Wave Farm Design: Preliminary Studies on the Influences of Wave Climate, Array Layout and Farm Control

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Abstract

In this paper preliminary results regarding the assessment of the energy absorption characteristics of an array of wave energy converters (also referred to as a wave farm) are presented. Regular and irregular waves are used as input in a frequency domain hydrodynamic model which allows iterations in the array layout and farm control strategy. Under such an approach each array element can be controlled independently while keeping the design objective (maximisation of the wave farm energy yield).

The approach is initially verified by comparing the solution of the radiation and diffraction problems for the array with the analogous results from a semi-analytic method developed at the University of Oxford. Additional regular wave simulations identify the most promising areas by quantifying the interaction factor as a function of the incident wave frequency and wave heading. Irregular waves which describe representative frequency spectra are then used as input to quantify the power absorption characteristics from an isolated wave energy converter. Finally, the same representative seas are used as input when evaluating the energy absorption by an array of wave energy converters, while iterating on the array layout and control settings associated with each array element.

The overall objective of the study is to quantify the influence of the array layout and farm control in the performance of a wave farm under the action of irregular waves. The results show that the energy yield is affected by such factors, hence these can be seen as key design drivers when considering the potential for reduction in the uncertainty and thus the cost of energy associated with a wave farm. Further studies may address additional constraints, either technical or economical. This study is expected to contribute to the development of specific modules of GH WaveFarmer, a tool that aims to optimise the design of wave farms.

Keywords: floating bodies; hydrodynamic interactions; wave farm.

Nomenclature

a	radius of the cylinder
A	wave amplitude
A_{ij}	added-mass matrix
B_{ij}	radiation damping matrix
c	clearance
C_{ij}	hydrostatic stiffness matrix
D_{ext}	applied (external) damping coefficient
f	wave frequency
g	modulus of the acceleration of gravity
k	wavenumber
h	water depth
$(h-c)$	draft of the cylinder
H_{m0}	significant wave height
M	Number of bodies per WEC
N	Number of WECs in the array
P	Absorbed power (kW/m)
$\bar{P}_n$	average absorbed power per WEC (irregular waves)
$\bar{P}$	average absorbed power by the array (irregular waves)
q	interaction coefficient
$S(f)$	frequency spectrum
t	time
T_{-10}	wave energy period
T_{peak}	peak (wave) period
x, y	Cartesian co-ordinate system
X_i	wave exciting force
Z	radiation impedance matrix
β	wave heading
Φ	velocity potential
ϕ	complex velocity potential
ξ	Response Amplitude Operator (WEC motions)
ω	wave angular frequency

Acronyms

BEM	Boundary Element Method
PTO	Power Take-Off
WEC	Wave Energy Converter

1 Introduction

The commercial development of wave energy in utility-scale projects will rely on the deployment of multiple units in an array, creating multi-MW wave farms. Several engineering challenges are associated with the definition and optimisation of a wave farm, namely the geometrical layout, electrical connections between array elements (and the link to shore), mooring configurations (if applicable), control aspects and hydrodynamic interactions between devices.

All of the above mentioned factors will have an impact on the energy yield. It is therefore essential that project developers have access to tools which are able to accurately assess and optimise the key factors which define the wave farm, maximising the energy output. Such tools should be seen as vehicles which allow project developers and / or investors to assess the suitability and potential of a given technology to a specific project, in a close parallel with the project development standard procedures which are available in wind energy.

Array interactions have been a recurrent R&D topic in offshore engineering over the last decades. Pioneering work has also been done in the wave energy field [e.g. 1-6], focusing primarily on linear arrays of point absorbers, equally and unequally spaced. In the following paragraphs of this section a short review of some selected publications is presented.

In [7] an overview of the key results related to the performance of wave energy converters when placed in arrays are presented, along with the fundamental principles behind some hydrodynamic interaction theories used in the field, such as the point absorber and the plane wave approximations. The latter are also approached, in more detail, in [8]. In summary the point absorber assumption neglects the scattered wave field, which is equivalent to assuming that $ka \ll 1$. The plane wave approximation can be summarised as a wide spacing assumption, resulting in an incident wave field made of plane waves for each array element. In [7] regular waves results for the interaction coefficient q for small arrays of heaving or surging buoys are presented as a function of the nondimensional spacing kd between array elements, for several wave headings β. The results illustrate the potential for constructive and destructive interference, i.e. the existence of wave frequencies for which the array arrangement is beneficial $(q > 1)$ or detrimental $(q < 1)$ with regard to farm energy yield. Although early work shows that a net gain may still be possible, in [6] it is mentioned that a practical strategy for the design of wave farms may involve the minimisation of the destructive interference effects. Due to these effects the necessity of a tool to accurately quantify the energy output of a wave farm becomes clear, as a means to reduce the uncertainty associated with the project development stage if commercial wave energy projects are to be considered.

The critical factors that condition the energy output and thus the q factor need to be identified and their influence quantified. Two that have been previously addressed under the action of regular waves are control [9] and layout [e.g. 10], which in practice will be conditioned by device access routes, mooring layout (if applicable) and overall footprint of the wave farm (linked with the number of devices and thus the power rating of the farm). The previously mentioned tool will need to be able to quantify the influence of all of these, plus additional factors such as real seas, device geometry and modes of motion linked with the wave power absorption (PTO modes).

