Optimisation of a Heterogeneous Array of Heaving Bodies

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Abstract

Many of the studies of the interaction factors for arrays of wave energy devices concern systems in which the mechanical damping matrix (R) is specified as exactly equal to the radiation damping matrix (B). Although this represents an 'optimal system of oscillating bodies, the damping and mass conditions required are difficult to engineer. Specifically, for the power generated by an array of bodies that each oscillate in a single mode, the mechanical damping matrix does not replicate the radiation damping which couples adjacent bodies, (R is diagonal whereas B is a dense matrix). Here, analysis is conducted using a frequency domain model based on hydrodynamic coefficients obtained from multi-body WAMIT analysis. We consider a five element array of hemispherical bodies in both terminator (beam-seas) and attenuator (head-seas) configurations in which power is generated from heave of each float only. Mass and mechanical damping are identified for each float to (a) maximize net power from the array and (b) minimize variation of average power across the array. We show that, by selecting appropriate values of damping for each float within the array, power capture can exceed that from an array in which damping is equal to the diagonal of either the radiation damping matrix or of the optimal damping matrix for a constant mass array.

Keywords: Wave Device; Array Interactions; Optimisation

Nomenclature

A	=	Added Mass [dense; $N \times N$]
A_w	=	Wave amplitude [single value]
B	=	Radiation damping matrix [dense; $N \times N$]
B_0	=	Radiation damping of isolated device [single value]
F	=	Excitation Force [$N \times 1$]
M	=	Total Float Mass ($M_m + M_{sup}$) [diagonal; $N \times N$]
M_m	=	Device displaced Mass [diagonal; $N \times N$]
M_{sup}	=	Supplementary Masses [diagonal; $N \times N$]
N	=	Number of elements within array [single value]
P	=	Power [$N \times 1$]
P_{net}	=	Net Power [single value]
R	=	Total Mechanical damping [diagonal; $N \times N$]
R_{opt}	=	Optimum R matrix for fixed mass array [$N \times N$]
S	=	Hydrodynamic Stiffness [diagonal; $N \times N$]
u	=	Velocity [$N \times 1$]
ω	=	Wave Frequency [single value]
ω_0	=	Natural Frequency [single value]
z	=	Complex (heave) response amplitude [single value]

1 Introduction

Many of the wave energy devices currently being developed use the motion of a floating buoy to drive an electric generator. Power absorption from individual devices is relatively small. However, theoretical studies have shown that array interactions may increase absorbed power, particularly at certain wavelength to spacing ratios [1]. This potential for increased power capture, as well as the possibility of higher rated power per installed device and reduced maintenance costs per unit of installed capacity has motivated the development of devices comprising closely spaced arrays of heaving floats (e.g. Manchester Bobber, Wavestar, Trident, Fred Olsen FO^3 Buldra). These are characterised by inter-device centre to centre spacing of the order of two diameters, power generation due to oscillation in heave only and a limited variation of mass. It is of particular interest to identify the maximum output that could be achieved from this type of system relative to the same number of devices in isolation. Much numerical analysis has been presented of arrays of devices but, in many cases, assumptions are made regarding both the mechanical damping and the oscillating mass of the system.

Applying linear wave theory and assuming small amplitude response, power absorption P by an array of wave devices is given by equation (1)[2]:

$$P = \frac{1}{8}F^{*}B^{-1}F - \frac{1}{2}\left(U - \frac{1}{2}B^{-1}F\right)^{*} B \left(U - \frac{1}{2}B^{-1}F\right) \quad (1)$$

where '$*$' represents the conjugate transpose, F the excitation force vector, B the full radiation damping matrix, and U the velocity vector. Modelling an array as a system of linear spring-mass-dampers, the velocity vector is a function of the device mass, stiffness and damping:

$$U = \frac{F}{(R+B) + i\omega\left(M + A - \frac{S}{\omega^2}\right)} \quad (2)$$

in which R is the mechanical damping matrix, M the system masses, A the frequency-dependent added mass component of the radiation force and S the hydrostatic stiffness. The diagonal matrices (R, M and S) consist of one diagonal element per device, and the vectors (U and F) one element per device. The non-diagonal elements of the dense matrices (A and B) are as a result of interaction effects within the device array (i.e. they represent the force on body j due to the motion of body i).

For an array of devices, the excitation force on each float will differ from that on an isolated float due to scattering of the incident waves by all devices in the array. Much of the published research on device arrays further simplifies the hydrodynamic forcing by assuming inter-device spacing to be large compared to the incident wavelength so that the interaction effects are so small that the excitation force is the same as if the devices were in isolation. This is acceptable when the device size is small relative to the wavelength (e.g. radius $a < 0.12L$)) but both the radiated and scattered waves may influence response for smaller spacing and so should be considered [3]. Using the point absorber large spacing approximation, the power absorbed by a 2×1 and 10×1 array (partially optimised by imposing a phase condition) was shown to be significantly greater than that of the same devices in isolation [4]. The theory was extended to the case of a fully optimal array in which phase and velocity conditions are applied to the devices [5] [6]. The optimum power absorbed by a 5×1 and 5×2 array was calculated by Thomas and Evans for a variety of separation distances, three angles of incidence and a single wave frequency [2]. Interaction factors greater than unity were found mostly in beam seas, except when the centre to centre separation (s) is small compared to the wavelength ($s < 0.42L$) for which higher interaction factors are observed in head seas. In beam seas maximum power absorption was shown to occur when the separation was just less than a wavelength.

