Analysis of potential power output of a device using a wave-by-wave approach

H.C.M. Smith[1], G.H. Smith[1], V. Venugopal[2] and T. Davey[2]

[1] Camborne School of Mines
University of Exeter, Cornwall Campus
Penryn, UK TR10 9EZ
E-mail: h.c.m.smith@exeter.ac.uk
g.h.smith@exeter.ac.uk

[2] Institute for Energy Systems
University of Edinburgh
King's Buildings, Mayfield Road
Edinburgh, UK EH9 3JL
E-mail: V.Venugopal@ed.ac.uk
Tom.Davey@ed.ac.uk

Abstract

The assessment of energy production from a wave energy converter commonly uses a stochastic approach. For short-term evaluations, a sea state can be represented in the frequency domain by an energy density spectrum. If a suitable frequency-dependent transfer function for the device is defined, an estimate of the total energy output for that specific sea state can be made. However, using a spectral averaging of the sea state in this way fails to account for short-term events that may significantly alter the output estimated using the spectral approach.

This paper uses a 'wave-by-wave' method to assess the potential energy production of a hypothetical device, using recorded time series of sea surface elevation. The device is modelled as a damped linear oscillator, with the assumption that the device can be rapidly 'retuned' to maximise its response to changing wave properties. The method involves calculating the power output for each individual wave in the time series for a range of tuning rates and power transfer function (PTF) bandwidths. The average power output for each time series is then compared with estimates made using the traditional spectral method. The results illustrate the extent to which the estimates of power output differ depending on the rate of tuning and bandwidth of the PTF.

Keywords: device tuning, power output, power transfer function, Wave Hub, wave power

Nomenclature

a	= constant
c_g	= wave group velocity
d	= damping term for wave energy converter (WEC)
d_{ps}	= WEC damping due to power take-off
d_w	= WEC damping due to device motion
F	= force acting on WEC
f	= wave frequency
f_c	= centre frequency of power take-off function
f_e	= inverse of wave energy period
g	= acceleration due to gravity
H	= wave height
H_{m0}	= significant wave height
K	= restoring force stiffness of WEC
m	= mass of WEC
m_n	= n^{th} spectral moment
N	= number of individual waves in the sea state
P	= power transported per unit wave crest length
P'	= output power of WEC
$\overline{P}$	= average power
PTF	= power transfer function
S	= slew rate of WEC
$S(f)$	= variance density function
T	= wave period
T_e	= energy period
y	= horizontal displacement of WEC
ε	= factors determining shape and response of PTF
ρ	= density of seawater
σ	= bandwidth of PTF
ω_0	= resonant angular frequency of WEC
ω_d	= damped angular frequency of WEC

1 Introduction

The concept of producing electricity from the energy of ocean waves has been the subject of ongoing research and development for over thirty years. In recent years, the industry has reached the point where full-scale wave energy converter (WEC) prototypes have been tested and proven in real seas, and numerous commercial projects are now in the planning and development stages.

A key aspect of any wave power development is predicting the power output levels of the devices deployed. Long-term predictions are needed in order to assess the economic viability of the project, but equally important are short-term operational predictions, to enable assessments of fluctuating levels of power output. All WECs will respond to incoming waves slightly differently, depending on their design and orientation. However, for all oscillating devices, the power generated will be a function of the frequencies of both the incoming wave and the oscillation of the device itself, which relates to the design and control of the power take off. A response curve, or power transfer function (PTF), giving the proportion of input power that will be converted at each frequency, can be defined for every device. Devices may be designed to operate at only one frequency, but the ability of the device to be 'tuned' to the peak energy frequency of the sea state in order to maximise the power output would increase the energy recovered from the sea state. The rate of tuning will vary with device; slow-tuning devices will adjust their resonant frequency over periods ranging from minutes to hours, whereas a fast tuning device would do so in a matter of seconds [1]. This type of fast response is difficult to engineer, but would produce the highest levels of power output.

The usual method of power prediction is a stochastic, or ensemble-average, approach. Recorded time-series of sea surface elevation are converted into the frequency domain to produce an energy spectrum that is representative of the sea state over the period of the record. Depending on the measurement process followed, this can be up to 30 minutes or longer. The power for each frequency of the spectrum can be calculated to produce a 'power spectrum' (described fully in Section 3) which, when multiplied by the device PTF, enables an estimate of the power output of the device for that sea state to be made. Over longer periods, sea states classified by their spectral wave height and period parameters can be represented in a bi-variate scatter diagram (usually significant wave height H_{m0} vs energy period T_e) giving their frequency of occurrence. By multiplying these frequencies of occurrence by the corresponding power outputs in developer-produced 'power matrices', the total power output over a specified period can be calculated. A particular problem with the above approaches is that representing a sea state as a spectrum or an H_{m0}-T_e pair eliminates any variation in individual wave characteristics, such as large waves or groups, that might influence the device performance.