In this paper a preliminary study which addresses the influence of control and layout in the wave farm energy yield under the action of irregular waves is presented. Irregular waves should be seen as another critical design factor which influences the design. Firstly comparisons with results from a semi-analytical method for an uncontrolled array of truncated cylinders previously presented in [11] are made, in order to verify the solution of the radiation and diffraction problems, with the latter allowing also a discussion related to the excitation forces associated with each array element. The optimal q for the array outlined in [12] is then presented as a function of ka and β, providing a benchmark and a starting point when extending the study to irregular waves. The findings of this paper are expected to contribute to the creation of a tool (GH WaveFarmer) that will assess, iterate and optimise the design of wave farms of N elements, for devices of arbitrary shapes and with an arbitrary number of bodies.

2 Motivation and Approach

The potential for constructive interference (i.e. the possibility of achieving an energy yield with a farm of N units which exceeds the output of N units working in isolation) has long been identified. Equally identified is the fact that it is not realistic to assume that such constructive behaviour will be present for the full range of incident wave frequencies, thus there will be destructive interference effects. The objective function when designing a wave farm is therefore twofold: 1) maximise the effects of constructive interference; 2) minimise the effects of destructive interference.

The motivation for this preliminary study is to assess the influence of several critical variables (wave climate, layout and control strategy) in the design of a wave farm. To verify the approach, which is based in the frequency domain and uses the solutions of the radiation (hydrodynamic coefficients) and diffraction (exciting forces) problems provided by a Boundary Element Method (BEM) solver (WAMIT), comparisons with semi-analytic methods [11] and [12] are firstly made for an array of uncontrolled devices. The same approach is then extended to a controlled array, in which each individual element can be independently controlled via an external damping term associated with the relative heave motion. The energy absorption results are a function of the incoming sea state.

The array configuration under study is schematised in Figure 1. It consists of a square array of four cylinders with radius a and draft $2a$, with centres equally spaced by $4a$. The water depth is also equal to $4a$. The regular wave results use a =1m, as in [11] and [12], whereas the irregular wave results use a = 10m, providing results which are more meaningful for wave energy conversion.

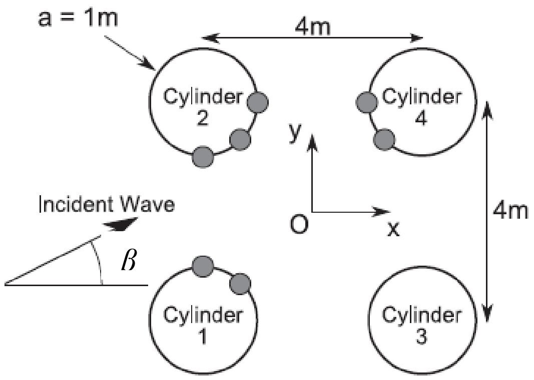

Figure 1: Initial array configuration (from [11]).

Linear wave theory is applied throughout this paper. As a result there are some well defined simplifying assumptions such as:

1. The free-surface and the body boundary conditions are linearised;
2. The fluid is incompressible and the flow is irrotational (potential flow): $\nabla^2\Phi = 0$, where Φ is the velocity potential;
3. Viscous effects like shear stresses and flow separation are not considered;
4. The bottom is assumed to be flat (and uniform);
5. Under these above mentioned assumptions all variables can be expressed as a complex amplitude times $e^{i\omega t}$ (regular waves, sinusoidal motions).

Under these assumptions, the velocity potential can be given by

$$\Phi = \mathrm{Re}\left\{\phi e^{i\omega t}\right\}, \tag{1}$$

where ϕ is the complex velocity potential, Re denotes the real part, ω is the angular frequency of the incident wave and t is time. By assuming the linear decomposition of the problem, the velocity potential can be obtained as the sum of the radiation and the wave exciting components,

$$\phi = \phi_R + \phi_S , \tag{2}$$

where ϕ_R is the radiation potential and ϕ_S the scattered potential, respectively given by

$$\phi_R = \sum_{j=1}^{6MN} \xi_j \phi_j , \tag{3}$$

and

$$\phi_S = \phi_0 + \phi_D , \tag{4}$$

where ϕ_0 is the velocity potential associated with the incident (incoming) waves and ϕ_D the diffraction potential. In Eq. (3) ξ_j are the complex amplitudes of oscillation in all the available degrees-of-freedom (6*MN*) and ϕ_j the corresponding unit-amplitude radiation potentials (those resulting from the body motion in the absence of an incident wave). Note that the summation is limited to 6*MN*, where M is the number of bodies per WEC and N is the number of WECs in the array. In the present case each array element has just one body and N=4, thus 6*MN* = 24. Even in cases where $M > 1$ it is unlikely that all 6*M* degrees-of-freedom are available, i.e. a multi-body WEC will necessary have locked modes, with power being absorbed in one or two relative modes (and thus a total of 7 or 8 degrees-of-freedom). It is important that the tools developed are able to cope with locked modes to reduce the computational effort while being flexible to allow all possible combination of locked modes.

