

Toward reliable production estimation from Wave Energy Converters

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Abstract

This paper presents a PhD project aimed to improve the reliability of production estimation of Wave Energy Converter from the performance description and the statistical description of a wave climate.

So far, a review of the different spectral parameters has been done. The parameters were then chosen regarding their relevance to wave energy conversion and how they combine to describe the directional spectrum. Using pseudo-deterministic methods (single summation), the generation of the sea states corresponding to the desired values of each parameter is an ongoing process.

Accurate measurement of the wave spectrum inside the tank is an important part of the project. A version of the MMLM [1] specially implemented to make use of deterministic waves is developed. Emphasis is put on the design of an optimal gauge array. The method is currently tested against virtual data and early results on the array design are presented.

Keywords: tank testing, production estimation, directional spectrum, Maximum Likelihood Method

Nomenclature

WEC = Wave Energy Converter

MLM = Maximum Likelihood Method

MMLM = Modified Maximum Likelihood Method

$S(\omega, \theta)$ = Directional energy spectrum

1 Introduction

Predicting the energy production of a wave energy converter relative to a location is an important need of the wave energy industry. However, it is still an uncertain process [2] due to the difficulty of statistically de-

scribing a wave climate and to use those statistical description with the performance characteristics of a Wave Energy Converter (WEC).

Reducing a sea state to its directional spectrum is a wide spread approach despite the fact that information is lacking about the relative phases between waves. Directional wave data are available from numerous places, and statistics are made from them to try characterising their wave climate. However, as their influence over the performance of Wave energy converters is not well defined, little is still used of those statistics. For many application of performance prediction, only statistics over significant wave height H_{m0} and energy period T_E are used matching corresponding scatter diagram and the power matrix of a device.

This paper is presenting a PhD project which aims to quantify the influence of several directional spectral parameters over the performance of different wave energy converters. For this, several types of wave energy converters will be tested in the Edinburgh Curved Wave tank against a large set of parameters. Guidance will be issued about the use of those parameters to reach more accurate performance estimation.

2 Roadmap toward better performance estimation

2.1 Motivation for better performance prediction

Current approach towards performance prediction relies mainly on matching H,T scatter diagram of a wave climate with the power matrix of the considered Wave Energy Converter (WEC). While advanced numerical models are developed to allow better estimation of a device performances, they are often specific to a device and not available in the public domain which limit their use by external parties and for device comparison in regards of a specific wave site.

However, the approach based on H,T scatter diagrams

is very limited as it disregards many aspect of wave climates, such as spectral shape, directional spreading and dual mode seas. These limitations could have an important effect on the precision of WEC production estimation [2, 3], can induce bias while comparing different devices, and thus increases the risk associated to the commercial deployment of wave energy converters. A extension of this approach to other parameters could reduce those associated risks without compromising much on its advantages.

2.2 Scope limitation of this project

The possible reach of this study is very wide: inclusion of many oceanographic feature could be justified and as a consequence, it is necessary to limit the scope of this research project. The choice to consider a phenomenon is taken both on its relevance to the study and the capabilities of the experimental facility to quantify its effect.

At this stage, it is not planned to introduce into this research:

- Transitional effect. Only phenomenon or parameters related to a static directional spectrum will be considered. Any transitional effect related to the constant evolution of the sea state is discarded. This decision is mainly motivated by the necessity to be consistent with current type of measurement given at a fix rate (for example, one spectrum every three hours).
- Wave-current interaction. Although it is a very discussed topic and a considered phenomenon for many current researches, it will not be taken into account mainly due to the lack of facility available during the time frame of this project. Plans are made to develop and test the hardware needed for this type of experimental work, but the knowledge and analysis techniques required to use them in this study will not be available. This topic can be seen as an opportunity for future extension of this project.
- Dual wave system only. Studies have been showing that a sea state could be composed of several wave systems, probably created by different meteorologic conditions at different location and converging together at the place of interest [3]. However, this research is going to be limited to mono and dual wave spectrum. Introducing more than two wave system will lead to a uselessly complex parametrisation that must be avoided. Data from [3] also show that 75% of the sea states are mono or dual spectra.
- This study is finally limited to deep water waves. The setting of the Edinburgh Curved Tank is the restricting factor in this limitation.

