Frequency domain techniques for numerical and experimental modelling of wave energy converters

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Abstract

This paper discusses the restrictions on frequency domain modelling of wave energy converters (WECs). It is shown that, for a model where the radiation is represented as causal, and where the control signal is not acausal, a frequency domain model is suitable for finding the post-transient, linear, causal response of a WEC. The common use of models that do not represent the casual nature of radiation (memory), or that include an acausal control signal, are identified in the literature. Arguments are presented for restricting both models to sinusoidal motion. Correct and incorrect applications of these restricted models to numerical and experimental work are discussed, with examples given from literature. Several papers are identified that could be interpreted as stating that frequency domain modelling of WECs is restricted to sinusoidal motion only. However, it is shown that these papers are in fact discussing the limitations of models where the control signal, or the WEC itself, are represented as mass-spring-damper systems.

Keywords: wave energy, frequency domain, monochromatic.

1 Introduction

The main purpose of this paper is to refute the idea that frequency domain modelling of wave energy converters (WECs) is restricted to sinusoidal motion. It is first necessary to define what is meant by *frequency domain*, and to show all of its possible interpretations. Section 2 shows how the frequency domain is used for signals (such as wave elevation), systems (such as WECs), and functions of frequency that are neither signals nor systems. There are several models that only correctly represent aspects of WEC behaviour when the input to the system is a monochromatic signal, i.e. a sinusoid. These restricted models are identified and explained mathematically.

One of these restricted models, introduced as a dry oscillator in section 3, has a frequency domain governing equation very similar in appearance to that for the model suitable for polychromatic behaviour, introduced as a wet oscillator. This similarity in appearance of the frequency domain descriptions of wet and dry oscillators is one reason why these concepts are not treated as distinct in the literature, as discussed in section 4. This blurred distinction impedes any discussion about the limits of the dry oscillator model. These limits are important to discuss, as the dry oscillator model is used by a significant minority in the wave energy community (see section 4), and several examples of its incorrect use appear in the literature.

The other restricted models involve a power take off (PTO) force. Section 5 presents an optimal PTO force, which only results in a causal system under sinusoidal conditions. Section 6 presents a mass-spring-damper PTO force, which is only optimal under sinusoidal conditions. Section 7 discusses the implications of the applicability of these PTO forces, and gives new and useful insights into experimental studies of optimally controlled WECs.

The similarity between the frequency domain representations of wet and dry oscillators makes it possible to interpret papers by Cummins [1] and Greenhow [2] as claiming that the frequency domain representation of the wet oscillator model is limited to sinusoidal motion only. Section 8 shows that these papers are in fact discussing the dry oscillator model introduced in section 3. A similar opportunity for misinterpretation appears in Niato and Nakamura's seminal paper [3]. It appears probable that this paper was identifying the limitations of the mass-spring-damping PTO introduced in section 6.

2 Frequency domain signals and systems

The term frequency domain has different meanings, depending on whether it is applied to functions, signals or systems. The term is often associated with the Fourier transform, of which there exist various versions.

2.1 Frequency domain signals

A signal is a function of time that contains information. It is described as time domain, because its domain, or independent variable, is time. Often the information in a signal is encoded in its frequency content, as is the case with sound and radio waves. The frequency content of ocean waves is also of interest, and sea states are often presented in the frequency domain. When describing a signal as frequency domain, this means that it is the Fourier transform of a time domain signal.

The Fourier transform of a real, i.e. not complex, signal results in a complex function of frequency. Two signals are required to represent a complex signal: either real and imaginary, or magnitude and phase; the latter being the most common. Various conventions for representing and reproducing ocean wave data, are shown in Fig. 1, and shall now be described.

Real seas

Here the word *real* means that the waves have not been artificially created in a basin. The surface elevation at any point is the result of waves converging from all directions. Some of these waves may have originated from distant storms and some from nearby winds. Ocean currents, local winds, wave steepness and bathymmetry will affect wave propagation. Wave elevation is a continuous function of time $\eta(t)$ defined at all values of time.

Measured spectra (A)

A measurement of the wave elevation is defined over a finite time period, known as the wave record. Modern wave records are discrete, so the wave elevation is sampled, $\eta[t_n]$. A wave record of N samples taken at intervals of Δt has a duration of $T_{DFT} = N\Delta t$. Some wave sensors allow an estimation of the contributions of waves from different directions. Where this is not available, the wave elevation is treated as though it were due to a polychromatic wave travelling in one direction. To perform a discrete Fourier transform (DFT) on aperiodic data, spectral noise reduction techniques are required to minimise leakage of energy into neighbouring bins. Windowing is a standard solution [4]. Fig. 1A shows that after multiplication with a tapered window, the record starts and ends at zero. Usually only the magnitude $|H[\omega_j]|$ is displayed, as the phase does not contain information that characterises the sea state.