The complex amplitudes of oscillation ξ_j are given by

$$\sum_{j=1}^{6MN}\Big[-\omega^2\left(M_{ij}+M_{ij}^E+A_{ij}\right)+ +i\omega\left(B_{ij}+B_{ij}^E\right)+\left(C_{ij}+C_{ij}^E\right)\xi_j\Big]=X_i , \tag{5}$$

where M_{ij} is the mass matrix, M_{ij}^E, B_{ij}^E and C_{ij}^E the externally applied mass, damping and stiffness matrices (respectively), A_{ij} is the added-mass matrix, B_{ij} is the radiation damping matrix, C_{ij} is the hydrostatic stiffness matrix and X_i the wave exciting force. Note that in this preliminary study M_{ij}^E and C_{ij}^E are everywhere null (i.e. inertia tuning by changing the mass of systems with e.g. ballast tanks is not considered and there is no externally applied stiffness component via e.g. a mooring line). A_{ij}, B_{ij} and X_i are calculated in WAMIT.

The absorbed power and associated control of the array and its elements are therefore limited to the influence of B_{ij}^E. In this preliminary approach the only elements of B_{ij}^E which are not null are the diagonal terms associated with the heave motion of each of the four array elements ($B_{33}^E, B_{99}^E, B_{15\ 15}^E, B_{21\ 21}^E$). The farm control strategy therefore involves four independent external damping terms. By normalising the complex amplitudes of oscillation ξ_j with the wave amplitude A, leading to $\bar{\xi}_j = \xi_j / A$, it is possible to derive an auxiliary absorber power P_{aux} which is given by

$$P_{aux} = \frac{1}{2} B_{ij}^E \omega^2 \left|\bar{\xi}_j\right|^2 , \tag{6}$$

which has dimensions of W/m^2. The usefulness of P_{aux} is particularly clear when evaluating the average absorbed power per farm element, $\bar{P}_n$, under the action of irregular waves (using the superposition principle). By definition the frequency spectrum $S(f)$ can be expressed by

$$S(f) = \frac{1}{2} \frac{A^2(f)}{df} , \tag{7}$$

and thus $\bar{P}_n$ can be given by

$$\bar{P}_n = \int 2 P_{aux} S(f)\, df \ . \tag{8}$$

Finally, the average absorbed power by an array can be quantified by

$$\bar{P} = \sum_{n}^{N} \bar{P}_n \ . \tag{9}$$

Note that in the irregular wave case B_{ij}^E is unchanged throughout the frequency range for each $S(f)$ – ensuring that the superposition principle is valid as the system remains unchanged from frequency to frequency. In practice this corresponds to a (conservative) control strategy that aims to tune the response to a particular sea state, described by $S(f)$. The objective function is the maximisation of $\bar{P}$, under all the previously mentioned constraints. In this paper the assessment is limited to the influence of the sea state, array geometry and control strategy.

The next section (Results) is divided into the several stages that lead to the preliminary assessment of relative influence of the several parameters listed above in the design of a generic wave farm of four cylindrical heaving WECs. Firstly the results associated with the verification of the approach with regard to the solution of the radiation and diffraction problems are presented, ensuring that the BEM method applied is consistent with the semi-analytical models. Secondly, optimal results for regular waves of various wave headings are presented for heaving and surging cylinders; such results allow comparisons with the subsequent results which are related to irregular waves (final two subsections of 'Results'): firstly a sensitivity analysis related to the damping coefficient as a function of the incident frequency spectrum for an isolated cylinder is presented, and secondly arrays of four cylinders (squared layout described in Figure 1) are assessed. In the latter each of the four cylinders is controlled via an independent damping coefficient (thus a total of four terms), and the average power absorbed by the array $\bar{P}$ is monitored. In other terms, and for each wave climate, the coupled influences of array layout and different farm control strategies consisting of a layer of four independent variables (damping coefficients) are assessed.

3 Results

Comparison with analytical solutions

The first set of results is presented in Figures 2 and 3, where the solutions of the radiation and diffraction problems derived in [11] and [12] are compared with new BEM predictions. The configuration under analysis is the array described in Figure 1. The comparisons are limited to the hydrodynamic coefficients (A_{ij} and B_{ij}) and the exciting force (X_i). With regard to the former, the comparisons made in Figure 3 use representative terms of the real and the imaginary part of radiation impedance matrix Z, respectively equal to B_{ij} and ωA_{ij}. As Equation (5) illustrates A_{ij}, B_{ij} and X_i are the only variables directly obtained from the BEM solution (the equation of motion is solved in a separate code, to ensure that controlled multi-body WECs can be accurately modelled).

The objective of these initial comparisons is to verify the approach, ensuring its applicability to the subsequent steps, namely when solving the equation of motion and adding the farm control layer(s). Figure 2 shows comparisons with regard to the wave exciting force (x, y, z components) for $\beta = 0$. The four cylinders are identified by the numbering listed in Figure 1, and the BEM results are marked with the 'WAMIT' legend. Overall the correlation between the analytical and the numerical results is very high, with most of the discrepancies occurring for low values of ka, and thus are more significant for longer waves and small WECs (i.e. small values of a). Results for other incident wave directions allow similar conclusions.

The radiation impedance matrix is [24 x 24], but in Figure 3 comparisons are limited to a reduced number of terms. Minor discrepancies between the analytical and numerical results are once again seen for low values of ka. These findings provide additional confidence in the approach and confirm its suitability to solve the radiation and diffraction problems in an array (which is particularly critical for more complex shapes). Recall that these are the only quantities derived from the BEM code and that the equations of motion are solved in a separate code; this approach is well suited to multi-body WECs.