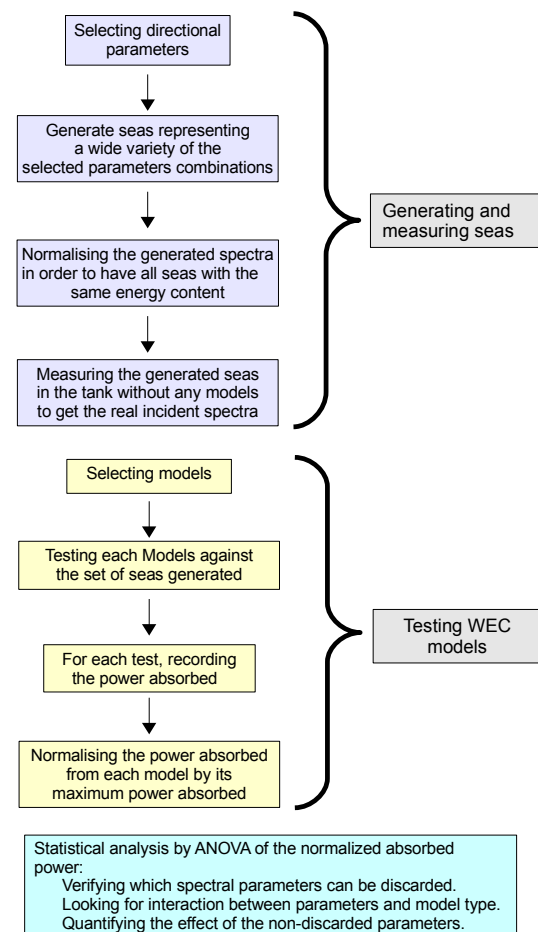


Figure 1: schematic of the general method

2.3 Expected Results

The main output of this study will be to validate the current performance estimation method based on H,T scatter diagrams and power matrices. This study will investigate if this approach is well suited for a wide range of devices and wave climates, and if it can be significantly improved by considering extra parameters.

If the later consideration is positive, guidance on wave measurements and device performance characterisation will be issued to reflect those potential improvements. The method itself, extended to more or different parameters (multi-dimensional scatter diagrams and power matrices), will be tested and its precision assessed. Finally, this study should contribute to reduce the associated risk with investment in wave energy by providing enhanced methods for performance estimation, as well as a fairer comparison between devices.

2.4 The overall method

The approach taken in this study is to explore the effect of different parameters accessible from a directional spectrum over a range of different wave model concepts. The research will not focus on the mechanisms in action but only in the parameters influence over the WEC concept performances. Tank testing is preferred over numerical simulations in order to be able to take into

account every effect induced by directional wave. This will allow the study not to make any simplification and linearisation about the WECs mechanical behaviour while many of the effects of directional seas are presupposed to be highly non-linear.

The WEC models will be exposed to a wide range of directional seas that will represent the widest combinations of values for the proposed set of selected parameters. The energy content of all the directional spectra will be normalised in order to simplify the performance comparison. Each test will be replicated up to five times with a different random phase spectrum to reduce any effect from sea state characteristics that cannot be determined with a directional spectra. The interest of those measurements is the performance variation of each WEC as a function of the incident spectrum and not the actual power absorption of the models. Reflecting this, the performance criterion selected for each test is the ratio between the observed power absorbed and the maximum observed power absorbed over all the tests.

2.5 Selecting spectral parameters

2.5.1 Criteria for choosing parameters

The main restriction applied to this study is that considered parameters should be available from the directional spectrum $S(\omega, \theta)$ characterising a sea state, and that it must be possible to turn into a statistic describing the wave climate of a location. The common spectral shape used in tank testing are statistical and normally not observed in a single directional spectrum. Hence, quantitative parameters that can be computed from any spectral shape need to be used.

2.5.2 Selected Parameters

The final list of parameters that will be included in this study is still under discussion. However, the parameters can be classified into three categories. In the first category are parameters that are not linked to any wave system, such as the energy period T_E . Those parameters are the ones traditionally used in scatter diagrams. In the second are parameters that are linked to a wave system, for example an energy period specific to a wave system can be defined [3], or its spreading. Finally, the third category groups the parameters that describe the relation between wave systems, such as an energy ratio between the systems. Those parameters can only be defined for dual wave system. The tentative list of selected parameters is presented in Table 1

The mean energy direction θ_M is not computed as it is assumed that the model WEC will either be omnidirectional or are self aligning themselves with it. In the case of single wave system spectrum, θ_M will be equal to 0° . $\theta_{M,n}$ is only considered as a wave system specific parameter in order to allow the comparison between the wave system positioning in the dual wave system spectrum.