This paper examines the use of a wave-by-wave, or time-domain, method for predicting the power output of a simple device, using a range of tuning rates. The results are compared with those obtained using more traditional spectral methods. The study utilises data recorded over a period of one month at the planned Wave Hub site in the southwest UK. An initial review of methods of wave data analysis and power prediction is given, followed by an overview of power transfer functions for WECs. The methodology for applying these techniques to the recorded wave data is then discussed, the results of the study are presented and conclusions drawn.

2 Wave energy converter power take-off function

Wave energy converters are complex hydrodynamic structures, with widely differing power transfer characteristics. However, it can reasonably be assumed that a simple device can be approximately modelled as a damped linear oscillator. Falnes [2] applies the general mechanical principles to the case of a WEC, and his methodology is summarised by Smith *et al.* [3]. If incident waves are assumed to be small and regular, and therefore sinusoidal, the WEC response can be given by the force equation

$$m\ddot{y} + d\dot{y} + Ky = F\cos\omega t \tag{1}$$

where y is the horizontal displacement. The mass m comprises the mass of the device and the added virtual mass of entrained water that moves as it oscillates. The damping term d includes both the drag of the device relative to the water plus wave radiation damping and internal friction (d_w), and damping of the power take-off system (d_{ps}). K represents the restoring force stiffness, and comprises components due to the characteristics of the WEC, the power take-off system, and potentially the mooring system.

From this equation, results for the undamped (ω_0) and damped (ω_d) angular frequencies, and the amplitude of motion (y) are found to be:

$$\omega_0 = \sqrt{\frac{K}{m}} \tag{2}$$

$$\omega_d = \sqrt{\frac{K}{m} - \frac{d^2}{4m^2}} \tag{3}$$

$$y = \frac{F\cos(\omega t + \varepsilon)}{\sqrt{\left(K - m\omega^2\right)^2 + d^2\omega^2}} = C\cos(\omega t + \varepsilon) \tag{4}$$

The power extracted by the device is given by $d_{ps}\dot{y}^2$, therefore from the result for y given in Eq. 4, an expression for the mean power can be found:

$$\begin{aligned}\overline{P} &= 0.5 d_{ps} C^2 \omega^2 \\ &= \frac{0.5 d_{ps} F^2 \omega^2}{(K - m\omega^2)^2 + (d_w + d_{ps})^2 \omega^2}\end{aligned} \tag{5}$$

Maximum power will be achieved when $\partial\overline{P}/\partial\omega = 0$ and $\partial\overline{P}/\partial d_{ps} = 0$, giving

$$\omega = \sqrt{\frac{K}{m}} \text{ and } d_{ps} = d_{\omega} \qquad (6)$$

i.e. maximum power is obtained at the resonant frequency, ω_0, and when the damping of the power take-off system matches the damping associated with the motion of the device.

Eq. 5 thus provides a power transfer function (PTF) for a simple WEC, providing an equation for output as a function of the frequency of oscillation of the device. The device can be tuned by adjusting the resonant frequency of the device by altering K or m. Changing the damping characteristics d will alter the bandwidth and peakiness of the PTF.

An alternative method was used by Wolf [4], in which the power transfer function is defined as a frequency-dependent 'absorption length', and relates the available power per crest length to the power extracted at each frequency. It can be approximated as a Gaussian distribution with bandwidth σ, centred on the frequency f_c, i.e.

$$PTF = \exp\left(-\frac{1}{2}\frac{(f - f_c)^2}{\sigma^2}\right) \qquad (7)$$

This definition of the PTF will be used for the calculations in this paper. It will be used in two forms. The first, PTF_1, will define a family of curves with varying bandwidth, but constant height. Therefore, as the bandwidth increases, the area under the curve will also increase, leading to greater power absorption. The form of PTF_1 is given in Eq. 7. However a PTF where the height remains constant as the bandwidth changes is not a realistic scenario for a wave energy converter. A second form, PTF_2, has therefore been defined in which the area under the curve remains constant; the height therefore decreases as the bandwidth increases. This is given by

$$PTF_2 = \frac{1}{a\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{(f - f_c)^2}{\sigma^2}\right) \qquad (8)$$

where a is a constant, chosen in this study to provide a PTF height of one (i.e. 100% absorption) for the narrowest bandwidth to be used of 0.01. Figs. 1 and 2 illustrate a family of fixed height and variable height PTF curves respectively, for a range of PTF bandwidths.

