

Comparison of measured shallow-water wave spectra with theoretical spectra.

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Abstract

Usually there is little knowledge of long-term wave conditions at prospective sites for wave-power plants, while the deep-water or open sea conditions may be more known and geographically less varying. However, many wave-energy plants are intended for water depths small compared to the wave length. A concept for assessing design waves at a near-shore site is to transform the off-shore wave spectra to the target site by a model for spectral wave-energy transfer over the actual bottom topography. The inshore spectra can be used for linear statistics of extreme waves, design wave loads and for assessment of power take off. In this context it is important to know the realism of used spectral forms.

Based on 58 measured wave spectra at 6 m water depth at the near-shore wind farm Bockstigen in the Baltic the most realistic spectrum was found to be the TMA spectrum, which is a JONSWAP spectrum modified for shallow water. Some few examples are given. Normally wave-energy devices would be placed in somewhat deeper water and water depth correction will be smaller

Keywords: Wave spectrum, measured shallow water waves, TMA spectrum

Nomenclature

E	= amplitude spectrum
S	= variance or wave spectrum
f	= frequency
ω	= angular frequency
T	= wave period
U	= wind speed
H_s	= significant wave height
α	= Phillip's constant
p	= index for spectral peak frequency
z	= index for mean zero-crossing period

Introduction

For the design of structures offshore or near-shore against wave loads and for assessment of power take-off it is necessary to have some realistic design waves. These could be regular design waves for extreme loads

or irregular waves for dynamic load cases, fatigue problems and power take-off. Based on measurements at Bockstigen the TMA shallow water spectrum is recommended here for assessing irregular waves. The TMA spectrum is a modified JONSWAP spectrum. (Hughes, 1984)

The wave measurements

Waves were measured at Bockstigen with a pressure and directional bottom mounted equipment (Valeport) and with a wave-radar (SAAB), the latter measuring the water surface elevation directly. Analysis of records from the same occasion shows that the pressure equipment mounted at 6 m water depth gives inaccurate information of the second-order double frequency waveform due to the attenuation of the dynamic pressure with depth, because, when amplifying the attenuated pressure signal by dividing with the attenuation factor, noise will also be amplified which is a problem when the attenuation factor is small. Also the second order dynamic pressure will attenuate with a factor corresponding to the first order "parent" wave. The wave-radar, on the other hand, gives the actual geometric waveform directly, with some inaccuracies at peaked, breaking waves. In both measurements it is difficult to differentiate between first-order free waves and second-order bound waves at the same frequency. Measuring in one point, the wave celerity cannot be evaluated. Examples of variance spectra of surface elevation from the two types of equipment at the same occasion are shown in Figure 1.

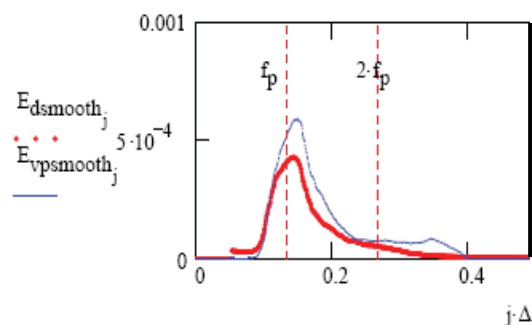


Figure 1 Smoothed amplitude spectra from Waveradar (thick graph) and Valeport (thin graph, truncated for attenuation factor below 1/20). The peak frequency is set to f_p .

Standard spectra

There are many wave spectra used for waves offshore in deep water, i.e. when the wavelength is larger than twice the water depth. See e.g. Gran (1992). A fundamental spectrum is the Pierson-Moscowitz spectrum, which should describe wave spectra for fully developed sea, or fully arisen sea (FAS), when a constant wind blowing infinitely long cannot increase the energy in the waves, but the energy transfer is balanced by dissipation. This spectrum is a one-parameter spectrum completely described by the wind speed:

$$S_{PM}(\omega) = \alpha g^2 \omega^{-5} e^{-0.74(\omega_o/\omega)^4} \quad \dots(1)$$

where $\alpha = 0.0081$ is Phillip's constant,
 g the earth acceleration,
 $\omega_o = g/(U_{19.5})$ and
 $U_{19.5}$ = the wind speed at the height 19.5 m above still water level.

See Figure 2 for examples of PM spectra for some wind speeds. Wind speed, mean period or significant wave height can be used as the input parameter to the one-parameter PM spectrum.