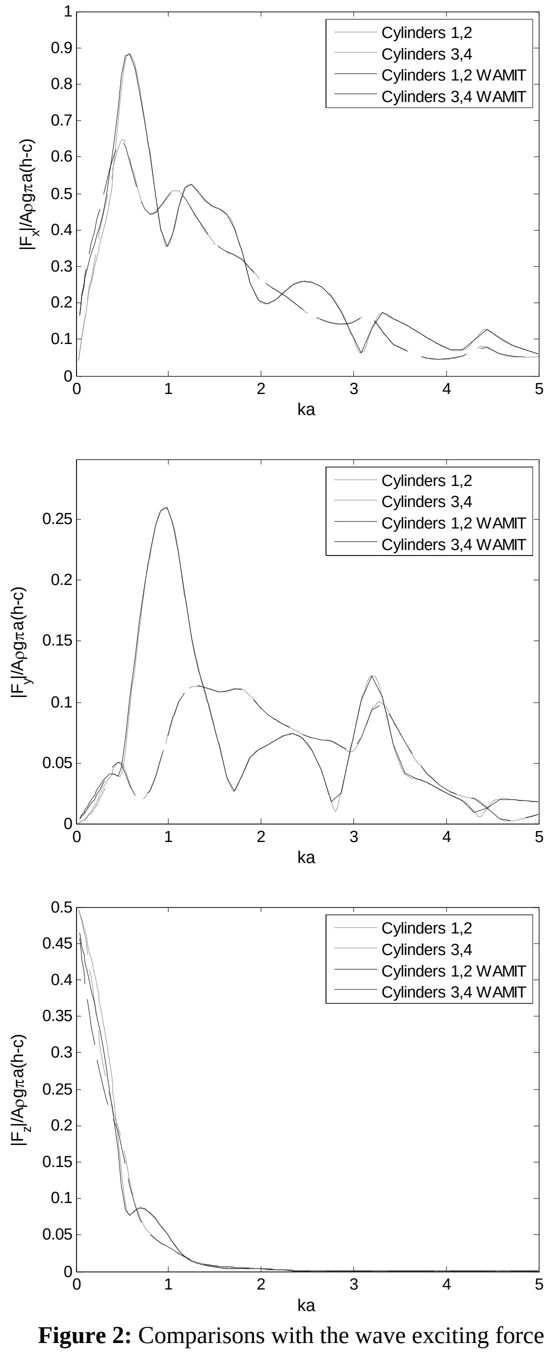

Figure 2: Comparisons with the wave exciting force results derived in [11] for $\beta = 0$

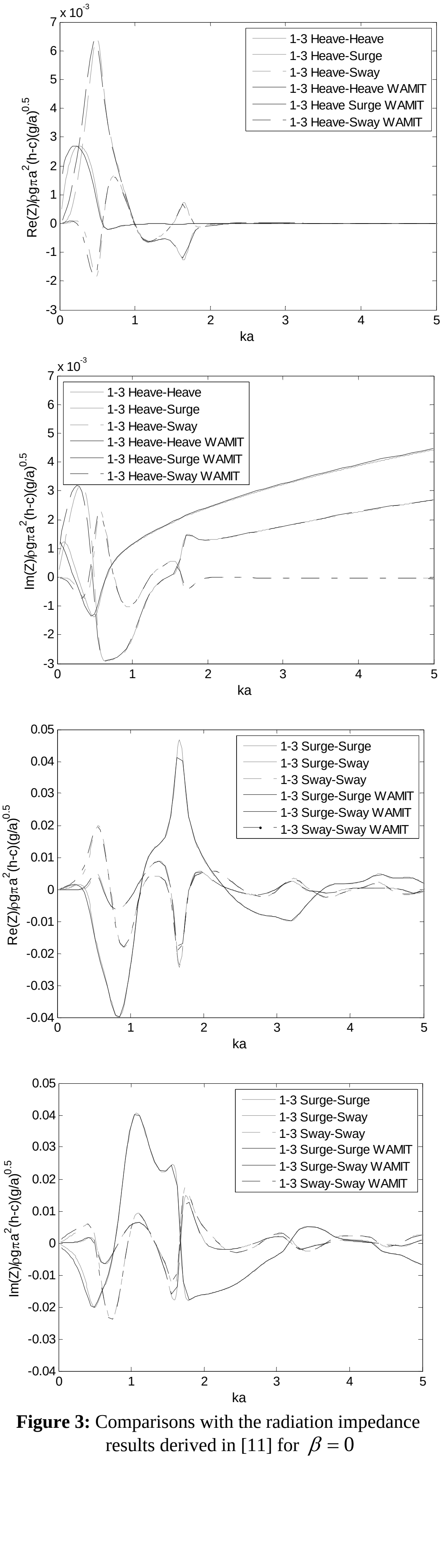

Figure 3: Comparisons with the radiation impedance results derived in [11] for $\beta = 0$

Results for complex geometries and multi-body WECs are more difficult to compare, as analytical solutions for such configurations are harder to derive. The usefulness of the BEM solution is clear in such cases, albeit it is important to ensure that adequate comparisons with experimental results (validation) is conducted, particularly if comparison with analytical results (verification) is not possible.