2.5.3 Discussion of parameters

The driving point is to define a system of parameters to describe a directional spectrum in the most concise way possible, keeping the point of view of the WEC performances. In that respect, parameters defining the relative position of the wave system in the directional spectrum (δT_E , $\delta \theta_M$) are thought to be important: it is not known how directional devices will align themselves in those dual spectrum, and their alignment is crucial regarding the energy they will be able to extract from each of the wave systems.

2.6 WEC models

For those tests, the interest is focused on the performance variation of the models as a function of the different selected wave parameters but not on the absolute performance of each of them. Hence, models are not chosen as a function of their performance but of their diversity. It is important to chose models that are intuitively sensitive to different wave characteristics in order to produce the most generic guidance possible. Finally, the WEC models should be fit for test in the Edinburgh Curved Wave Tank, so they are necessarily deep water models.

At this stage, three models representing different type of devices are planned to be tested:

- The Modified Edinburgh Duck. This model has been amply described by [5, 6]. The duck is categorised as a terminator [7] as its largest dimension is perpendicular to the main wave direction of propagation. This makes it a directional device that should align itself with the incident wave spectrum.
- The Sloped UPS Buoy [8]. It is a point absorber, converting energy both from the surge and heave motion of the water particle. This sensitivity to the surge motion makes it a directional device that must align itself with the incident wave spectrum.
- An oscillating water column. An omnidirectional device is needed in this set of WEC models. Oscillating water columns (WC) are well known and studied and should be a good contender for the omnidirectional devices.

Together, these three models represent a wide range of offshore devices. It is kept as an option to use two identical OWCs to check the influence of the selected spectral parameters over the simplest possible array of devices. It is seen as an important aspect to consider in this study that a single device can be omnidirectional but arrays are likely not to be.

3 Advance on the MMLM

One of the important aspect of this project is the ability to accurately measures the incident directional spectra in the tank. The quality and significance of the results

Table 1: list of selected parameters

Parameter	Symbol	Unit	Physical Meaning
<i>omnidirectional parameters</i>			
significant wave height	H_{m0}	meter	
energy period	T_E	second	linked to the energy in deep water P_T $P_T = \frac{\rho g}{4\pi} T_E m_0$
Peak number	P_N	-	number of wave systems
<i>wave system specific parameters</i>			
Local significant wave height	$H_{m0,n}$	meter	significant wave height of the wave system
Local energy period	$T_{E,n}$	second	energy period of the wave system
Local mean energy direction	$\theta_{M,n}$	degree	mean energy direction of the considered wave system
Mean Direction Variability	τ_n	degree	variability in frequency of the mean direction of the considered wave system
Local angular spread factor	σ_n	-	wave spreading factor of the wave system. Based on a definition in [4]
Local frequency spread factor	ν_n	-	frequency spreading factor of the wave system.
<i>relational parameters between wave systems</i>			
Energy ratio	μ	-	$\mu = \frac{T_{E,2} H_{m0,2}}{T_{E,1} H_{m0,1}}$
Period Difference	δT_E	second	$\delta T_E = T_{E,2} - T_{E,1}$
Mean direction difference	$\delta \theta_M$	degree	$\delta \theta_M = \theta_{M,2} - \theta_{M,1}$

will highly depend on the measurement quality of the incident wave fields.

Many measuring method for directional wave spectrum are described in the literature, based on different principle. A good review of them is available in [9]. However, little work has been done to measures spectra in reflective wave fields and they are normally not developed for the special case of a wave tank environment. As a consequence, a version of the Modified Maximum Likelihood Method (MMLM) first described by [1] has been implemented using work done by [10, 11]. The following section describes the adaptation needed for best performances in the Edinburgh Curved Wave Tank as well as early results on virtual wave elevation data.

3.1 Adaptation to wave tank environment

3.1.1 discrete frequency spectrum

There is fundamental differences between working in a controlled environment such as the Edinburgh Curved Wave Tank and working with ocean data. Specially, when using pseudo-deterministic wave generation method [12] the wave elevation time series at a wave probe is periodic whereas wave elevation records from ocean wave are aperiodic.