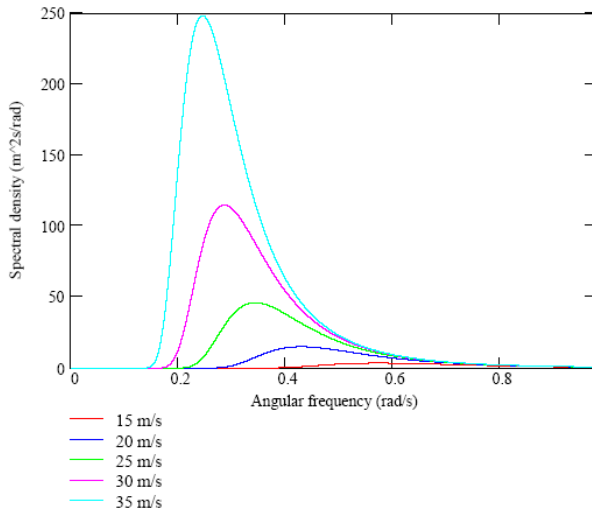


Figure 2. PM spectra for some wind speeds.

Mostly the sea state is, however, not fully developed as the wind speed and direction change, the fetch is too short, or the wind duration is not long enough, especially for strong winds and high waves. Then two-parameter spectra for developing seas can be used, e.g. some in which the wave height and frequency are the parameters. This was originally proposed by Bretschneider and offers more flexibility, because the energy of the spectra can be placed at arbitrary locations on the frequency axis with the demanded significant wave height. Such spectra belonging to the PM-family are also, somewhat incorrectly, referred to as PM-spectra. Widely used spectra are the ISSC spectra (ISSC = International Ship Structures Congress, 1964).

Two-parameter spectra still give too little freedom to reproduce realistic spectra of developing sea. In 1973 Hasselmann *et al.* published the five-parameter JONSWAP spectrum, which was one of the results from the Joint North Sea Wave Project.

$$S_{JONSWAP}(\omega) = S_{PM}(\omega) \gamma^e^{-\frac{1}{2} \left(\frac{\omega - \omega_p}{\sigma(\omega) \omega_p} \right)^2}, \quad \dots(2)$$

where γ is the peak enhancement factor,

ω the angular frequency,

ω_p the modal angular frequency (peak of spectrum)

$\sigma(\omega) = \sigma_a$ if $\omega < \omega_p$, “standard deviation” of the peak enhancement factor to the left and

$\sigma(\omega) = \sigma_b$ if $\omega > \omega_p$, “standard deviation” of the peak enhancement factor to the right.

α and ω_o of $S_{PM}(\omega)$ are also parameters to be chosen.

By Hasselmann originally recommended values are, when the fetch, F , and the wind speed, U_{10} , at 10 m height is used:

$$\gamma = 3.30$$

$$\sigma_a = 0.07$$

$$\sigma_b = 0.09$$

$$\alpha = 0.076 F_o^{-0.22}$$

$$\omega_p = 7\pi(g/U_{10})F_o^{-0.33} \text{ and}$$

$$F_o = gF/(U_{10})^2.$$

For the above JONSWAP spectrum

$$T_{01} = 1.073 \quad T_{02} = 0.834 \quad T_p.$$

The JONSWAP spectrum is in common use for design of drilling platforms in the offshore industry because it offers more flexibility with its five parameters, and can produce realistic spectra. The parameters are then chosen from wave statistics combined with systematic parameter fitting.

Note in Figure 3 the different characteristics of the two spectra with the sharp peak of the JONSWAP spectrum on top of the PM spectrum. In Figure 3 the JONSWAP spectrum has a larger variance and thus larger significant wave height. In Figure 4 the JONSWAP spectrum and the PM spectrum are given the same variance and significant wave height but then the PM spectrum is shifted to a higher peak period.

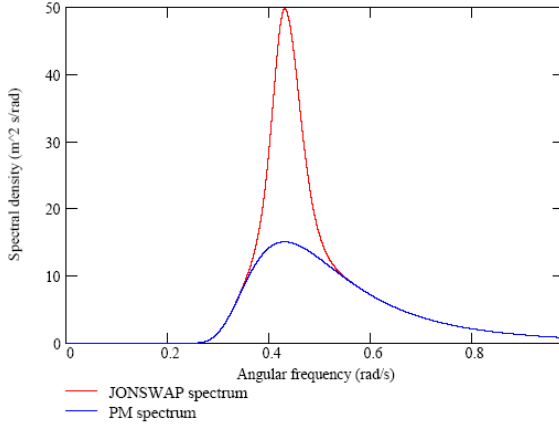


Figure 3. A JONSWAP spectrum and PM spectrum with the peak frequency, $\omega_p = 0.878$ $\omega_0 = 0.431$ rad/s.