Interaction factor for regular waves

The assessment of the performance of an array of WECs has been approached to date mostly under the action of regular waves, in the vast majority of the studies available in the literature [e.g. 1-10]. Even with significant limitations (quite often the peak values of the interaction factor q are obtained under unrealistic amplitudes of motion) such an approach is still useful to assess the main constructive and destructive interference areas and the wave headings that most contribute to constructive effects. Nevertheless it is important to emphasise that conclusions from regular waves tests will always be limited, and the following subsections of the present paper address more realistic conditions (namely the influence of irregular waves and suboptimal control settings).

In this subsection the interaction factor q, which corresponds to the ratio between the power absorbed by the array and N times the power absorbed by a single isolated device (recall that N is the number of WECs in the array) was calculated [13] as a function of the nondimensional wave frequency ka and the wave heading β, for two scenarios: in Figure 4 the cylinders can absorb energy in heave, whereas in Figure 5 the energy absorption mode is surge.

In Figure 4 the row of peaks along $\beta = \pi / 4$ corresponds to the array being oriented diagonally, in which near trapping was observed at certain wavenumbers. The central peak corresponds to q = 2.3, i.e. the array of four cylinders is absorbing about nine times what a single cylinder of the same dimensions could at this wavenumber.

If the same optimal conditions were applied to absorption in surge, the results would show very high values of q near $\beta = \pi / 2$, as an isolated surging cylinder can not absorb any power at this wave heading. The array can still absorb some small power at this heading, so the q factor tends to infinity.

Figure 5 shows a suboptimal interaction factor, reflecting a more practical configuration for absorption in surge. The q factor was redefined to assume that the isolated surging cylinder always surges in the ideal direction. As expected, when the cylinders can only surge perpendicular to the wave heading ($\beta = \pi / 2$), very little power can be absorbed. Conversely, performance is best when they oscillate with the wave heading. A significant amount of power can, however, still be absorbed even when the cylinders oscillate at an angle to the incoming wave.

Although the regular wave investigation only allows limited conclusions, its usefulness is clear in Figures 4 and 5: such assessment of q leads to the identification of areas and scenarios which may be starting points for further, more refined studies which address e.g. the effects of irregular waves. A preliminary study (limited to absorption in heave) on the influence of representative seas in the energy yield from the array is presented in the following subsections. Particular attention is given to the $\beta = \pm\pi / 4$ case, which can be physically associated with either a change in the dominant direction of the incoming sea or a change in the layout of the array.

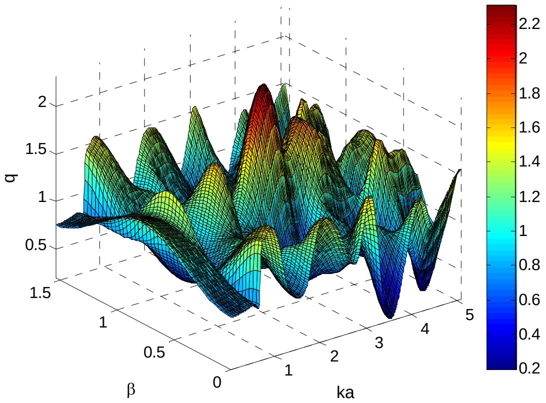

Figure 4: Interaction factor q – cylinders absorbing in heave

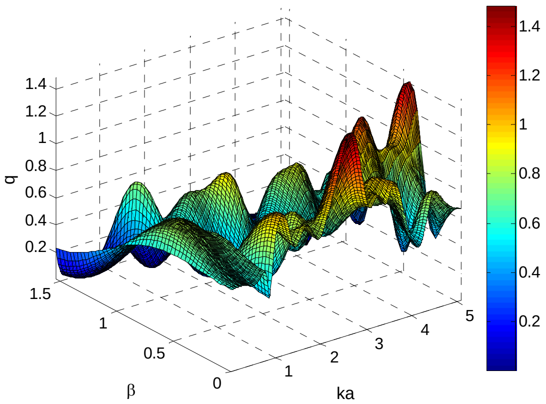

Figure 5: Interaction factor q – cylinders absorbing in surge

Suboptimal damping settings for irregular waves (single WEC)

In order to design a wave farm a realistic representation of the sea state needs to be used as input to the simulation. A preliminary assessment of the influence of this additional factor is done in the following subsection, but first it is relevant to quantify the power absorption characteristics of a single WEC (acting in isolation) under the same irregular waves in order to compare its response with that of each farm element.

In this study the influence of different omnidirectional spectra (and different dominant wave directions, in the array layout) was assessed. Two spectral parameters, H_{m0} and T_{-10}, plus the shape of the frequency spectrum, influence the results and thus these parameters were used to define the irregular wave input. Table 1 summarises the frequency spectra that were assessed. In the two last cases the peak period T_{peak} is specified, and its value set equal to the period leading to the peak capture width (a regular wave result). Such assessment tries to evaluate the effect of the incoming sea in the device response, which given the simplicity of the design is narrow banded with regard to wave frequency.