As waves are periodic when generated with pseudo-deterministic methods (excluding noise), it is possible to use discrete fourrier transform on the wave elevation records without the need of preprocessing them and get the exact values for each bin and not an estimation.

In the Edinburgh wave tank, wave are generated using single summation technique [12, 13]. The typical spectra used for this thesis will have a frequency band $\Delta F = 0.0312$ Hz and 32 wave fronts per frequency band. This lead to a wave period of 1024 s and a Δf between each wave components of $9.7e^{-4}$ Hz.

Two directional energy spectra can be defined following this generation technique: the general discrete direc-

tional energy spectrum (frequency step ΔF) for which the energy in each set (ω_i, θ_j) is noted $S_{i,j}$, and an expended spectrum, with a frequency resolution Δf , for which the energy in each set $(\omega_{i'}, \theta_i)$ is noted $s_{i',j}$.

The followed method to compute the general spectra from wave records taking into account the nature of the wave generated is:

- Compute the relevant cross spectra from the wave elevation records at the Δf frequency resolution.
- Apply the MMLM computation steps from those cross spectra to get the spectral estimates $s_{i',j}$ (see [1]).
- Scale the obtained spectrum to ensure that the energy at each frequency is equal to the average energy in the auto-spectra at this frequency.
- Compute the measured the $S_{i,j}$ for each frequency band and direction of propagation from the spectral estimate $s_{i',j}$ of the expended spectrum.

$$S_{i,j} = \sum_{i'=u}^{u+32} s_{i',j}, \quad \text{with } u = i * 32 \quad (1)$$

3.1.2 discrete angular spectrum

It is theoretically possible to enhance further more the precision of the MMLM method by applying the same principle to the angular discretisation of the spectrum. In the case of generated waves, the direction of each component is precisely known. By choosing an angular discretisation of the plan ω, θ matching the one used to generate the spectra, it is possible to avoid any 'smearing' effect along the angular dimension.

However, this prior knowledge of the generated wave is only applicable if the tank can generate the wave direction of propagation accurately enough. In the present

case, the angle between each wave component is $\approx 3^\circ$ meaning that any error in the direction of propagation $> 1^\circ$ will prevent the use of this adaptation. A study in the tank shew that the tank is accurate enough in the $[-20^\circ; +20^\circ]$ range (error $< 1^\circ$) [14], but out of it the direction of propagation are not precise enough. As a consequence, it is not possible of making full use of the pseudo-deterministic characteristics of the generated waves.

3.2 Effect of geometric constraints

An other aspect of wave measurement in a wave tank are the geometrical constraint imposed to the probe positioning. About wave measurement near a reflector, [10] characterised area relative to the probe positions, the wave length and the record duration regarding the need to use methods taking phase locking into account or not. The authors conclude that in some cases, method integrating phase locking are required as other are loosing accuracy due to the reflection, but they will produce spurious peaks. Those spurious peaks are the consequence of uncorrelated noise close to theoretical nodes between incident and reflected waves. Their location in the directional spectrum plan (ω, θ) can be predicted for each probe position.

Fig. 2 shows the working zone of this study relative to the zones presented by [10]. It shows that, considering the tank geometry and the spectrum characteristics, experiments will be done exactly in the zone where phase locked method such as the MMLM are required but with generation of spurious peaks. It is necessary to remove those artifacts from the estimated spectrum prior to compute any spectral statistics. This is done by applying a mask to the estimated spectrum, preserving the real estimates but selectively erasing the spurious peaks where no energy should be found.

3.3 Results with virtual data

3.3.1 Virtual data characteristics

At each probe location $M(x, y)$, the virtual wave elevation is generated using single summation technique [12, 13]. Reflection from a wall is simulated by adding the elevation at the point $M_s(x_s, y_s)$ symmetric of $M(x, y)$ relative to the reflective wall, assuming a reflection coefficient r_c . To simulate the measured inaccuracy in wave front direction of propagation (see 3.1.2), random error of up to 1° are introduced for each component. Finally, random noise equal to 5% of the maximum wave component amplitude is added as well as real electronic noise recorded from the wave probe at still water. The final expression for the elevation at $M(x, y)$ is:

$$\eta(x, y, t) = \sum_i [a_i \cos(-\omega_i t + k_i(x \cos \theta_{i,r} + y \sin \theta_{i,r}) + \phi_i) + r_c a_i \cos(-\omega_i t + k_i(x_s \cos \theta_{i,r} + y_s \sin \theta_{i,r}) + \phi_i)] + \epsilon_i \quad (2)$$