It is generally beneficial for a wave device to maximise power output in the range of wave frequencies expected at the deployment site. For a known wave frequency (ω), power output P can be modified by changing the float form to adjust the excitation force, the radiation damping, B, the added mass, A or the hydrodynamic stiffness, S. Within an array, these hydrodynamic terms can be modified by altering the inter-float spacing or the geometry of individual floats. The spacing and the geometry would however be difficult to vary efficiently to match a varying wave frequency. It is more straightforward to adjust either the mechanical damping, R, or the total mass of the float, M. However, large variation of mass is not straightforward to achieve and, for the case of neutrally buoyant devices, a change of mass would also result in a change of immersed geometry. Although varying R is not the most effective way to manipulate the power absorbed by an isolated device, it may have a significant impact on the response and hence power within a closely spaced array since the forcing on other floats due to radiated waves will be modified.

Power output is maximum when the second and third terms of equation (1) vanish (i.e. when $U = \frac{1}{2}B^{-1}F$). This is satisfied only when the force and velocity are in phase (i.e. $S = M\omega^2$) and when the mechanical damping matrix R is identical to the dense radiation damping matrix B. Whilst these conditions are of theoretical interest, they are difficult to apply to a real world system such as the closely spaced arrays mentioned above and so the interaction factors following from these assumptions are of limited relevance. Since the added mass matrix is dense, a dense mass matrix M is required. This is not possible, particularly over the range of wave periods of interest. A dense R matrix could theoretically be produced by varying the mechanical damping applied to each float based on the motion of other floats, although this procedure would not be straightforward to implement. A simpler approach would be to apply specific damping values to each float. This would only produce a diagonal damping matrix but represents most of the closely spaced wave energy device array systems currently in development.

It has been shown that if R is selected such that $R = C \cdot diag(B)$ where C is a constant, the maximum power absorbed by a 5×1 array of hemispheres in both beam and head seas (for a range of s and ω values) is achieved by taking $C = 1$ [7]. The focus of the present study is to identify appropriate values for the diagonal mechanical damping matrix R of an N body system that either maximise net power output at each frequency or provide equal power output over a range of frequencies. A linear array of $N = 5$ heaving hemispheres (of radius a) with a centre to centre separation distance of $4a$ is considered. A detailed description of the analysis is given in Section 2. In Section 3, R values are selected for the 5×1 array in order to get maximum net power in both beam (section 3.1.1) and head seas (Section 3.1.2). An alternative to varying the mechanical damping on each float is to change the individual float masses. The mass variation required to maximise net power output is briefly considered in Section 3.2. Finally, in Section 4 an approach is outlined for identifying the diagonal mechanical damping matrix that would provide a specified value of power output for the array; this provides the damping distribution if the maximum power output is known in advance.

2 Selection of Mechanical Damping

Accounting for the incident, diffracted and radiated waves, linear wave theory can be used to predict the velocity potential, and hence loads and forces on a device. Various numerical codes are now available for computing these parameters for arbitrary geometries (for example WAMIT [8] and AQWA). The array geometry considered here is a 5×1 array of hemispheres with separation distance $s = 4a$ (see Figure 1). From the body geometry, the displaced mass, M_m, is calculated as $M_m = \rho\frac{2}{3}\pi a^3$. Throughout this section, float mass remains constant for all floats and all frequencies. For motion in heave, elements of the hydroynamic stiffness matrix, S are given by the change of buoyancy per unit displacement; calculated by $S_{ii} = \delta\rho g$ where δ is the volume of an infinitesimally small section of the body at the water-plane. If the float is not straight-sided, water-plane area would change with float displacement but this is neglected such that S is approximated as $S = \pi a^2 \rho g$.

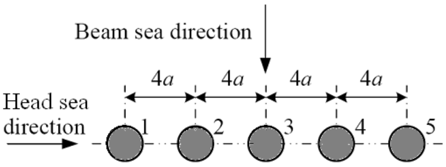

Figure 1: Diagram of 5×1 array of hemispheres of radius a with a centre to centre separation distance of $s = 4a$; beam and head sea directions are also indicated

For an array of devices, the power output can be determined by equation 1. Differentiating this with respect to the matrix R and equating with zero would determine the damping matrix which would result in the maximum net power output for a fixed mass array. This procedure is not straightforward for unknowns in both power and damping and so several alternative approaches are considered. One approach is to apply $R = B$ for a fixed mass matrix even though this is only known to give maximum power when $M = A - S/\omega^2$. An alternative method for obtaining an R matrix that would maximise power output from an array of fixed mass floats for a particular frequency is based on the output of an isolated device. Equation 3 gives the power which can be absorbed by a mechanical damper applying a damping force R to an isolated device. The differential with respect to R of equation 3 can be equated with zero to determine R_{opt}; the value of R which results in the greatest power for an isolated device (equation 4) [6].

$$P = \frac{\frac{1}{2}R|F(\omega)|^2}{(R+B(\omega))^2 + \omega^2\left(M + A(\omega) - \frac{S}{\omega^2}\right)^2} \tag{3}$$

$$R_{opt} = \sqrt{B^2 + \omega^2\left(A + M - \frac{S}{\omega^2}\right)^2} \tag{4}$$

By substituting matrices A, B, M and S to equation 4 in place of the intended scalars a (nearly) optimum damping matrix can be obtained for an array. This approach is not expected to give the maximum attainable power output but does account for use of a fixed mass matrix. A further problem arises when using $R = B$ and $R = R_{opt}$ as these are both dense matrices and so are difficult to implement. Herein $R = diag(B)$ denotes the diagonal matrix whose diagonal elements are the same as those of B. A possible method to predict the diagonal R matrix which will result in the maximum net power is to take either $R = diag(B)$ or $R = diag(R_{opt})$. The $R = diag(B)$ and $R = diag(R_{opt})$ damping values are shown in Figures 2 and 3 for the beam seas case. In Figure 4 the power for systems with $R = R_{opt}$, $R = diag(R_{opt})$, $R = B$ and $R = diag(B)$ are all shown for a 5×1 array of hemispheres with constant mass, $M = 2M_m$. Away from the peak, the array with $R = diag(R_{opt})$ results in greater power than the array with $R = diag(B)$, however both have the same values near the peak.