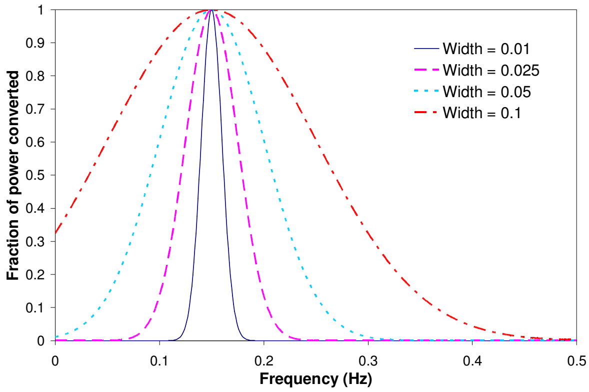

Figure 1: Fixed height power transfer functions used in estimation of power output.

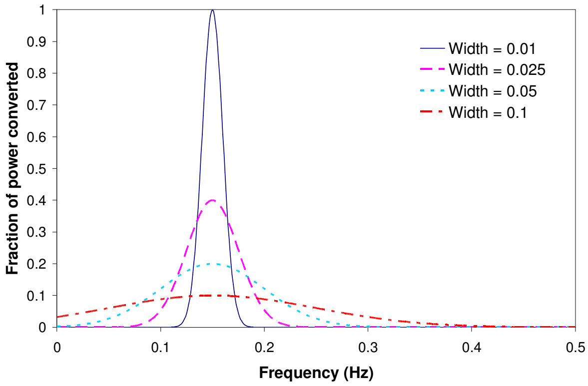

Figure 2: Variable height power transfer functions used in estimation of power output.

3 Wave analysis and power estimation

A critical aspect of selecting a location for a wave energy development, or for evaluating wave energy devices once deployed, is a full understanding of the local wave resource. This will vary significantly from site to site, with different locations exhibiting different characteristics. For example, Hawaii typically experiences large, regular swell waves with consistent characteristics. Regions such as the North Sea have very irregular sea states, strongly influenced by the wind conditions. In other regions, such as the western UK and Ireland, mixed sea states comprising both wind and swell waves are often found.

Spectral analysis

Resource assessments commonly comprise measurements via wave buoy. Sea surface elevations are recorded over a fixed period, e.g. 20 minutes, during which the sea state is considered to be stationary. This time series is then processed to produce a spectrum of energy density in the frequency domain, $S(f)$. Spectral moments, m_n, defined as

$$m_n = \int_0^{\infty} f^n S(f)df \qquad (9)$$

can be used to calculate parameters such as the significant wave height, H_{m0}, and the energy period, T_e, i.e.

$$H_{m0} = 4\sqrt{m_0} \quad (10)$$

$$T_e = \frac{m_{-1}}{m_0} \quad (11)$$

It can be shown (e.g. [5]) that the power transported by a wave per metre of crest length is

$$P = \rho g \int c_g(f) S(f) df \quad (12)$$

where c_g is the group velocity of the wave. In deep water, this becomes

$$P = \frac{\rho g}{4\pi} \int f^{-1} S(f) df \quad (13)$$

If a power transfer function for a device is defined as $PTF(f, \{\varepsilon\})$, where ε represents factors determining the shape and response of the power transfer function, the output power, P', for a particular sea state can be estimated by multiplying the PTF by the power spectrum and integrating over the frequency range, i.e.

$$P' = \frac{\rho g}{4\pi} \int f^{-1} S(f) PTF(f, \{\varepsilon\}) df \quad (14)$$

This is defined as the spectral method of power output estimation and is illustrated in Fig. 3, using the PTF form given by PTF_1 (Eq. 7), with the centre frequency f_c set equal to the inverse of the energy period of the sea state.

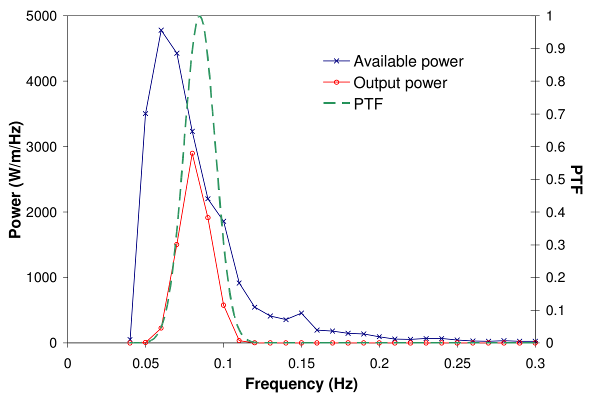

Figure 3: Modification of the power density spectrum by multiplication by the PTF.

Spectral analysis is also used when considering long-term datasets. The frequency of occurrence of H_{m0} and T_e pairs, calculated from spectra, can be summarised in a scatter diagram, with height and period 'bins' selected to match those in a device 'power matrix'. A power matrix, produced by the device developer, provides the power output for each H-T pair. Multiplying the power for each sea state within the power matrix by the corresponding frequency of occurrence enables the total annual energy production to be calculated.

Wave-by-wave analysis

Measuring individual waves from a time series is not a novel method of calculating wave statistics, indeed prior to the availability of computers for calculating power from spectra, this was the most common method of obtaining wave statistics. However, as a method for estimating the power output of a wave energy device, it has the potential to provide greater accuracy, particularly for fast-tuning devices, than the spectral method described above.