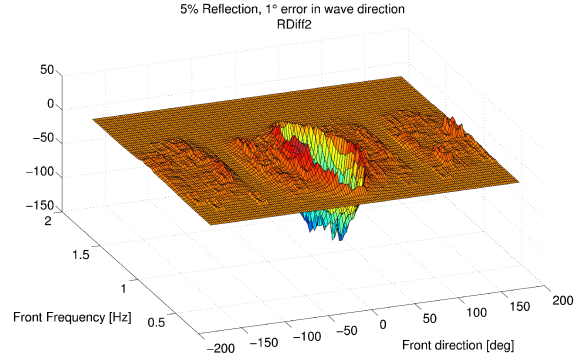


Figure 3: Difference between generated and computed spectra. $r_c = 5\%$ and $\pm 1^\circ$ error in wave direction.

with: $\theta_{i,r} = \theta_i + \delta \theta_i$

$\delta \theta_i$ randomly chosen in $[-1^\circ; +1^\circ]$

ϵ_i randomly chosen in $0.05 * [-\max(a_i); +\max(a_i)]$

$r_c \in [0; 1]$, the reflection coefficient.

The ϕ_i are chosen randomly for each component, whereas the a_i depends on each directional spectra. Details for the ω_i and θ_i can be found in [12].

3.3.2 general observation on the MMLM results

It is difficult to assess the quality of the measurement from a spectrum computation. The method performances can be assessed trough many parameters that can be conflictual. However, some general comments can be done by comparing the generated and measured spectra. The first important point is that when generating virtual data without including reflection ($r_c = 0$) and no error in the wave direction of propagation, no significant difference is found between the generated and measured spectra. This shows that any substantial difference observed on imperfect data are due to those imperfection (reflection, noise, error in wave propagation) and not on the method itself.

Finally, while using imperfect virtual data, a general trend can be observed: the MMLM consistently compute broader spectra than the generated (Fig. 3), leading to overestimation of the spreading parameter. This behaviour is consistent with observation from [15].

3.3.3 Comparison between MLM and MMLM

While implementing a method to measure directional wave spectra in a controlled and reflective environment, it is an intuitive decision. However, it is interesting to compare the results from this adapted method with the results from generic method such as the standard MLM. For this, both methods have been ran again a large set of virtual data generated as described in 3.3.1. The set of virtual data contains spectrum with three levels of reflection (0%, 10% and 25%) and three types of spectra (single, dual with aligned peaks and dual spectrum with non aligned peaks). For each possible combination of reflection level and type of spectrum, five replications are done

