}|}{\rho g A}\left(\frac{k}{2\pi r}\right)^{1/2} = k|z|\frac{|\hat{F}_{ext}|}{\rho g A}\beta, \tag{39}$$

where A is the incident wave amplitude and β a parameter dependent on the wave frequency and the distance r. In the case of a distance from the wave source, equal to the wave length, i.e., $r \approx \lambda$, (where the near body radiation potential is insignificant) the coefficient β assumes the form

$$\beta = \left[\frac{k}{2\pi(\lambda + a)}\right]^{1/2} \cong \frac{k}{2\pi} = \frac{1}{\lambda}, \tag{40}$$

in which, according to the point absorber condition, the radius of the active surface, a, was neglected comparing to the wave length.

It is important to relate the radiated wave amplitude with the incident wave amplitude as, generally, the last one is more convenient. For this purpose, the radiated power, given by *Eq. 36*, can be rewritten, according to *Eq. 33*, by

$$P_r = \frac{\rho g^2}{4\omega}2\pi r|A_r|^2, \tag{41}$$

where it was assumed, as before, that, according to the large distance from the radiated wave source $(r \geq \lambda)$, the circular wave can be considered plan. On the other hand, the optimum power radiated by a heaving axisymmetric *WEC* might be written by

$$P_{r_{opt}} = \frac{J}{k} = \frac{2\rho g^2 D(kh)}{8\omega k}|A|^2. \tag{42}$$

Equalizing *Eq. 42* and *40* it is possible to relate the optimal radiated wave amplitude, A_r, and the incident wave amplitude, A, through

$$|A_r| = \left(\frac{r}{\lambda}\right)^{-1/2}\frac{|A|}{2\pi}, \tag{43}$$

in which it was considered, as before, the deep water condition, $D(kh) \approx 1$. Assuming once more a distance between the radiated wave front and the source point equal to the wave length $(r = \lambda)$, both amplitudes (radiated and incident wave) can be related by the expression

$$|A_r| = \frac{1}{2\pi}|A|. \tag{44}$$

Therefore, introducing *Eq. 44* into *Eq. 39*, the non-dimensional excitation force can be described by

$$|\hat{F}^*_{ext}| = \frac{A}{|z|}\frac{1}{Sk^2} = \frac{A}{|z|}\frac{1}{\pi(ka)^2}. \tag{45}$$

Non-dimensional Active Surface Parameters

For the particular case of a partially submerged heaving *WEC*, the assessment of the infinite added mass coefficient, shown by *fig 5*, allows to verify that its behaviour can be well approximated by the displayed simple relation, within the most interesting range of the relative excursions $|z|/a$ (from *0.2* to *1.0*) from the *WEC*'s design perspective. Outside this range, the device vertical excursion is too low, and therefore not appropriate for energy absorption, or too high, and, consequently, unrealistic from the physical point of view.

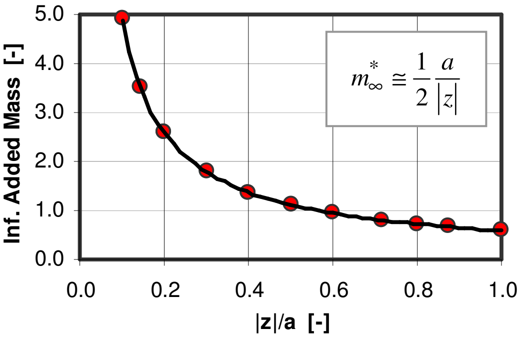

Figure 5. Non-dimensionalised infinite added mass, m_∞^*, vs. relative device excursion $|z|/a$ for geometries schematically represented by fig.1.

Introducing the approximate infinite added mass relation into *Eq. 30*, the non-dimensionalised excitation force becomes,

$$\left|\hat{F}_{ext}^*\right| = \frac{e^{2k|z|}\left(1-\frac{1}{2}ka\right)}{\frac{\pi}{2}\frac{A}{|z|}(ka)^2 + e^{3k|z|}}. \tag{46}$$

Introducing *Eq. 45* into *46*, the outcome is a relation between the non-dimensional radius, *ka*, and depth, *k*|*z*|, of the active surface with the relative amplitude of the vertical excursion, $|z|/|A|$, written by

$$\frac{A}{|z|} = \frac{(2-ka)}{\frac{A}{|z|} + \frac{2}{\pi(ka)^2}e^{3k|z|}} e^{-k|z|}. \tag{47}$$

Taking into account that for a point absorber $ka \ll 1$ and larger displacements are, usually, required, the previous equation can be simplified assuming the form

$$\frac{A}{|z|} \cong \pi(ka)^2 e^{-k|z|}. \tag{48}$$

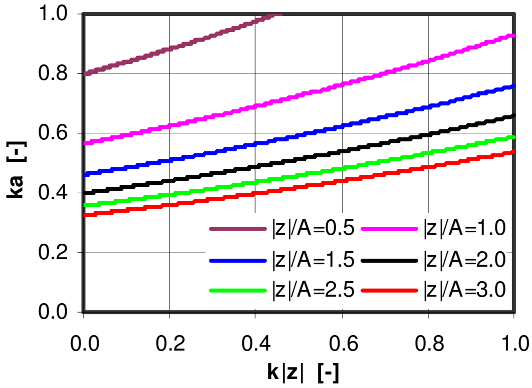

Figure 6. Non-dimensional relation between the optimal active surface radius, *ka*, and depth, *k*|*z*|, for several relative depths, |*z*|/*A*, for the geometries schematically represented in fig. 2 or 5.