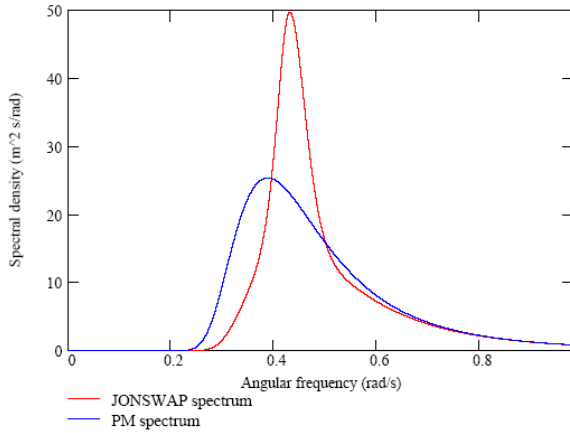


Figure 4. A JONSWAP spectrum and PM spectrum with the same significant wave height H_s .

Gran (1992) gives the following recommendations for the parameters in the JONSWAP spectrum based on field data (Houmb and Overvik, 1977).

$$\gamma = 7(1 - \frac{0.0027}{\alpha}) = 7(1 - 2.18 \cdot 10^{-5} \frac{g^2 T_z}{H_s^2}) \quad \dots(3)$$

(Sic. Dimensionally incorrect)

$$\sigma_a = 0.07$$

$$\sigma_b = 0.09$$

$$T_p = \frac{T_z}{\sqrt{\frac{5 + \gamma}{10.89 + \gamma}}} \quad \dots(4)$$

$$\alpha = \frac{30 H_s^2 \pi^4 f_p^4}{g^2 (5 + \gamma)} \quad \dots(5)$$

This spectrum parameterisation needs two input parameters: mean period, T_z , and significant wave height, H_s , and is the JONSWAP model originally implemented in Vidyn (Carlén, 2001)

While the JONSWAP spectrum originally was developed for developing sea in deep water, the waves in shallow areas are often waves coming in from deeper

areas into an area where the waves are much affected by the limited water depth. Hughes (1984) gives for such cases the following modified JONSWAP spectrum in shallow water called the TMA spectrum. It is based on the fact that low-frequency or, equivalently, long-period waves must have a limited height in shallow water. Therefore the spectrum is multiplied by a function for limited depth Eq. (6) and Figure 5. See also Jensen (2002) who expanded the application to directional spread and superimposed current.

$$\phi(\omega, h) = \begin{cases} 0.5(\omega\sqrt{h/g})^2 & \text{if } \omega\sqrt{h/g} < 1 \\ 1 - 0.5(2 - \omega\sqrt{h/g})^2 & \text{if } 1 \leq \omega\sqrt{h/g} < 2 \dots(6) \\ 1 & \text{if } \omega\sqrt{h/g} \geq 2 \end{cases}$$

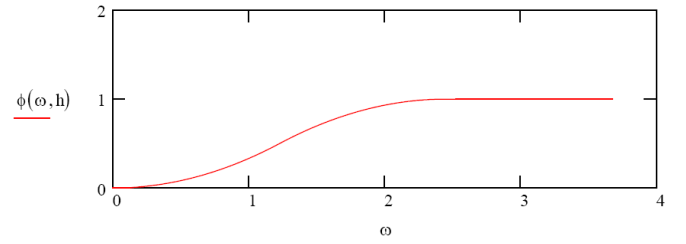


Figure 5. The limited depth function in the TMA spectrum as function of angular frequency, ω , and depth, h .

The expression for the TMA spectrum is then:

$$S_{TMA}(\omega) = \alpha g^2 \omega^{-5} \phi(\omega, h) \exp \left[-\frac{5}{4} \left(\frac{\omega_p}{\omega} \right)^4 + (\ln(\gamma)) \exp \left(-\frac{1}{2} \left(\frac{\omega - \omega_p}{\sigma(\omega) \omega_p} \right)^2 \right) \right] \quad \dots(7)$$

Hughes gives e.g. the following possible expression for the parameters α and γ in the TMA spectrum based on shallow-water data.

$$\text{with } \varepsilon = \frac{H_s}{4L_p}, \quad \dots(8)$$

$$\alpha = 16\pi^2 (\varepsilon)^2 \quad \gamma = 6614 (\varepsilon)^{1.59} \quad \dots(9)$$

where L_p is the wavelength at the peak frequency for the water depth at hand and H_s the significant wave height.