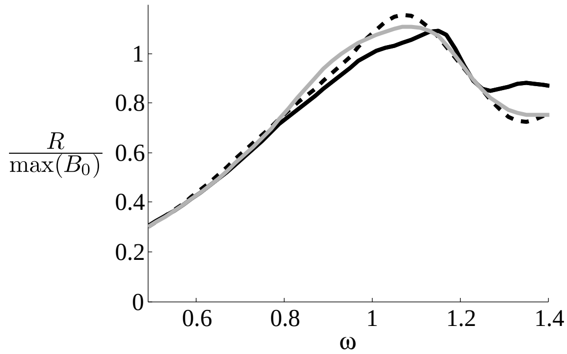

Figure 2: $R = diag(B)$ values for a 5×1 array of hemispheres in beam seas: solid black line = float 1 (and 5); solid grey line = float 2 (and 4); dashed line = float 3

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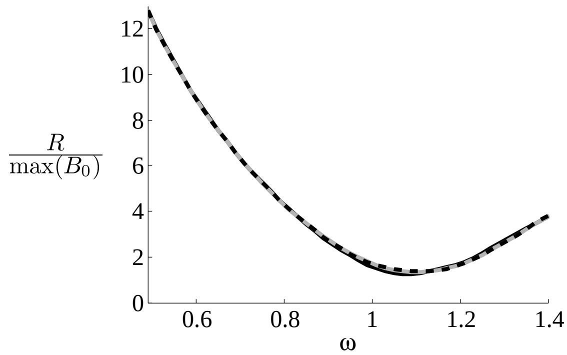

Figure 3: $R = diag\left(R_{opt}\right)$ values for a 5×1 array of hemispheres in beam seas (see thick dashed line in Figure 4 for resulting P_{net} values): solid black line = float 1 (and 5); solid grey line = float 2 (and 4); dashed line = float 3

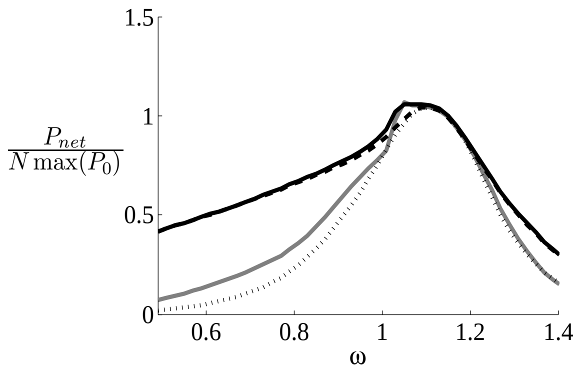

Figure 4: P_{net} for a 5×1 array of hemispheres in beam seas: solid black line, $R = B$; solid grey line, $R = R_{opt}$; dashed line, $R = diag\left(R_{opt}\right)$; dotted line, $R = diag(B)$

If $R = C\,diag(B)$ for some constant, C, for an 5×1 array of hemispheres with constant mass, the greatest power is achieved by taking the constant to be $C = 1$ [7]. This can be seen in Figure 5, which also shows that a constant mass array with $R = C\,diag(B)$ can produce at most the same power as when $R = diag(R_{opt})$. If $C < 1$ then the array is underdamped and will achieve less power at all frequencies than when $R = diag(R_{opt})$. For $C \geq 1$, the maximum array power is equal to that when $R = diag(R_{opt})$ is applied. As damping is increased ($C > 1$), the peak power reduces and the peak is shifted to a lower frequency.

For an $N \times 1$ array, the matrices in Equation (1) can be expanded to produce a linear system of N equations. Conditions on the net power, P_{net} , individual float powers, P and the range of available R values can be included in the system of equations. If all floats are assumed to provide equal power at a given wave frequency, the system of N equations has only N unknowns, and hence the R values can be determined directly. This approach is discussed in Section 4. Designers are often interested in the maximum net power. In this problem there are $2N$ unknowns and only N equations (plus inequalities to define variable constraints). Here, a Least Squares analysis is applied, using the Gauss Newton Algorithm, to solve for a unique value of mechanical damping for each float such that net power is maximised at each frequency. Results are shown for both the $\max(P_{net})$ case with no restrictions on the magnitude of mechanical damping R, and for the more practical case in which R is limited to a finite range (double that of the radiation damping of an isolated device, $R < 2\max(B_0)$). Wave frequencies of 0.02 rad/s increments were tested from 0.49 to 2.3 rad/s to obtain net power outputs greater than 40% of the maximum power output of an isolated device. This lower limit is not satisfied over the entire frequency range and so data presented in this paper focuses on the range 0.5 to 1.4 rad/s only. For frequencies outside this range, alternative methods could be used to achieve a higher P_{net} value such as adjusting float mass as discussed briefly in Section 3.2.

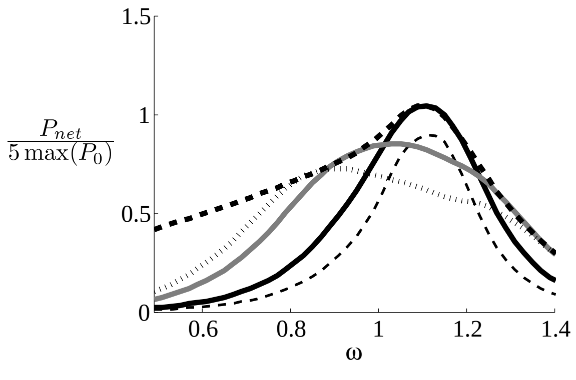

Figure 5: P_{net} for a 5×1 array of hemispheres in beam seas: thick dashed line, $R = diag\left(R_{opt}\right)$; thick dotted line, $R = 5diag(B)$; grey solid line, $R = 3diag(B)$; black solid line, $R = diag(B)$; thin dashed line, $R = 0.5diag(B)$

3 Maximum Net Power

For most wave device developers it is of some importance to determine the maximum power output that could be obtained from a device and to understand the conditions required for this output. In this Section, maximum net power output is obtained for a five element array of equal and constant mass floats in both beam and head seas by applying different values of mechanical damping to each member of the array. Variation of individual float mass whilst applying equal damping is also briefly considered.