Individual waves were measured in each record using the zero-upcrossing method, as illustrated in Fig. 4. An assumption was made that each wave was a regular wave with height H_i and period T_i. The power transported per unit crest length for each wave was calculated using the deep-water power equation for regular waves,

$$P_i = \frac{\rho g^2 H_i^2 T_i}{32\pi} \quad (15)$$

The average power for each sea state is therefore

$$\bar{P} = \frac{\sum P_i}{N} \quad (16)$$

where N is the total number of individual waves in the sea state time series.

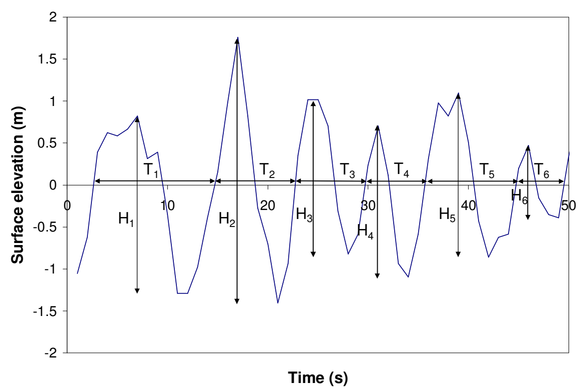

Figure 4: Illustration of the identification of individual wave heights and periods in the time series record.

To calculate the power converted using the wave-by-wave method, the power of each wave is multiplied by the PTF in one of its two forms. However, whereas to achieve maximum power conversion for the spectral method, the centre frequency of the PTF was matched to the energy frequency f_e (i.e. the inverse of the energy period) of the sea state, the aim when using the wave-by-wave method is to achieve a centre frequency as close as possible to the frequency of the individual wave. This will change with each wave, and hence the concept of device tuning, and in particular the *rate* of tuning, becomes critical. In this paper, the tuning rate has been defined as a rate of change of frequency, or slew rate, S , where

$$S = \frac{\Delta f}{\Delta t} Hz/s \qquad (17)$$

Thus one can characterise a device of slow response with a low slew rate, whilst one capable of changing its system characteristics rapidly will have a higher slew rate.

For the calculation of total power converted for a particular record, the power output for the first wave is calculated with a PTF centre frequency that is the inverse of the energy period for the sea state, i.e.

$$P_1^{'} = \frac{\rho g^2 H_1^2 T_1}{32\pi} PTF(f, f_e, \sigma) \qquad (18)$$

where H_1 and T_1 are the height and period of the first wave. It is assumed that the device is then retuned over the period of the first wave, with the centre frequency moving toward the frequency of the next wave. The power output of the second wave is computed thus:

$$P_2^{'} = \frac{\rho g^2 H_2^2 T_2}{32\pi} PTF(f_2, f_{c2}, \sigma) \qquad (19)$$

where

$$f_{c2} = f_e \pm T_1 \cdot S \qquad (20)$$

The sign in Eq. 20 is selected so that the centre frequency of the transfer function moves toward the frequency of the next wave. If the slew rate is high enough, the centre frequency will equal the frequency of the next wave, leading to maximum power conversion.

This process is then repeated on a wave-by-wave basis for every wave in the record. The average power output for the record is therefore

$$\overline{P}^{'} = \frac{\sum P_i^{'}}{N} \qquad (21)$$

4 Methodology

The spectral and wave-by-wave methods of power output estimation were applied to one month of sea states recorded at the Wave Hub site at approximately 50m depth (see Fig. 5) using a Seawatch Mini wave buoy.

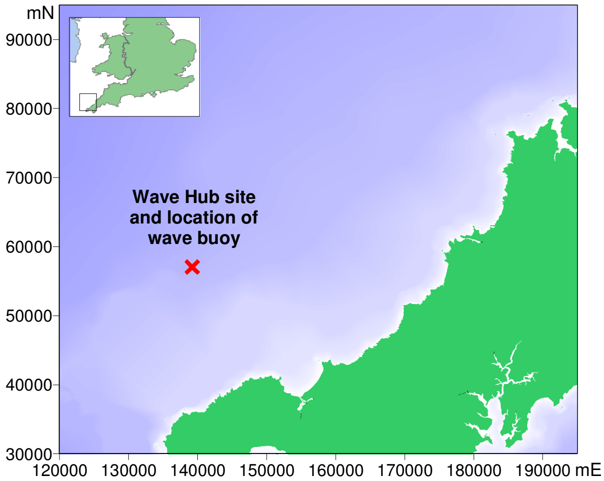

Figure 5: Location of the wave buoy used to collect data at the Wave Hub site.