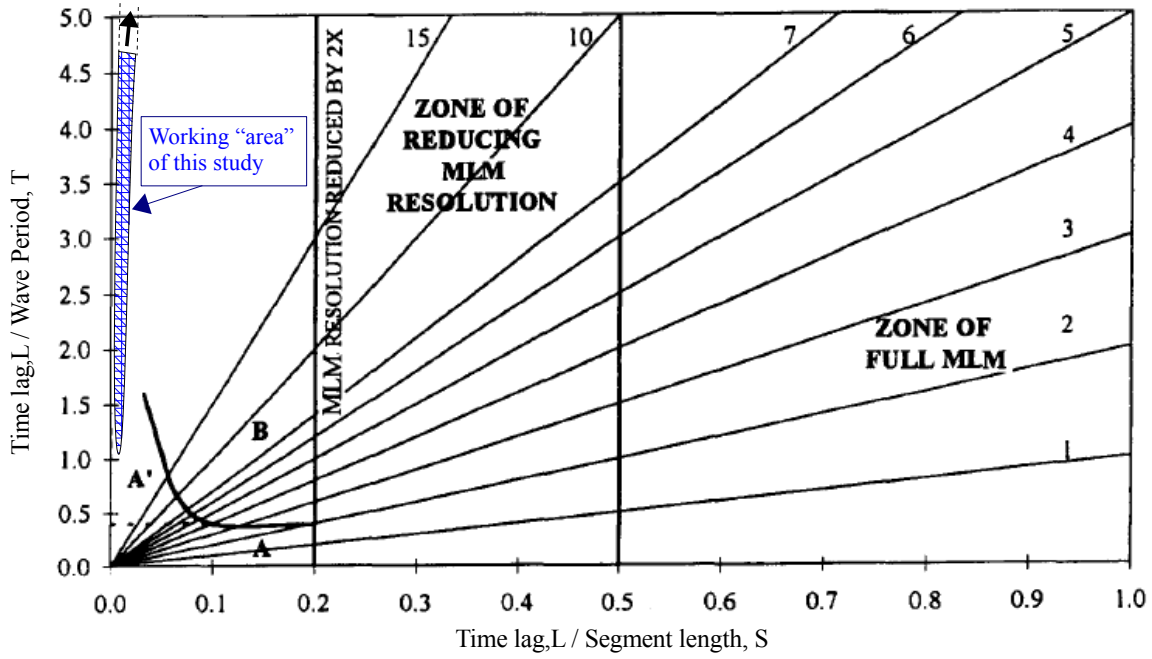


Figure 2: Graphs from [10] representing the zone in which phase locked or non phase locked method should be applied. The hatched zone shows the working zone of this study.

using different random phase spectrum and the spectrum are computed with the MLM and the MMLM. The performance criterion P_c used for the evaluation of each method is the error between the computed mean direction of propagation and the generated one.

$$\theta_M(\omega_i) = \sum_j S(\omega_i, \theta_j) / \sum_{i,j} S(\omega_i, \theta_j) \quad (3)$$

$$P_c = \sqrt{\frac{\sum_i (\theta_{M,c}(\omega_i) - \theta_{M,g}(\omega_i))^2 S_M(\omega_i)}{\sum_i S_M(\omega_i)}} \quad (4)$$

with: $\theta_M(\omega)$, the mean direction as

a function of the frequency

$\theta_{M,m}$, the measured mean direction

$\theta_{M,g}$, the generated mean direction

$S_M(\omega, \theta)$, the measured directional spectrum

$S_M(\omega)$, the measured omnidirectional spectrum

The results are analysed using R ([16]) to separate and quantify the influence of each factor (Method, Reflection level, Spectrum Type, Processing Type). Fig. 4 shows some evidence of better accuracy from the MMLM compared to the MLM. Fig. 5 shows the interaction between the Method and the Reflection level: while results are comparable between the two methods without reflection, the MMLM results are less degraded when the level of reflection increases. Those results are in line with the expectation, as the MMLM should behave like the MLM in a non reflective environment.

A formal analysis of the error distribution fitting a multilinear model taking into account some interaction between the factors was done. An Analysis of variance

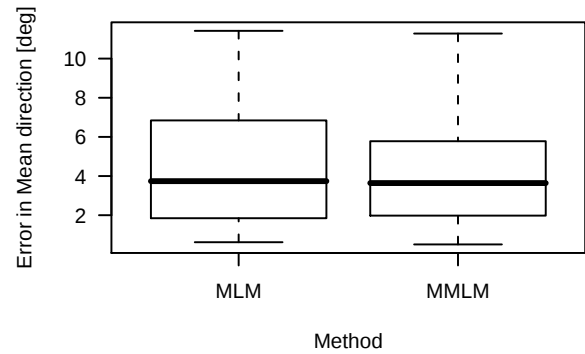


Figure 4: box plot for the Method factor, representing the min, max, Q1, Q3 and median for each method.

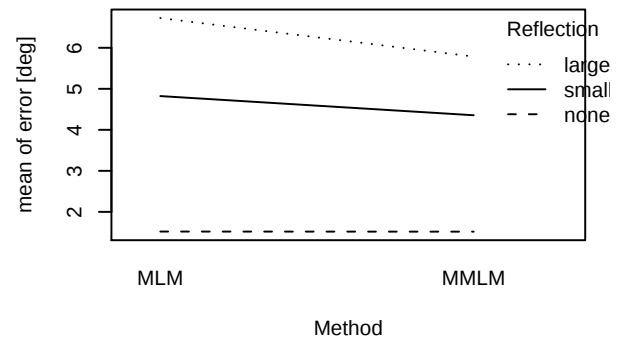


Figure 5: interaction plot for the factor Method and the Reflection level.

of this model shows strong evidence that the method has an influence over the results, as well as the interaction between the method used and the reflection level. This brings confidence in the choice of a method taking phase locking into account instead of a more generic one.