Generalised spectra (B)

Measured spectra can be used to create a set of generalised spectra for a given location. Spectra with common shapes, as in Fig. 1A, are grouped together as one generalised spectrum. The phase information is discarded. An example of a group of generalised spectra is the South Uist 399 data set [5]. The annual occurrence of each generalised spectrum is assumed to be the same as that in the measured data.

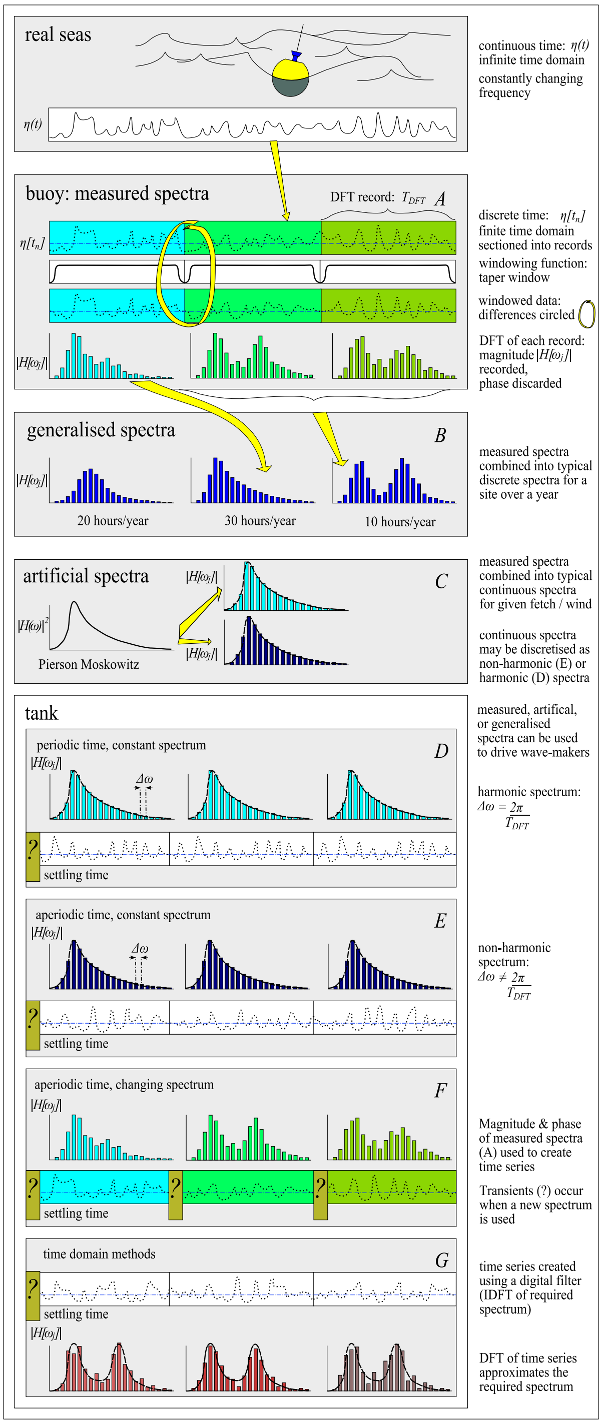

Figure 1: The DFT is used to A: characterise measured wave elevation, B and C: generalise and classify measured spectra, D-G: synthesise wave elevation from measured, artificial, or generalised spectra.

Artificial spectra (C)

Measured spectra are used to derive artificial spectra, which are heuristically derived functions of parameters such as wind speed, in the case of the Pierson Moskowitz [6] (PM) spectrum. By convention the spectral density, also known as *the spectrum*, is shown. The spectrum is continuous. To generate finite duration time series, it must first be discretised. Fig. 1C shows that it can be discretised into harmonic or non-harmonic spectra.

Tank waves: harmonic spectra (D)

The inverse discrete Fourier transform (IDFT) is a common way of synthesising drive signals for wavemakers [7]. The resulting time domain signal is periodic: it repeats with the DFT record. A measured wave elevation can be recovered from the frequency domain magnitude and phase, but the ends of the record will be lost due to windowing. A representative time series for a generalised or artificial spectra can be synthesised using randomly generated phase. A continuous spectrum such as a PM spectrum must first be discretised to look like a DFT. In a DFT, the frequency bins are harmonically related; each bin is an integer multiple of the first bin.