3.1 Damping Variation

A 5×1 array of neutrally buoyant hemispheres of constant mass ($M = 2M_m$) is studied to determine the values of mechanical damping R that are required so as to achieve maximum net power. Although optimum power at each frequency would also require variation of mass to match the natural period of the device to the wave conditions, the discussion in this section concerns an array of equal mass floats. Comparison is drawn to two benchmark cases: i) a system in which a dense mechanical matrix is optimal for fixed mass floats (termed $R = R_{opt}$ by Equation 4), and ii) the diagonal of the same radiation damping matrix (termed $R = diag(R_{opt})$). The dense damping matrix would provide optimal output but is difficult to apply whereas the diagonal mechanical damping matrix can be implemented. To obtain the diagonal damping matrix which maximises net power, the system of $N = 5$ equations obtained from the expansion of Equation 1 is analysed using a Gauss-Newton algorithm subject to a constraint on net power (P_{net}). Damping distributions are obtained for 100 values of P_{net} in equal increments ranging from $0.4\max\left(P_{net}\left(R = diag(R_{opt})\right)\right)$ to $\max\left(P_{net}\left(R = diag(R_{opt})\right)\right) + \left(2 \times 10^5\right)$ over the full range of frequencies. For each P_{net} value, the minimum and maximum frequencies (ω_1 and ω_2 respectively) at which solutions are determined are calculated. The ω_1 or ω_2 which correspond to the maximum P_{net} value are then stored. These P_{net} values are plotted against their corresponding ω_1 and ω_2 frequencies to give a comparison to the $P_{net}(diag(R_{opt}))$ and $P_{net}(R_{opt})$ plots of net power. The R values required to achieve these P_{net} values are also shown.

3.1.1 Beam Seas

The first case tested is a 5×1 array in beam seas with the central float denoted float 3. In this case, due to symmetry the damping values selected are the same on floats 4 and 5 as on floats 1 and 2 respectively. Figure 6 shows the net powers achieved both when the range of permitted R values is unlimited and when the system of equations is modified to account for a restriction on the R values such that $R < 2\max(B_0)$. The corresponding values of mechanical damping for each float are shown in Figure 7 and 8 respectively.

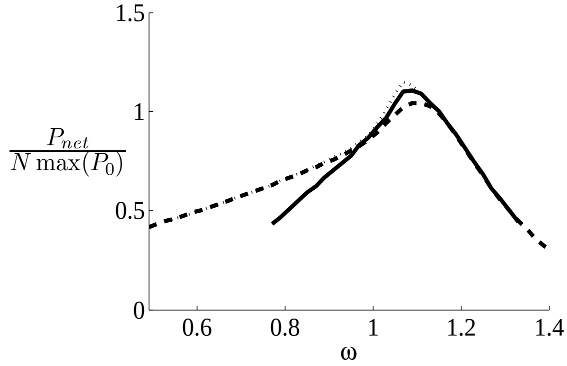

Figure 6: P_{net} for a 5×1 array of hemispheres in beam seas: thick dashed line, $R = diag\left(R_{opt}\right)$; Thick solid line, R values less than $2\max(B_0)$ selected so as to give the maximum P_{net} at each wave frequency, ω; Thin dotted line, unlimited range of R values selected to give maximum P_{net}

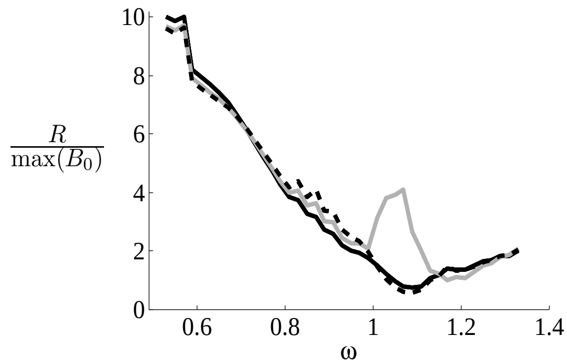

Figure 7: R values chosen from an unlimited range in order to give the maximum P_{net} value at each wave frequency for a 5×1 array of hemispheres in beam seas (see thin dotted line in Figure 6 for resulting P_{net} values): solid black line = float 1 (and 5); solid grey line = float 2 (and 4); dashed line = float 3

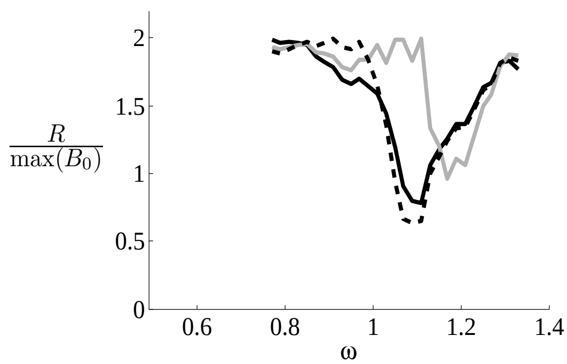

Figure 8: R values less than $2\max(B_0)$ chosen in order to give the maximum P_{net} value at each wave frequency for a 5×1 array of hemispheres in beam seas (see thick solid line in Figure 6 for resulting P_{net} values): solid black line = float 1 (and 5); solid grey line = float 2 (and 4); dashed line = float 3