The dataset comprised time series of sea surface elevation recorded at 1 Hz in bursts of just over 17 minutes (1024 data points) every two hours. The same data was available as processed spectra over a frequency range of 0.04 – 0.5Hz, at 0.01Hz frequency spacing. Fig. 6 illustrates the range of H_{m0} and T_e values recorded over the period, showing a significant variation over the month, including one major storm event.

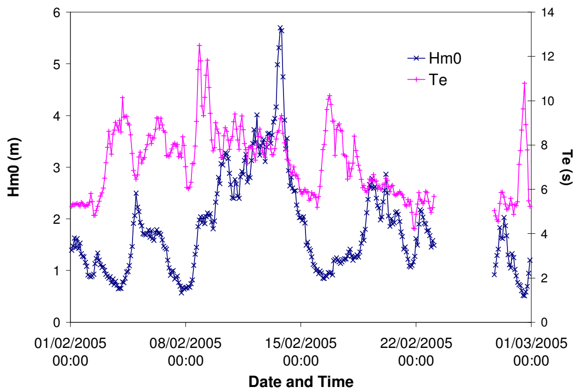

Figure 6: Significant wave heights and energy periods of the datasets used in the power output assessment.

For both the spectral and wave-by-wave calculations, the bandwidth of the PTF was varied over four values: 0.01, 0.025, 0.05 and 0.1. Initially a fixed height PTF was used (as illustrated in Fig. 1), followed by a variable height PTF with the area under the curve remaining constant, and height decreasing with bandwidth (as illustrated in Fig. 2). For the wave-by wave approach, the PTFs were applied at six different slew rates: 0.0001, 0.001, 0.0025, 0.005, 0.01 and 0.05Hz/s.

5 Results

The power transported per metre of crest length for each sea state, calculated using the spectral method, was initially compared with the average power for each time series found using the wave-by-wave method (Fig. 7). It can be seen that the spectral method produces higher estimations of power. Values calculated using the wave-by-wave method range from 6% to 54% lower, averaging 24% lower over the month. This is a surprisingly large discrepancy. However, it is surmised that this is due to the comparatively large number of low-period waves which occur in the time series and contribute very little power to the sea state. When dividing the total power from all waves by the number of waves to obtain the average power in the sea state (Eq. 16), the result is disproportionally influenced by these small waves. Venugopal and Smith [6] investigated the effect of filtering out low period waves, and found it had very little effect on the power available for extraction. A similar approach was followed here, with all waves where $T < 3s$ eliminated from the record. When the average power for each sea state was re-calculated, the results were closer to the spectral values, averaging 13% lower over the month. The filtered power results are illustrated in Fig. 8.

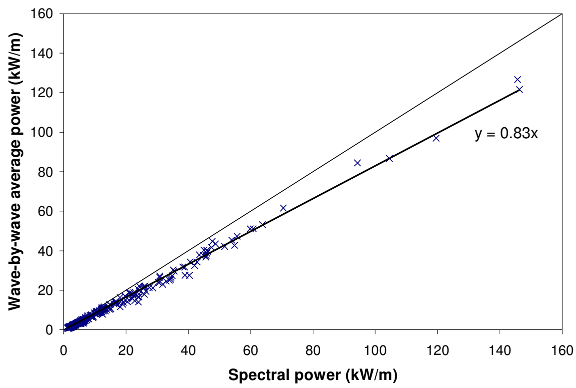

Figure 7: Comparison of power transport calculations using the spectral method and wave-by-wave approach for each sea state, with all waves accounted for in the wave-by-wave approach.

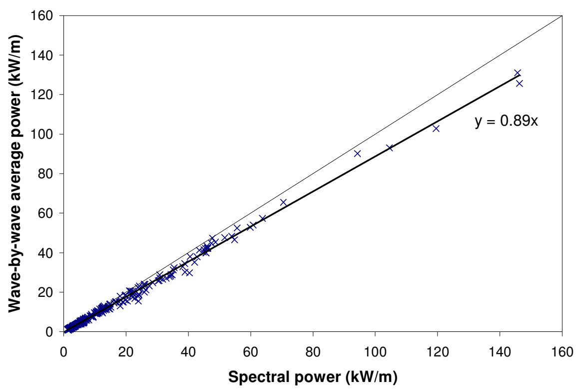

Figure 8: Comparison of power transport calculations using the spectral method and wave-by-wave approach for each sea state, eliminating waves with a period < 3s in the wave-by-wave approach.

For each wave record, the estimated power output was found using both the spectral and wave-by-wave methods. Calculations were performed for each PTF bandwidth and, in the case of the wave-by-wave method, each slew rate. Initially, all waves were accounted for in the wave-by-wave calculations, including those with low wave periods. The proportion of power output relative to the total power for each sea state, as calculated above, was then found. The variation in the average proportion of output power with PTF bandwidth and slew rate over the month is illustrated in Figs. 9 (fixed PTF height) and 10 (variable PTF height). For both forms of the PTF, the spectral method produced the lowest fractional power conversion, regardless of PTF bandwidth. However, this could be because the total power calculated using the spectral method was significantly higher than that found using the wave-by-wave method. It was found that repeating the process using the variable height PTF with the filtered wave-by-wave record, i.e. only including waves with a period > 3s, had very little impact on the results (Fig. 11). The maximum increase in the fraction of power converted was 5%, with most results increasing by less than 2%.