Example

In Table 1 and Figure 6 below comparisons for a case at 6 m water depth at Bockstigen are given. Results from the PM spectra with measured inputs of wind velocity, S_{PMU} , measured mean period, S_{PMTz} , or measured significant wave height, S_{PMHs} , are given together with results from the deep-water JONSWAP spectrum with measured H_s and T_z ; and the shallow-water TMA spectrum with measured H_s and peak period, T_p . The spectra with significant wave height as input give approximately the correct significant wave

height as output, as expected, but the wave energy is very differently distributed over the frequency.

Table 1. Significant wave-height output yielded from the spectra with measured $H_s = 0.664$ m, $T_z = 5$ s, $T_p = 5.60$ s and $U_{19.5} = 6.81$ m/s (8 m/s at the anemometer height 37 m) as input.

Spectrum	S_{JONSWAP}	S_{PMTz}	S_{PMHs}	S_{PMU}	S_{TMA}
H_s (m)	0.66	1.49	0.66	0.99	0.58

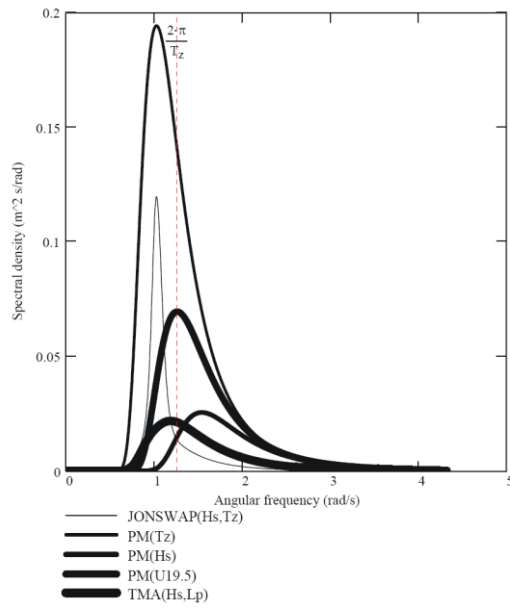


Figure 6. Variance density spectra for the JONSWAP spectrum with H_s and T_z as input, PM spectra with T_z , H_s or $U_{19.5m}$ as input, and TMA spectrum with H_s and f_p as input.

Comparison with measured spectra

In Figure 7 below spectra from the Wave radar and the Valeport measurements are compared with the TMA spectrum using the recommended parameterisation with input of measured significant wave height, $H_s = 1.55$ m and peak frequency $f_p = 0.135$ s estimated by inspection from the wave radar measurement. Looking at Figure 6 and Figure 7 it is obvious that only the TMA spectrum, of the spectra investigated here, is capable of reasonable approximation of this sea state at the Bockstigen site. The conformity between the measured spectrum and the TMA spectrum may be improved if the parameters are adjusted for each measured spectrum.

The length of each wave record was 1.046 h, sampling interval 17.4 Hz, 10^{16} -1 data points. The evaluation of the spectra was performed by a pure FFT transformation of the wave records, deleting the first n ($=101$) component amplitudes, then forming a discrete variance spectrum and averaging arithmetically over n components centred around the calculated spectral component. Almost no variance was lost in the process.

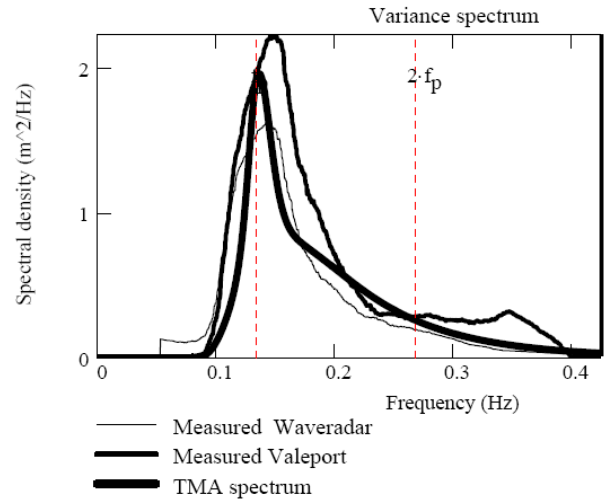


Figure 7. Smoothed density spectra of Waveradar and Valeport measurements compared with the TMA spectrum with standard parameterisation and input significant wave height, $H_s = 1.55$ m and peak frequency $f_p = 0.135$ Hz estimated from the wave radar measurement.