Over a wide range of frequencies, appropriate selection of mechanical damping can improve power output relative to a system based on radiation damping. If we assume that an arbitrary value of mechanical damping may be applied, net power output is increased compared to the $R = diag(R_{opt})$ case over the range $0.95 \leq \omega \leq 1.15$ rad/s (Figure 7), with a 12% increase at the peak ($\omega = 1.07$ rad/s). Outside of this range of frequencies the power obtained using the unlimited range of R closely follows that of $R = diag(R_{opt})$. In order to achieve these net power increases at low frequencies, damping values up to $10.15\max(B_0)$ are required. Such large mechanical damping values could be difficult to implement in practice. However, for wave frequencies over the range $0.99 \leq \omega \leq 1.33$ rad/s similar power levels can be obtained with damping variation of only zero to double that on an isolated device ($0 < R < 2\max(B_0)$). For lower frequencies ($\omega < 0.9$ rad/s) increased damping would be required to maximise power output but, although lower than the $R = diag(R_{opt})$ case, the levels attained are still twice that given by applying the diagonal of the radiation damping matrix (47% increase at $\omega = 0.77$ rad/s relative to applying $R = diag(B)$). Close to the peak, power is increased by around 7% relative to $R = diag(R_{opt})$.

The damping distribution for the limited R case is identical to that of the unlimited case except where the values on float 2 (and 4) are capped at $2.0\max(B_0)$. At low frequencies ($0.49 \leq \omega \leq 0.99$ rad/s) the relationship between R and ω can be closely approximated by that of the $R = diag(R_{opt})$ case. When a restriction is imposed on the R values, it is not possible to achieve a power greater than $0.4\max(P_0)$ below $\omega = 0.77$ rad/s as higher damping is required. P_{net} values are however found within the range $0.77 \leq \omega \leq 0.99$ by setting the R values on all floats to be close to the maximum allowed value ($2\max(R_0)$).

In the frequency range $0.99 \leq \omega \leq 1.15$ (i.e. close to resonant frequency of an isolated device), P_{net} values greater than those of the $R = diag(R_{opt})$ case are obtained for both the limited and unlimited R cases. In this range disparate values of mechanical damping are required for both cases, such that R_{11} is similar to R_{33} whereas R_{22} is much greater. In the unlimited R case this constitutes a damping discrepancy of up to a factor of four between adjacent floats. Since excitation and motion is harmonic, this implies that there must be a considerable difference between the response amplitude of adjacent floats. This is considered further in Section 5. In both cases, for $\omega > 1.15$, a greater value of R is required on floats 1 and 3 than on float 2 (and 4).

An increase in P_{net} value has been achieved by selecting R values from an unlimited range and R values from the limited range of $0 < R < 2\max(B_0)$ compared to the P_{net} values obtained by setting $R = diag(R_{opt})$. Although a greater increase can be seen in the unrestricted case, a significant increase in P_{net} values is still obtained across the frequency range for the restricted case compared to when $R = diag(B)$.

3.1.2 Head Seas

In beam seas (Figures 2 to 8) a five element array can be considered as a symmetric system and this simplifies the analysis. Furthermore, the excitation force on all floats differs due to the wavefield scattered by each float but is similar in terms of phase. For a head seas array, symmetry cannot be assumed since the phase of the incident excitation force varies with distance along the array. The same Gauss-Newton iterative procedure is employed to obtain the mechanical damping values for each float within a five-element array arranged in head seas. Float dimensions and mass are as given in the previous section and the maximum radiation damping and peak power output from an isolated device (B_0, P_0) are again used to normalise the data.

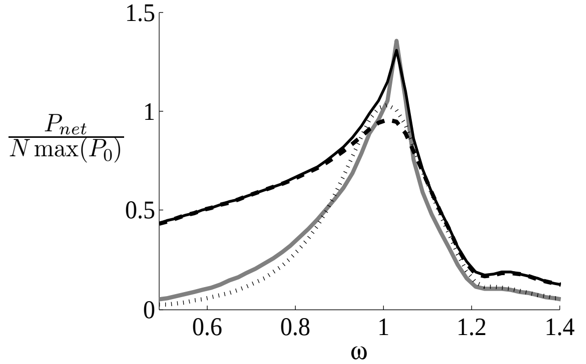

Figure 9: P_{net} for a 5×1 array of hemispheres in head seas: solid black line, $R = B$; solid grey line, $R = R_{opt}$; dashed line, $R = diag(R_{opt})$; dotted line, $R = diag(B)$

The exact peak in the power obtained using the iterative results is not calculated due to the frequency increments used, however it is clear that the peak occurs close to $\omega = 1.3$ rad/s. Near $\omega = 1.3$ rad/s, the R values selected from the unlimited R range are below $2\max(B_0)$, so limiting the R values to be below this limit produces little variation in power at the peak. The R values increase with increasing float number for all frequency below $\omega = 1.1$ rad/s when the choice of R is unlimited and $\omega = 0.9$ rad/s when $0 \leq R \leq 2\max(B_0)$ (Figures 10 (solid black line) and 11), and decrease with float number at all frequencies greater than $\omega = 1.5$ rad/s for the unlimited R case and $\omega = 1.07$ rad/s when $0 \leq R \leq 2\max(B_0)$. For the unlimited R case, net power is calculated within the frequency range $0.49 \leq \omega \leq 1.19$ rad/s and is greater than that of $R = R_{opt}$ in the range $0.95 \leq \omega \leq 1.15$ rad/s. Restricting the permitted range of R values to $0 \leq R \leq 2\max(B_0)$ reduces the frequency range for which values were obtained to $0.67 \leq \omega \leq 1.19$ rad/s, and for which the net power is greater than that of $R = R_{opt}$ in the range $1.01 \leq \omega \leq 1.15$. For the unlimited R case, the percentage increase in P_{net} at $\omega = 1.03$ rad/s (the peak) compared to when $R = diag(R_{opt})$ is 18% and approaches the power outputs obtained with a dense mechanical damping matrix.