Formula / shape	*Case Id.*	H_{m0} *[m]*	T_{-10} *[s]*	T_{peak} *[s]*	β *[rad]*
Bretschneider	1	2	7		$-\pi/2$, $-\pi/4$
	2	4	7		
	3	2		10.205	
	4	4		10.205	

Table 1: Definition of the frequency spectra tested

Note that the different dominant directions β are irrelevant in the single body case, as the cylinder is axisymmetric. Table 1 applies also for the array simulations under irregular waves (see the following subsection), hence the inclusion of β. Further studies can approach different spectral shapes (e.g. Pierson-Moskowitz, JONSWAP) or real (measured) seas.

Rather than searching for the absolute optimum with regard to energy capture under the influence of the sea states described in Table 1 (recall that the optimal solution is typically associated with large and unrealistic amplitudes of motion) suboptimal solutions following a conservative control approach were evaluated. It is recognised from inception that such a constrained optimisation exercise will not result in the absolute maximum in terms of energy absorption, yet it will result in an achievable estimate. The cylinder is controlled via a single control term (external damping, D_{ext}), allowing an assessment of the sensitivity of the solution to variations in the control setting under the action of realistic seas. In the case of the array layout, each cylinder is controlled independently (hence the farm control layer has four terms). To obtain more representative dimensional results the radius of the cylinder a was altered to 10m, and the other geometrical properties listed in Figure 1 apply unless otherwise stated. The single cylinder iterations provide the benchmark for the calculation of the interaction factor q under the influence of irregular waves.

Furthermore, it is interesting to record not only the value of the maximum absorbed power, averaged over the frequency spectra specified in Table 1 (calculated from Equation 9), but also the damping coefficient D_{ext} associated with such a situation, for comparison with the individual damping coefficients associated with each array element when deriving the maximum power absorbed by the farm.

Table 2 summarises the maximum power and the associated damping coefficient for the four cases identified in Table 1. Figures 6 and 7 illustrate the power absorption characteristics of the heaving cylinder, as a function of the wave period and external damping coefficient for selected sea states. Note that the absorbed power is presented in Figures 6 and 7 with dimensions of MW/Hz, corresponding to the absorbed power associated with each monochromatic component, which is then integrated over the entire period range allowing the quantification of the averaged absorbed power per damping coefficient (and thus the identification of the maximum absorbed power per sea state).

As Table 2 shows the narrow frequency response of the heaving cylinder is clear: the maximum absorbed power in cases 3 and 4 is about 2.64 times higher than that associated with cases 1 and 2, which illustrates the suitability of the WEC to the longer peak period as expected from its capture width characteristics. The wave period also dictates the external damping coefficients D_{ext} associated with the maximum absorbed power for each sea state. As a result, cases 1-2 and 3-4 achieve the maximum $\overline{P}_n$ for the same D_{ext}. Finally, it should also be noted that the peak power (in MW/Hz) is not necessarily associated with the maximum absorbed power (in kW) over the entire sea, and thus the two events can (and in for the cases listed in Table 1 do) occur for different damping settings.

These results are particularly relevant for comparisons between the array output and the output of single cylinder times N (number of WECs). Also, the external damping values that were obtained can be seen as a starting point for the farm control strategy iteration, which involves four independent damping terms, associated with each array element.

Case Id.	*Max* $\overline{P}_n$ *[kW]*	D_{ext} *[kNs/m]*
1	39.2	590
2	157.0	
3	103.5	640
4	414.1	

Table 2: Maximum absorbed power (per sea state) by an isolated cylinder absorbing in heave with an external damping coefficient D_{ext}

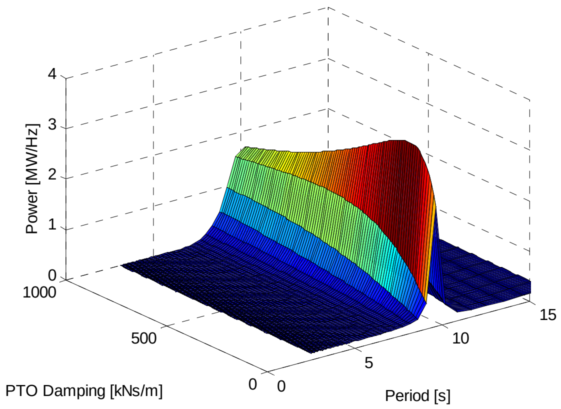

Figure 6: Power absorption characteristics of a single heaving cylinder – Case Id. 1

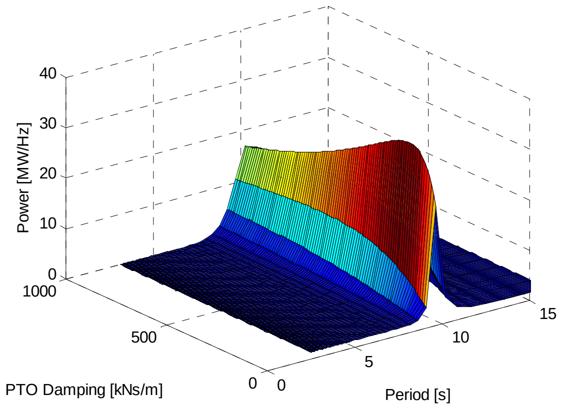

Figure 7: Power absorption characteristics of a single heaving cylinder – Case Id. 4

Suboptimal damping settings for irregular waves (arrays of WECs)

This subsection presents the first preliminary assessment of the combined influences of the incoming sea state, array layout and control strategy in the wave farm energy yield. Ultimately, it is expected that such fundamental research can benefit the creation of components in software package which are able to quantify the interaction effects, iterate and optimise the wave farm design, taking these and other operational constraints into account (e.g. electrical cable length, mooring strategy, minimum / maximum distance between each WEC, etc.).