Spectra from 58 one-hour wave radar measurements have been compared with TMA spectra with standard parameterisation and input significant wave heights, H_s , and peak frequencies, f_p , estimated from the measurements. By looking at these 58 graphs one can conclude that for high waves (> 1.5 m approximately) originating from steady winds in one direction the TMA spectrum gives a good fit. See Appendix below “Date 2004-07-11 00.00” for a typical example. For lower waves the fit is poorer. See “Date 2004-06-25 23.00 hrs”, but can be improved by adjusting γ . For waves originating from turning or strongly changing winds with two spectral peaks, theoretical spectra with two peaks should be adapted for shallow water. See “Date 2004-07-10 00.00 hrs”. All 58 spectra are documented in Appendix 2 in Bergdahl *et al.* (2006), which will be distributed on request

Conclusion

It is concluded that only the TMA spectrum, of the spectra investigated here, is capable of reasonable approximation of the sea state at the Bockstigen site. The conformity can be improved if the parameters are adjusted for each measured spectrum.

Acknowledgements

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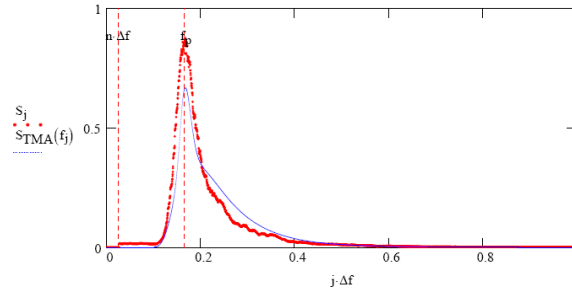
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Appendix: Three measured occasions

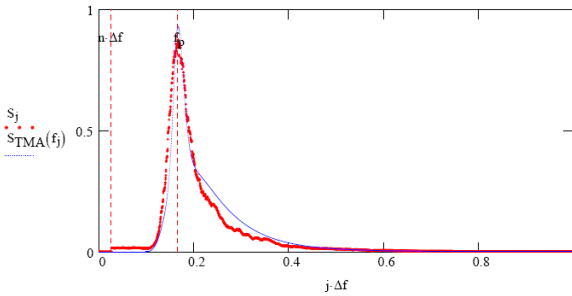
Date 2004-06-25 23.00 hrs

$H_s = 1.06$ m, $f_p = 0.165$ Hz, $\alpha = 6.447 \cdot 10^{-3}$,

Standard $\gamma = 2.14$



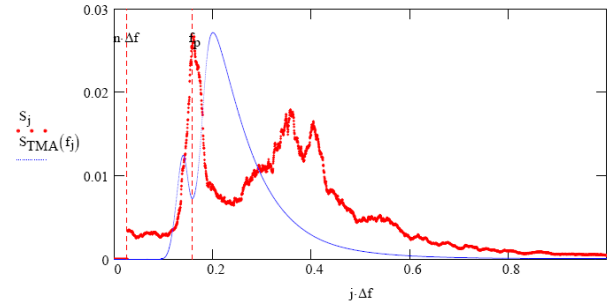
Adjusted $\gamma = 3$ for better fit



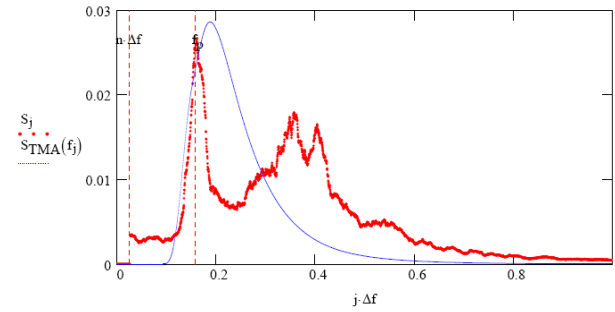
Date 2004-07-10 00.00 hrs

$H_s = 0.31$ m, $f_p := 0.16$ Hz, $\alpha = 5.006 \cdot 10^{-4}$

Standard $\gamma = 0.281$



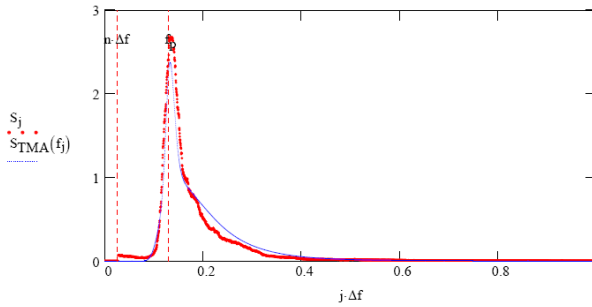
Adjusted $\gamma = 0.9$ for better fit



Date 2004-07-11 00.00

$H_s = 1.638$ m, $f_p = 0.131$ Hz, $\alpha = 8.913 \cdot 10^{-3}$

Standard $\gamma = 2.773$



Adjusted $\gamma = 3.1$ for better fit