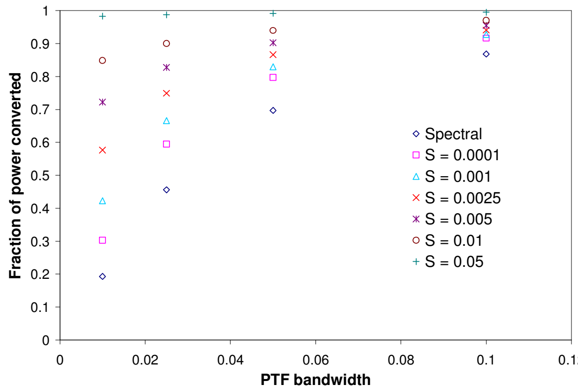

Figure 9: Variation in the average fraction of wave power converted with PTF bandwidth and slew rate for fixed height PTFs.

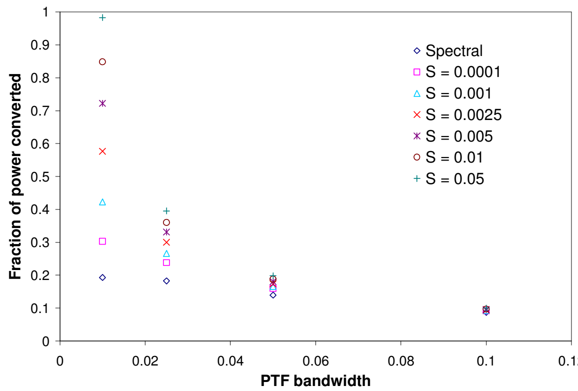

Figure 10: Variation in the average fraction of wave power converted with PTF bandwidth and slew rate for variable height PTFs.

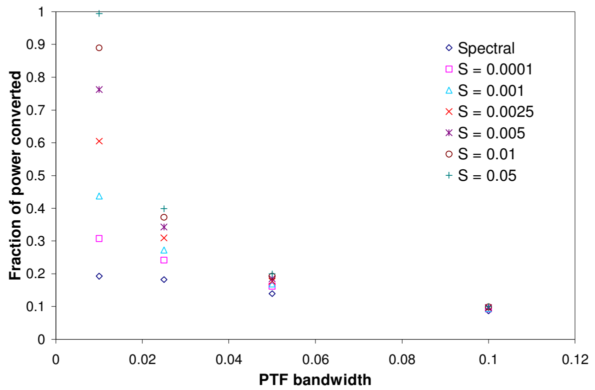

Figure 11: Variation in the average fraction of wave power converted with PTF bandwidth and slew rate for variable height PTFs. Calculations using the wave-by-wave approach do not include waves with a period < 3s.

When the average *total* power outputs over the month are compared (Fig. 12), although the spectral method still produces the least power at narrow PTF bandwidths, the power output is slightly higher than the output for the wave-by-wave method at the widest bandwidth of 0.01. However, for both the total power output and the fractional power converted, the trend in the results is very similar. At narrow PTF bandwidths, high levels of power conversion are achieved with high slew rates – effectively, the spectral peak is almost perfectly matched with the centre frequency of the PTF, providing close to 100% power conversion. As the slew rate decreases, power conversion levels also drop. This trend continues with increasing PTF bandwidth when fixed height PTFs are used, except the power conversion increases for the lower slew rates. However, when the more realistic variable height PTFs are applied, the power conversion drops sharply as the bandwidth increases and the maximum value of the PTF is correspondingly lower. At the widest bandwidth value of 0.1, there is minimal difference in the fraction of power converted for any of the methods used.

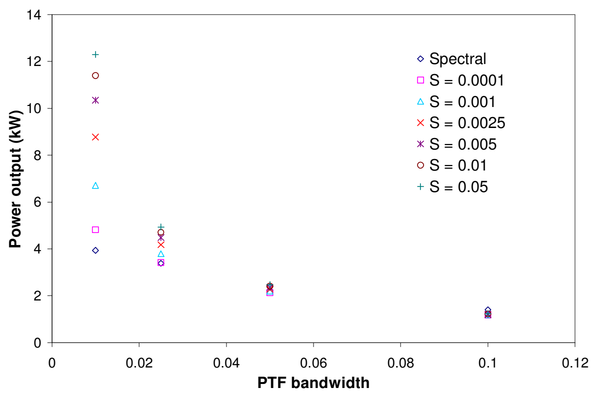

Figure 12: Variation in the total power output with PTF bandwidth and slew rate for variable height PTFs.