According to the previous relation, *fig. 6* shows for several relative amplitude of the vertical displacement $|z|/|A|$, the non-dimensional radius, *ka*, versus the non-dimensional depth, *k*|*z*|, of the active surface. Therefore, it allows to identify the dimension of the active surface which maximizes the power absorption of an axisymmetric heaving point absorber. For this purpose, it is necessary to know the local wave climate and so the most relevant spectral wave frequency as well as the maximum required stroke of the device. Generally, the relative displacement amplitude, $|z|/|A|$, should be amid *1* and *3* as higher values will introduce considerable non-linear dissipative effects. Usually devices in which the *PTO* equipment consists of linear generator, which requires low forces and high displacements, the optimum dimension of the active surface is smaller than the one which optimizes the power absorption of a device equipped with hydraulic *PTO*. In this type of device the *PTO* system usually requires high forces and low displacements. Thus, a higher area of the active surface to optimize the energy extraction is also required. The base of this study is to ensure good radiation capabilities through tiny distances between the device top and the free surface. This means that *k*|*z*|<<*1*, and so, an additional simplification can be applied to *Eq. 48* transforming it into

$$\frac{A}{|z|} \cong \pi(ka)^2. \tag{49}$$

It is important to emphasize that the purpose of this study is to identify, approximately, the area of the active surface which maximizes the energy absorption of a heaving axisymmetric point absorber *WEC*. Therefore, it was assumed a pressure centre (dynamic and static) as close to the free surface as possible, keeping the hydrostatic coefficient constant to avoid non-linear effects. Then, the analysed *WEC* bottom corresponds to a flat base which minimizes the pressure centre, as required, but it is not the most suitable to reduce viscous effects. Even though, we believe that it is possible to obtain an equivalent flat bottom for different and more suitable shapes from the point of view of viscous reduction, if the same centre of the dynamic pressure is kept, as schematically represented by *fig. 7*. The methodology consists of performing a non-linear evaluation of several bottom shapes to identify the one which reduces the viscous dissipation and also minimizes the increment, δ, of the dynamic pressure centre.

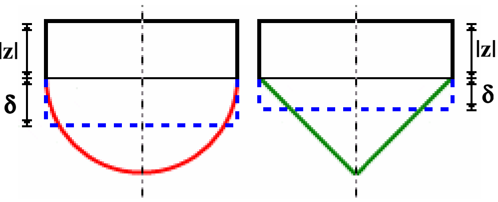

Figure 7. Potential shapes of the WEC bottoms for reduction of viscous effects (red or green) and respective flat bottoms at identical depth of the dynamic pressure centre (blue).

Then, *Eq. 48* can be slightly modified to take into account the additional depth of the dynamic pressure centre, δ, assuming the form

$$\frac{A}{|z|} \cong \pi(ka)^2 e^{-k|z+\delta|} . \qquad (50)$$

According to the described method it seems to be achievable a balance between the device radiation capabilities (and then its capacity to absorb energy), which is reduced with the depth increment of the dynamic pressure centre, and the related device viscous dissipation.

Conclusions

The objective of this work was the characterization of the active surface of an axisymmetric heaving point absorber. Accordingly, the main conclusion achieved was that it seems to be possible defining, for maximum energy extraction, a relation between the non-dimensionalised radius and depth of the active surface with the relative excursion of the device. Then, the size required to optimize the energy absorption for the main frequency of the wave spectrum can be easily recognized.

Even if the considered geometric bottom simplification (flat device bottom) does not minimize the viscous dissipation, it seems to be possible to expand the analysis to more suitable bottom shapes. For this purpose, a non-linear evaluation of several bottom shapes should be performed to identify the one which, reducing the viscous dissipation, does not increase significantly the depth of the dynamic pressure centre. Therefore, it was presented the modified expression that relates the non-dimensionalised radius and depth of the active surface with the relative device excursion for the prescribed additional depth of the dynamic pressure centre. This additional depth is imposed by the most suitable bottom defined to minimize the viscous dissipation.

Finally, even if this analysis was focused on axisymmetric point absorber *WEC*, an extension to different devices with different energy extracting modes, considering a similar methodology, could be performed.

References

[1] Aquadyn User manual. Laboratoire de Mecanique des Fluides, École Centrale de Nantes, 1987.

[2] M. Alves, H. Traylor and A. Sarmento. “Hydrodynamic Optimization of a Wave Energy Converter Using a Heave Motion Buoy”. Proc. 7th European Wave and Tidal Energy Conference, Porto, Portugal, 2007.

[3] P. Ricci, M. Alves. “Optimisation of the Geometry of Wave Energy Converters”. Proc. of the International Conference on Ocean Energy, Bremerhaven, Germany, 2006.

[4] J. Falnes. “Ocean waves and oscillating systems. Linear interactions including wave-energy extraction”. Cambridge University Press, 2002.