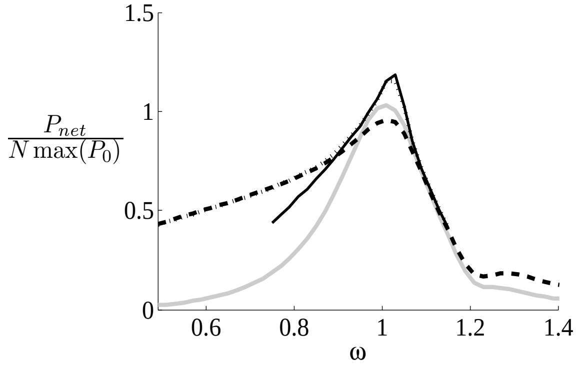

Figure 10: P_{net} for a 5×1 array of hemispheres in head seas: Thick dashed line, $R = diag(R_{opt})$; Grey solid line: $R = diag(B)$; Black solid line: R values less than $2\max(B_0)$ selected so as to give the maximum P_{net} at each wave frequency, ω; Thin dotted line: Unlimited range of R values selected to give maximum P_{net}

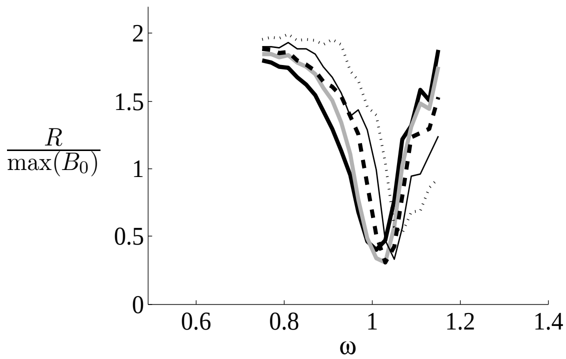

Figure 11: R values less than $2\max(B_0)$ chosen in order to give the maximum P_{net} value at each wave frequency for a 5×1 array of hemispheres in head seas (see thick solid line in Figure 10 for resulting P_{net} values): solid black line = float 1; solid grey line = float 2; dashed black line = float 3; thin black line = float 4; dotted black line = float 5

3.2 Varying Mass in Beam Seas

The system of equations formed by the expansion of Equation 1 is analysed here in order to determine the M_{sup} values required to get maximum net power. As the mass is not constant a comparison to the diagonal of the radiation damping matrix ($R = diag(B)$) is employed. Note that mass is changed without variation of immersed volume. Figure 12 (dotted line) shows the predicted net power when the range of permitted M_{sup} values is unlimited (corresponding mass distribution not shown); Figures 12 (thick solid line) and 13 show the results when the supplementary mass is restricted to the range $0.75M_m \leq M_{sup} \leq 1.25M_m$.

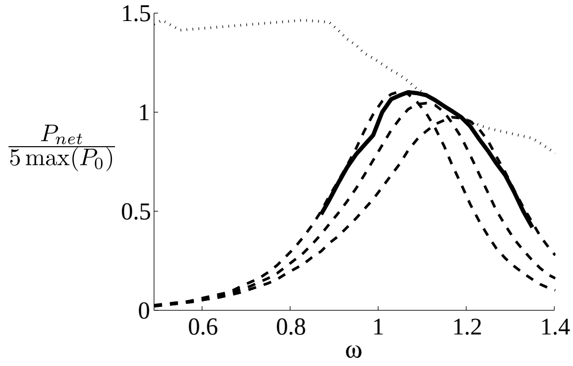

Figure 12: P_{net} for a 5×1 array of hemispheres in beam seas: dashed line- $R = diag(B)$ for case where $M_{sup} = 1.25M_m$, $M_{sup} = M_m$ and $M_{sup} = 0.75M_m$(from left to right); thin solid line: $R = B$; Thick, solid line: $R = diag(B)$ with M_{sup} values selected from within the range $0.75M_m \leq M_{sup} \leq 1.25M_m$ so as to give the maximum P_{net} at each wave frequency, ω; Thin dotted line: M_{sup} values from unlimited range

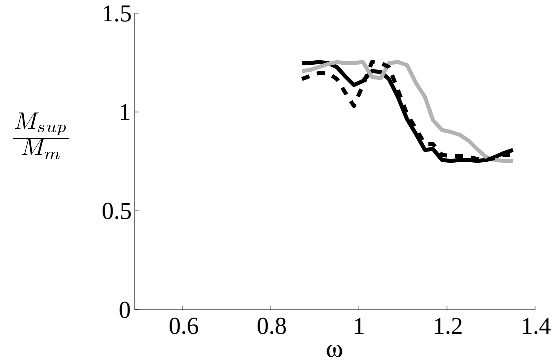

Figure 13: M_{sup} values between $0.75M_m$ and $1.25M_m$ chosen to give the maximum P_{net} value at each wave frequency for a 5×1 array of hemispheres in beam seas (see thick solid line in Figure 12 for resulting P_{net} values): solid black line = float 1; solid grey line = float 2; dashed line = float 3

When M_{sup} is unlimited, P_{net} values are obtained (i.e. power output exceeds 40% of that from an isolated device) within the range $0.49 \leq \omega \leq 1.59$ rad/s, and for the limited M_{sup} range, $0.87 \leq \omega \leq 1.35$ rad/s. When M_{sup} is restricted, an increase in net power of 65.8% is achieved at $\omega = 1.35$ rad/s, 8.4% at $\omega = 1.07$ rad/s (the peak) and 26.9% at $\omega = 0.87$ rad/s. For this range of frequencies, unique masses are required for each of the three floats. The peak increase is similar to that obtained using selected values of mechanical damping (compare Figure 12 to Figure 6). A similar increase is also observed over the lower frequency range although changing damping appears to be more effective at frequencies below 0.95 rad/s. For higher frequencies a greater increase of power is obtained by varying mass than damping.