The results thus far have only considered the average results over the month of sea states. As was illustrated in Fig. 4, the wave parameters were highly variable throughout the month. The effect of different types of sea state on the results was therefore investigated further.

Four individual sea states were identified from the data record as representative of swell-dominated, wind-dominated, bi-modal and mixed sea states. Their spectra are illustrated in Fig. 13. The results for each of these sea states are shown in Figs. 14a – d, enabling an examination of what effect the spectral shape has on the levels of power conversion. Only results using variable height PTFs are shown to provide a more realistic scenario.

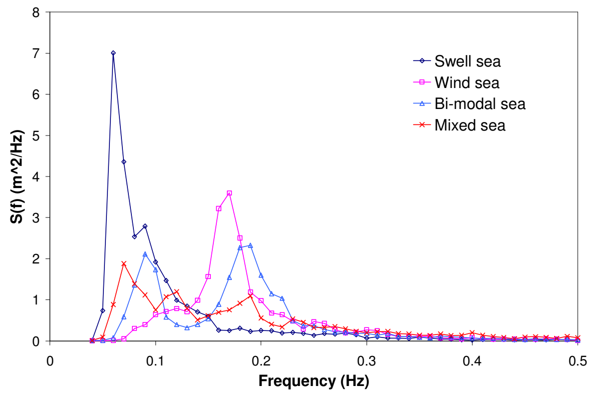

Figure 13: Energy density spectra for the four different sea states to be examined in more detail.

The results illustrate the difference in power conversion between sea states with one clearly defined spectral peak, i.e. the swell and wind seas, and those comprising two or more separate wave systems, the bi-modal and mixed seas. Certain features are consistent for all four sea states and with the average results above, for example the spectral method consistently producing the lowest power conversion, and all methods producing similarly low levels of power conversion at the widest PTF bandwidth. However, there are some key differences between the sea states with one clearly defined spectral peak, i.e. the swell and wind seas, and those with multiple peaks. The key difference can be seen at the narrowest PTF bandwidth of 0.01. Results for all four sea states for this bandwidth are summarised in Fig. 15. With the exception of the highest slew rates, the power conversion for the mixed sea states is considerably lower for the multi-peaked sea states than for the swell and wind seas. The power conversion for the wind-sea, where spectral energy is spread over a wider range of frequencies, is also lower than for the swell sea state. However at the highest rate of tuning, the spectral shape becomes almost irrelevant, with all sea states achieving power conversion of close to 100%

It is also interesting to note that for the spectral method of calculating power output, the maximum power conversion for multi-peaked sea states occurs at a wider PTF bandwidth of 0.05 which is able to capture more power across a wider part of the spectrum.

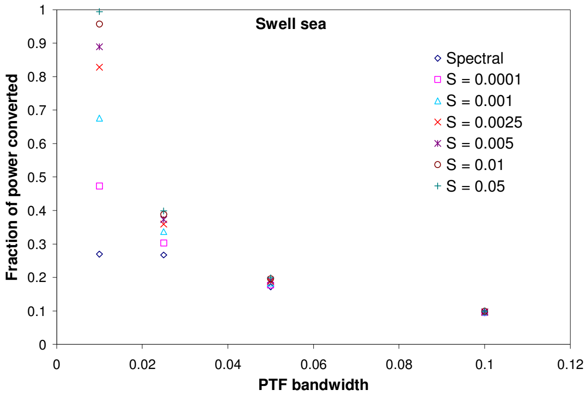

Figure 14a: Variation in the average fraction of wave power converted for a swell-dominated sea state.

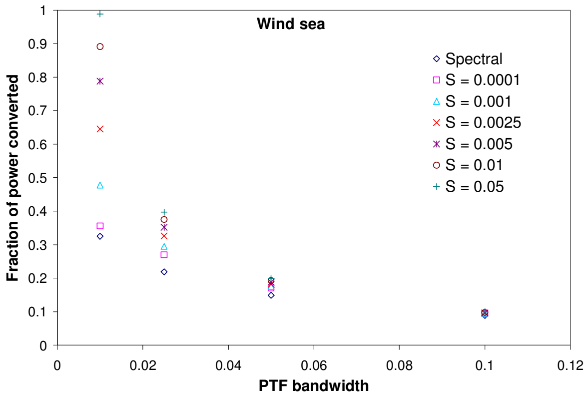

Figure 14b: Variation in the average fraction of wave power converted for a wind-dominated sea state.