In this preliminary assessment three aspects are particularly critical: 1) the influence of the layout of the array in the energy yield under the action of irregular waves; 2) the potential to further improve the energy yield by adjusting the control of each farm element (farm control strategy); 3) the quantification of the interaction factor q for irregular waves in the above mentioned scenarios. As in the previous subsection a suboptimal solution (one term control strategy per WEC – external damping) was implemented, to obtain more conservative estimates. All the geometrical properties of the array are listed in Figure 1, although as for the isolated cylinder case the radius a was altered to 10m. The study was focused on the square array of four elements, and the influences of the sea sates described in Table 1, the farm control strategy and two dominant wave directions ($\beta = -\pi/2$ and $\beta = -\pi/4$). The latter case corresponds physically to the array being oriented diagonally (hence a layout change). Table 3 summarises the key results: the maximum averaged absorbed power $\bar{P}$ and the interaction factor q. The suboptimal farm control strategy iterates on each of the four external damping coefficients which control the four array elements with step changes of 10kNs/m.

Case Id.	β *[rad]*	*Max* $\bar{P}$ *[kW]*	q
1	$-\pi/2$	146.6	0.93
1	$-\pi/4$	153.4	0.97
2	$-\pi/2$	586.2	0.93
2	$-\pi/4$	613.7	0.98
3	$-\pi/2$	380.1	0.92
3	$-\pi/4$	392.9	0.95
4	$-\pi/2$	1520.5	0.92
4	$-\pi/4$	1571.8	0.95

Table 3: Wave farm absorbed power and interaction factor (irregular waves) under a suboptimal control strategy

It should be emphasised that, as in the isolated cylinder case, the control strategy that was implemented is suboptimal, as a single external damping value is implemented throughout the sea state on each WEC (passive control per sea). A more complex control approach would involve changing this control term for each incoming wave period or for each representative wave group (active control). Further complexity is introduced by controlling more than just the external damping (e.g. stiffness) or by absorbing in more than one degree-of-freedom. More complex WECs are likely to involve more than one body, and thus the potential to explore other control and layout configurations is also higher.

This methodology deliberately tries to obtain more achievable estimates than those linked with optimal control, although it is recognised that improvements can be obtained via the implementation of less simplistic approaches. The main value of this preliminary assessment is in the quantification of the combined effects of layout and control changes under irregular waves in the farm energy yield, even in such a simplistic case. As Table 3 shows there is an average increase of 4% due to the (coupled) layout and control iterations (when changing β from $-\pi/2$ to $-\pi/4$), a figure which would have a significant impact in the farm's energy yield and thus on the revenue of a wave farm. It is further encouraging to acknowledge that such result was derived in a simplified scenario (simple geometry, single body per WEC, one term controller per WEC).

It is also possible to compare the average power $\bar{P}$ with the power that would be absorbed if all the array elements were controlled with the same external damping coefficient, set equal to the value derived in the previous subsection when analysing an isolated cylinder. The average drop for all scenarios is around 1%, in this case exclusively due to the absence of fine tuning of the farm control strategy.

Figures 8 and 9 show the type of quantitative assessment that can be made when evaluating different control approaches for each case. When a wide range of damping settings is analysed, zooming out in such figures allows the assessment of the implications of under and over-damped scenarios, while zooming in allows the fine tuning of the four term farm control layer to become more evident.

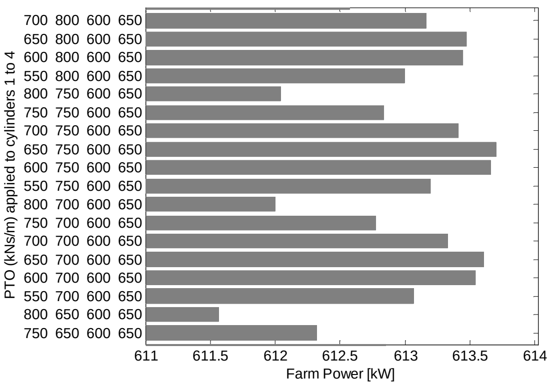

Figure 8: Wave farm power absorption characteristics (Case 2: $H_{m0} = 4\text{m}$, $T_{-10} = 7\text{s}$, $\beta = -\pi/4$)

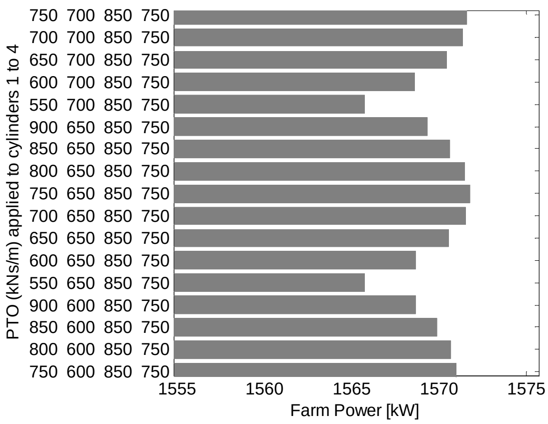

Figure 9: Wave farm power absorption characteristics (Case 4: $H_{m0} = 4\text{m}$, $T_{peak} = 10.205\text{s}$, $\beta = -\pi/4$)

Finally, it is also interesting to compare the values of the damping coefficients associated with each of the four cylinders and compare such values with the analogous coefficient linked to the isolated cylinder case. For example, for case 2 ($H_{m0} = 4\text{m}$, $T_{-10} = 7\text{s}$) with $\beta = -\pi/4$ the suboptimal damping setting for cylinders 1 to 4 are 650, 750, 600 and 650 kNs/m (see Figure 8), which can be compared to the 590 kNs/m (thus up to 27% smaller than the maximum value applied in the array case). Similar variations are obtained for the case listed in Table 3. This range may induce changes in the engineering specifications of the PTO, when considering the implementation of the full range of damping coefficients in a real application.