4 Constant net power

To maximise net power output from an array, it is advantageous to apply different values of mechanical damping to each float within an array. A consequence of this is that there is also a large variation in power output between each float (not shown but up to a factor of more than three in the two arrays considered here). It can be beneficial to a wave energy device developer for all the devices within an array to produce equal power. It is possible to select damping values in order for all of the devices to achieve the rated power for a range of frequencies. When the matrices in Equation 1 are expanded a system of N equations can be obtained such that equation $i \in 1 : N$ is given by

$$0 = P_i R_{ii}^2 + \phi_i R_{ii} + \psi_i \tag{5}$$

where

$$\phi_i = 2P_i B_{ii} - \frac{1}{2}F^2 + 2\sum_{j\neq i}^{N} [B_{ij} P_j]$$

$$\begin{aligned}\psi_i &= \alpha_i P_i + \sum_{j\neq i}^{N} [\beta_{ij} P_j] \\ &= \begin{pmatrix} \omega^2 \left(M - \frac{S}{\omega^2} + A_{ii}\right)^2 + B_{ii}^2 \\ + \sum_{j\neq i}^{N} \left[\omega^2 A_{ij} A_{ji} + B_{ij} B_{ji}\right] \end{pmatrix} P_i \quad + \\ &\quad \sum_{j\neq i}^{N} \left[\begin{pmatrix} 2\omega^2 \left(M - \frac{S}{\omega^2} + A_{ii}\right) A_{ij} \\ +2B_{ii} B_{ij} \\ + \sum_{k\neq j\neq i}^{N} \left[\omega^2 A_{ik} A_{kj} + B_{ik} B_{kj}\right] \end{pmatrix} P_j \right]\end{aligned}$$

If the power on all floats is known, then P_i, and hence ϕ and ψ become constants, and Equation (5) becomes a directly solvable quadratic equation. Roots are obtained by applying:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

this gives:

$$R_{ij}(\omega, P) = \frac{1}{2P_i}\left(-\phi_i \pm \sqrt{\left(\zeta_i^2 - 4P_i \psi_i\right)}\right)$$

When searching for the maximum net power however, the power values on the individual floats are not known. As a result, Equation (5) is not directly solvable, and an iterative solver is required, as in the previous sections. If the power is equal on all floats, then for $i \in 1 : N$, $P(i) = P^d =$ constant, hence ϕ_i and ψ_i simplify such that Equation (5) becomes:

$$0 = P_d R_{ii}^2 + \zeta_i R_{ii} + \eta_i \tag{6}$$

where ζ_i and η_i are the constants:

$$\zeta_i = 2\left(\sum_j^N [B_{ij}]\right) P_d - \frac{1}{2}F_i^2$$

$$\eta_i = \left(\begin{array}{c} \omega^2 \left(M - \frac{S}{\omega^2} + A_{ii}\right)^2 + B_{ii}^2 \quad + \\ \sum_{j\neq i}^N \left[\begin{array}{l} \omega^2 A_{ij} A_{ji} + B_{ij} B_{ji} \\ +2\omega^2 \left(M - \frac{S}{\omega^2} + A_{ii}\right) A_{ij} \\ +2B_{ii} B_{ij} \\ +\sum_{k\neq j\neq i}^N \left[\omega^2 A_{ik} A_{kj} + B_{ik} B_{kj}\right] \end{array} \right] \end{array} \right) P_d$$

Equation (6) is directly solvable by applying the quadratic equation such that:

$$R_{ij}(\omega, P_d) = \frac{1}{2P_d}\left(-\zeta_i \pm \sqrt{\left(\zeta_i^2 - 4P_d \eta_i\right)}\right) \quad (7)$$

The direct method described can be used to determine the R values required to achieve specific power values on all floats within an array. In the case of achieving maximum net power, the distribution of power across the array would first need to be determined.

5 Discussion

For the five-element array of equal mass floats considered in Sections 1-3, a significant increase of net power output can be achieved by appropriate selection of mechancial damping for each member of the array rather than by matching mechanical damping to radiation damping. The combinations of mechanical damping and power output presented are obtained using a Gauss-Newton iterative procedure to obtain constant values of net power over a range of frequencies. As such, the range of frequencies at which R values can be found is limited by the range of constant P values specified. To limit computational cost a restricted net power range ($0.4\max(P_{net}(R = diag(R_{opt})) < P_{net} < \max(P_{net}(R = diag(R_{opt})) + 200000$) and only 100 increments of constant net power are analysed. These limits mostly affect the low- and high-frequency ranges over which power output is low (less than 40% of an isolated device). This corresponds to a narrower frequency range ($0.8 < \omega < 1.3$ rad/s) when damping is limited to double that on an isolated device (Figure 2) since very high damping is required to increase output at low frequencies. Over these ranges it would be more appropriate to vary natural period (as in Section 3.2) raher than attempt to optimise output by varying damping across the array. Within the range for which solutions are obtained, clear trends are observed in both the value of net power and the damping applied to each float.

Average power is calculated simply as $P = \frac{1}{2}R|\frac{z}{i\omega}|^2$ where z is the complex response amplitude. If z is to remain constant across the array then it is simple to calculate R values that will be result in specific P values. The increased values of P_{net} achieved in this paper, as well as the constant P_{net} values, require a large variation in response across the array. This is demonstrated in Figures 14 and 15 in which the response amplitude ratio, $RA = \frac{z}{A_w}$ where A_w is the wave amplitude, is shown for the beam seas array in which R values were selected so as to achieve maximum P_{net} across the range of frequencies. The response amplitudes required to achieve the P_{net} values at the peak in the unlimited case (Figure 14) are very large, although as the device is assumed to have a draft of 5m, the maximum predicted response amplitude of 3.205 on float 1 (in beam seas) is theoretically possible (in the unit amplitude waves considered here), however the basic assumption that the variation in water plane area is negligible would be incorrect with such a large variation in draft for a hemisphere. In order to predict more achievable P_{net} values a modification to the model would be required so as to account for a variation in stiffness [9]. When the R values are restricted so that $0 \le R \le 2\max(B_0)$, not only are the damping values more easily achieved, but the response amplitude ratio is much more reasonable.