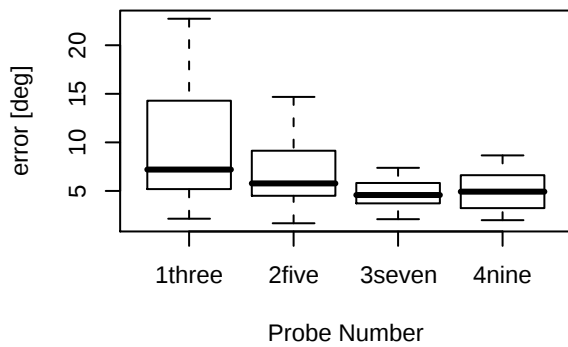


Figure 6: box plot for the ProbeN factor, representing the min, max, Q1,Q3 and median for each probe number.

3.3.4 Influence of array parameters over the method performances

The mathematical implementation of the method do not leave much freedom but many parameters are left to be fixed without strong guidance. Compared to wave ocean measuring, a wave tank gives full control over the number of wave gauges and their spatial arrangement. Array design is a long standing problematic that does not have a very defined solution [17].

Determining the number of gauges used is a first important step in array design. First approach is to used as many as possible, but that is clearly at the expense of simplicity and computing time. In order to quantify the effect of the number of probes, the MMLM has been run against virtual data (see 3.3.1) for arrays of 3, 5, 7 and 9 probes. The probe coordinates were selected randomly in a $1.5 \times 1.5 \text{ m}^2$ square which centre was located 3 m from the reflective wall. Three different types of spectrum (3.3.3) were used to check any interaction and a 10% reflection level is used. For each combination of spectrum and probe number, five replications were made with different probe coordinates and different phase spectra. For each computation, additional information about the array were stored: its swept area, the mean length of the segments, the standard deviation of each segment and the number of peripheral probes (the probes subset of an array needed to described its swept area). Those extra information will help to design the final arrays. The performance of each array using the MMLM method is measured by the error between the computed generated mean direction of propagation (Eq. 4).

A linear model explaining the observed error with the three factors ProbeN (probe number), Spectrum (type of Spectrum) and Processed (smooth or raw data) and their interaction is analysed by ANOVA. The results shows that no interaction between the factors should be retained, and the influence of smoothing the data is negligible. Both the type of spectrum and the number of probes have a strong influence on the observed error in mean direction.

Fig. 6 shows the effect of the number of probes over the method accuracy. The method accuracy seems to improve while increasing the number of probes up to seven,

but there is no obvious gain from seven to nine probes. Early analysis shows that a minimal swept area of 0.2 m^2 , whereas the mean segment length, the segment length standard deviation and the number of peripheral probes do not seems to have much influence on the method accuracy.

Following those results, a set of seven probes array with a minimal swept area will be generated and tested, before starting testing the method and array in the Edinburgh Curved Wave Tank.

4 Conclusion

A PhD project is underway to quantify the influence of several directional spectral parameters over a range of WECs. This will be achieve trough experimental testing in the Edinburgh Curved Wave tank of several WEC models. Using tank testing instead of numerical modelling will allow the results to be free of any assumption of linearity and over simplification. The expected results from this project is a set of guidance to make better use of the directional wave measurement available under the perspective of WEC performance estimation.

An important aspect of this project is the accurate measurement of the incident wave spectrum in the experimental facility. A version of the MMLM specially developed to make use of the characteristics of the pseudo-deterministic generated waves is currently under test against virtual data simulating the tank imperfection. Early results shows that the implemented method is a significant improvement compared to a traditional MLM in a reflective environment. Current research are now focused in optimising the gauge array design for optimal performance in the Edinburgh Wave Tank.

Future works will consist of testing and validating the spectrum measurement method in the tank and start the test sessions of each wave energy converter model selected for this project.

Acknowledgements

The first author acknowledges the support of the EPSRC Supergen Marine programme (PhD scholarship). Dr Ian Bryden acknowledges the support from Scottish Funding Council of his position within the the Joint Research Institute in Energy with the Heriot-Watt University which is a part of the Edinburgh Research Partnership in Engineering and Mathematics (ERPem).

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