Under these simplified assumptions the interaction factor q is always below 1 for all the sea states tested, which places the farm design objective as the minimisation of the effects of destructive interference. Further iterations which can include other variables such as the array spacing (regular or irregular), may mitigate such aspects, and even alter the design objective (if $q > 1$ the goal is to maximise the effects of constructive interference).

4 Final Remarks

A preliminary assessment of a simplified WEC array was conducted, quantifying the effects of layout and control iterations under the action of irregular waves. To increase confidence in the numerical predictions, the fundamental hydrodynamic properties were firstly benchmarked against analytical results, and a regular wave investigation was also carried out to outline the areas of interest. The ultimate objective of this approach is the creation of a software tool that assesses and quantifies the interaction between the several array elements, optimising the output. Particular attention is given to forces (wave induced and externally applied) and the wave farm's power absorption characteristics, allowing comparisons with N times the output of an isolated WEC (N being the number of WECs in the array).

For simplicity the study was focused on a simple geometry (circular cylinder), a limited (and fixed) number of WECs (four), and a suboptimal control approach which allowed only one term to be controlled per WEC (external applied damping in heave). Even in this simplified scenario, layout and control iterations led to an average increase of 4% in the farm's energy absorption properties, a figure that would have a significant impact in the energy yield. Furthermore, the suboptimal nature of the control strategy (deliberately tested to ensure that conservative estimates were obtained) limited the potential of the approach. For example the individual damping coefficients are kept fixed for each sea state (passive strategy, suited to tune a WEC in a statistical sense, i.e. for the duration over which the sea state is characterised); a more complex methodology would involve changing these settings for the next set of incoming waves (active strategy).

Further studies can address (among other variables) variations in the number of WECs (farm installed capacity), WEC spacing (which is limited for operational and non-technical reasons), and other externally applied forces (e.g. mooring loads). The implementation of alternative algorithms such as those described in [8] and [14] may also provide a means to reduce the computational effort associated with the design of a wave farm.

References

[1] Budal, K. Theory for absorption of wave power by a system of interacting bodies. *Journal of Ship Research*, Vol. 21, 248-253, 1977.

[2] Falnes, J. and Budal, K. Wave-power conversion by point absorbers. *Norwegian Maritime Research*, Vol. 6, 2-11, 1978.

[3] Evans, D. Some theoretical aspects of three-dimensional wave-energy absorbers. *Proc. First Symposium on Wave Energy Utilization*, Gothenburg, Sweden, pp. 77-113, 1979.

[4] Falnes, J. Radiation impedance matrix and optimum power absorption for interacting oscillators in surface waves. *Applied Ocean Research*, Vol. 2, 75-80, 1980.

[5] McIver, P. Some Hydrodynamic Aspects of Arrays of Wave-Energy Devices. *Applied Ocean Research*, Vol. 16, 61-69, 1984.

[6] Thomas, G. and Evans, D. Arrays of three-dimensional wave-energy absorbers. *Journal of Fluid Mechanics*, Vol. 108, 67-88, 1981.

[7] McIver, P. The Hydrodynamics of Arrays of Wave-Energy Devices. *Wave Energy Converters – Generic Technical Evaluation Study (Annex B1)*, Report to the Commission of European Communities, 1993.

[8] Mavrakos, S. and McIver, P. Comparison of Methods for Computing Hydrodynamic Characteristics of Arrays of Wave Power Devices. *Applied Ocean Research*, Vol. 19, 283-291, 1998.

[9] Justino, P. and Clément, A. Hydrodynamic Performance of Small Arrays of Submerged Spheres. *Proc. 5th European Wave Energy Conference*, pp. 266-273, 2003.

[10] Fitzgerald, C. and Thomas, G. A preliminary study on the optimal formation of an array of wave power devices. *Proc. 7th European Wave Energy Conference*, 2007.

[11] Siddorn, P. and Eatock Taylor, R. Diffraction and independent radiation by an array of floating cylinders. *Ocean Engineering*, Vol. 35, 1289-1303, 2008.

[12] Yilmaz, O. and Incecik, A. Analytical solutions of the diffraction problem of a group of truncated cylinders. *Ocean Engineering*, Vol. 25, 385-394, 1998.

[13] Siddorn, P. Wave energy absorption by arrays of oscillating bodies. MEng Final Report, University of Oxford, 2007,

[14] Li, Y. and Mei C. C. Multiple resonant scattering of water waves by a two-dimensional array of vertical cylinders: Linear aspects. *Physical Review*, Vol. 76, Article Number 016302, Part 2, 2007.