When tested experimentally, it has been shown that the amplitude of the motion of the free surface between fixed cylinders within a linear array is substantially less than linear theory predicts [10]. This is due to practical restrictions such as friction. The linear predictions presented in this paper may therefore not occur in physical tests. All of the work in this paper has focused on linear wave response. According to linear wave theory any irregular wave can be created by superposing a selection of linear (sinusoidal) waves but array response is not straightforward to predict unless response is assumed to be optimal. It is clearly important to understand how arrays behave in irregular waves but such an extension is not considered here.

A similar increase in average power output can also be achieved by varying the masses of individual devices within the array. For simplicity, it is assumed that mass can be varied without variation of float draft such that the hydrodynamic parameters do not vary with mass. Clearly this is not possible with all devices since implementation would require an external force (non-buoyancy) to support the additional mass. Even if this is available, an unlimited variation of mass is clearly impractical. However, over a small range (±25% shown in Figure 8) mass variation would be straightforward to implement for structure supported devices. For an axisymmetric, straight-walled buoyant device the hydrodynamic parameters would not change significantly due to a 25% variation of draft although this variation may be more significant within an array.

A system by which both the mass and the damping could be varied within limited ranges would allow for even greater manipulation of the power output. Using the current method to determine such M and R values would require numerous iterations, and would be computationally inefficient. To reduce computation time, the current method could be superseded by a method which applies a relationship between the R values determined in this paper and the diagonal elements of the radiation damping method.

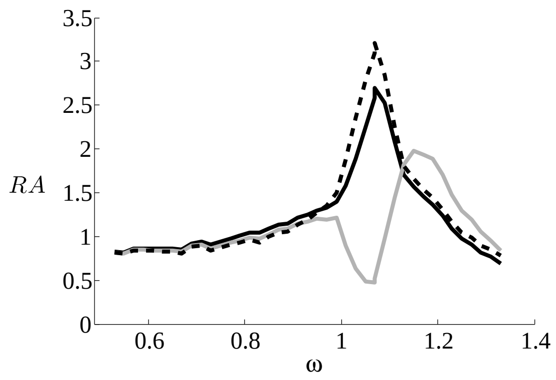

Figure 14: Response Amplitude Ratio, *RA*, for a 5×1 array of hemispheres of radius 5 m in beam seas with *R* values chosen from an unlimited range to achieve maximum net power (P_{net}, thin dotted line Figure 6): solid black line = float 1; solid grey line = float 2; dashed line = float 3

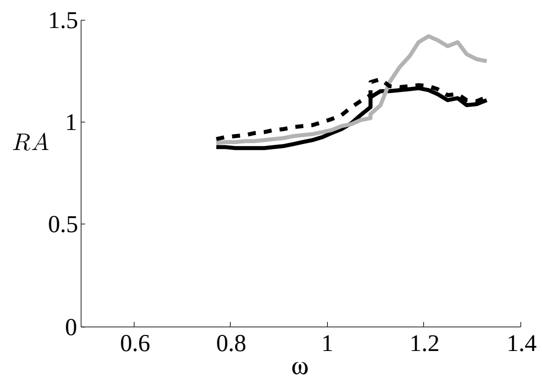

Figure 15: Response Amplitude Ratio, *RA*, for a 5×1 array of hemispheres of radius 5 m in beam seas with *R* values chosen from the range $R \leq 2\max(B_0)$ to achieve maximum net power (P_{net}, thick solid line Figure 6): solid black line = float 1; solid grey line = float 2; dashed line = float 3

6 Conclusions

This paper has investigated whether an appropriate diagonal mechanical damping, *R*, matrix can be applied in order to achieve maximum or constant net power over a range of wave frequencies. Three approaches for obtaining a diagonal damping matrix have been considered based on i) the radiation damping matrix, ii) the optimal damping matrix for an array of equal mass floats and iii) an iterative approach. For a particular frequency, the values of mechanical damping required on each float within the array that maximise net power output do not appear to be directly related to the diagonal elements of the radiation damping matrix ($R = diag(B)$). Over most of the frequency range, the diagonal of the optimal damping matrix for a fixed mass system provides maximum net power. In this case there is minimal difference between the damping of each float. However, close to the maximum power output from an isolated device, greater net power can be obtained by using damping values that are determined by an iterative approach. A least squares analysis has been conducted using the Gauss-Newton algorithm. If an arbitrary value of mechanical damping can be applied, a 12% increase is achieved at the frequency corresponding to the greatest net power for beam seas, and an 18% increase in head seas. When a restriction is placed on the range of *R* values, $R \leq 2\max B_0$, the peak increase reduces to $\sim 7\%$ in beam seas, A similar iterative approach has also been used to determine a unique mass for each float in order to maximise net power. Assuming mechanical damping to be equal to the diagonal of the radiation damping matrix, similar increases of net power are found if the mass of each float is varied by less than 25%. The iterative process used is adequate for solving for either mechanical damping or mass to optimise net power. However, the process is computationally inefficient, particularly for optimisation of both mass and damping. An analytic approach for obtaining damping has also been described which provides damping assuming that the power output of individual floats is known. Although it remains to be determined whether these findings can be translated to real world applications, they do represent the behaviour of a realistic closely spaced array in which float mass cannot change significantly and damping is applied to only the heave oscillation of each float.

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