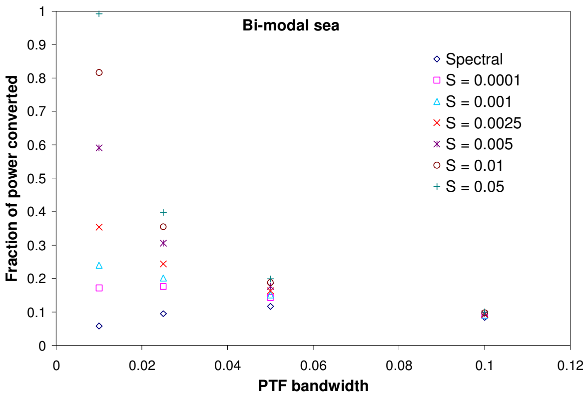

Figure 14c: Variation in the average fraction of wave power converted for a bi-modal sea state.

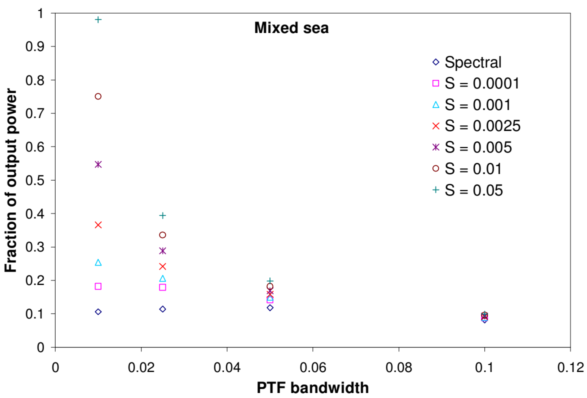

Figure 14d: Variation in the average fraction of wave power converted for a mixed sea state.

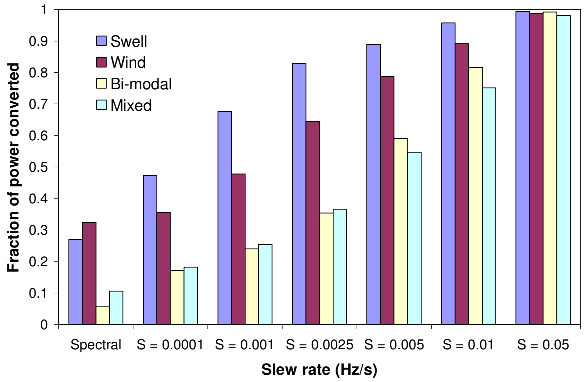

Figure 15: Fraction of wave power converted for four different sea states with a PTF of 0.01.

6 Conclusions

This paper has introduced a new method of calculating the power output for a simple wave energy device, which allows the tuning rate of the device to be taken into account. The method makes a number of assumptions, including a consideration of each individual wave as a regular wave, and is thus not an accurate means of determining the average power of a particular sea state. It has been found to under-estimate the power when compared with spectral calculations, although the discrepancy is substantially reduced when low-period waves are eliminated from the calculation. It is, however, a useful tool for calculating the proportion of power converted.

The wave-by-wave method applied here is somewhat arbitrary in that it does not represent the detailed time response of an actual device. It serves however to provide a relatively simple method to ascertain the desirability of a specific 'tuning rate' within a given sea state. The use of this method has particularly highlighted the potential gains in power output for a device with a high rate of tuning and a narrow PTF bandwidth. As the tuning rate decreases and the bandwidth increases, the results approach those obtained using the traditional spectral method. It is also highly applicable in multi-peaked sea states, when a calculation of power output using the spectral approach with a PTF tuned to the energy frequency of the sea state produces particularly low levels of power conversion. The wave-by-wave method illustrates that as the device tuning rate increases, the power conversion for multi-peaked sea state approaches, and is eventually comparable to, the output for clearly defined single-peaked spectra.

Acknowledgements

This work is a part of the EquiMar project and the authors are grateful for the funding by the European Community's Seventh Framework Programme FP7/2007-2013 under grant Agreement No. FP721338. The authors would also like to thank the Southwest Regional Development Agency (SWRDA) for providing the Wave Hub datasets. The first and second authors gratefully acknowledge the Peninsula Institute for Marine Renewable Energy (PRIMaRE) and Scottish and Southern Electricity who support their respective posts.

References

[1] Carbon Trust website: http://www.carbontrust.co.uk/technology/technologyaccelerator/ME_guide2.htm [accessed 20/4/09]

[2] J. Falnes. *Ocean Waves and Oscillating Systems.* Cambridge University Press, 2002

[3] G.H. Smith, V. Venugopal and J. Fasham. Wave spectral bandwidth as a measure of available wave power. *Proc. 25th International Conference on Offshore Mechanics and Arctic Engineering*, Hamburg, 2006.

[4] D. Wolf. *Sensitivity of power output to wave spectral distribution*. Report for Seapower Europe Ltd, 2002.

[5] M.J. Tucker and E.G. Pitt. *Waves in Ocean Engineering*. Elsevier Ocean Engineering Series, Volume 5, 2001.

[6] V. Venugopal and G.H. Smith. The effect of wave period filtering on wave power extraction and device tuning. *Ocean Engineering*, 34: 1120-1137, 2007.