Tank waves: non-harmonic spectra (E)

In a DFT the bin number indicates the number of complete cycles made by a sinusoid of that frequency within the DFT record. For a non-repeating signal, none of the component sinusoids must complete a full number of cycles within the record. This is done by discretising the continuous spectrum at non-integer bin numbers, so the bins are not harmonically related. When used to synthesise a time series, this is not a member of the inverse Fourier transform family. It is nevertheless a useful method [8] for synthesising aperiodic time series.

Tank waves: non-repeating spectra (F)

Wave elevation could be synthesised with a different spectrum for each successive record, but this technique is not common. Fig. 1F shows an attempt to reconstruct elevation from the magnitude and phase of successive measured spectra (Fig. 1A). The original elevation cannot be recreated exactly as the spectra represent windowed time series, and new transients (indicated by a question mark: *?*) arise when each new spectrum is used.

Tank waves: white noise filter (G)

To generate a drive signal for a wavemaker in the time domain, the white noise filtering method is used [9]. Noise is passed through a filter, which is the IDFT of the required spectrum.

2.2 Frequency domain systems

The generalised representation of a system is a process that converts an input (excitation) into an output (reponse), as shown in the middle row of Fig. 5. When the system is time invariant, the same input always gives the same response. When the system is linear, superposition applies: for an input that is the sum of several signals, the output is the sum of individual responses to the constituent input components. The general form of governing equation for a linear time-invariant system is:

$$x(t) * h(t) = y(t) \tag{1}$$

where $x(t)$ is the excitation, $y(t)$ is the response, and $h(t)$ is the impulse response, which characterises the system. The impulse response is a function of time and can be described as instantaneous, causal, anticausal, or acausal (Fig. 2). As the convolution in (1) involves the multiplication of $x(t)$ and the time reversed $h(t)$, an impulse response with non-zero values at positive times gives a response based on the recent past of the input. Likewise, an impulse response with non-zero values at negative times gives a response based on the near future of the input. When the impulse response only has a non-zero value at $t = 0$, then the response is simply the instantaneous value of the input, scaled by a constant.

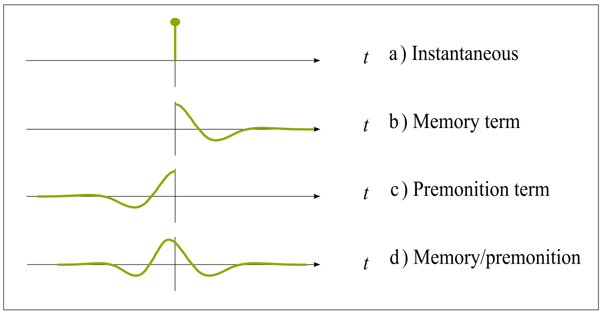

Figure 2: Types of impulse response functions a) instantaneous b) memory/ causal c) premonition / anticausal d) memory and premonition / acausal.

The response $y(t)$ has transient and post-transient components. The transient response depends upon the initial values of $y(t)$ and is independent of the frequency content of $x(t)$, while the reverse is true of the post-transient response. In stable systems the transient response decays, leaving only the post-transient response.

A linear time-invariant system may also be represented in the frequency domain. The general form of the governing equation is:

$$X(\omega)H(\omega) = Y(\omega) \tag{2}$$

where the frequency domain excitation $X(\omega)$, the transfer function $H(\omega)$, and the frequency domain response $Y(\omega)$, are the Fourier transforms of $x(t)$, $h(t)$, and the post-transient component of $y(t)$. Convolution in the time domain corresponds to multiplication in the frequency domain. As convolution takes a long computational time compared to performing Fourier transforms, the post-transient response is often calculated in the frequency domain (2).

2.3 Frequency domain functions

A frequency domain function is any function where the independent variable is frequency. Frequency domain signals and systems are both frequency domain functions. Here the domain is the frequency of the component of a polychromatic signal or system. Some frequency domain functions are neither signals nor systems. These are properties of sinusoidal waves, such as group velocity, wavelength and period, or any parameters measured in sinusoidal waves, not derived using a Fourier transform, such as power generated, volume of overtopping water, or monochromatic capture width. Here the domain is the frequency of the sinusoidal wave.

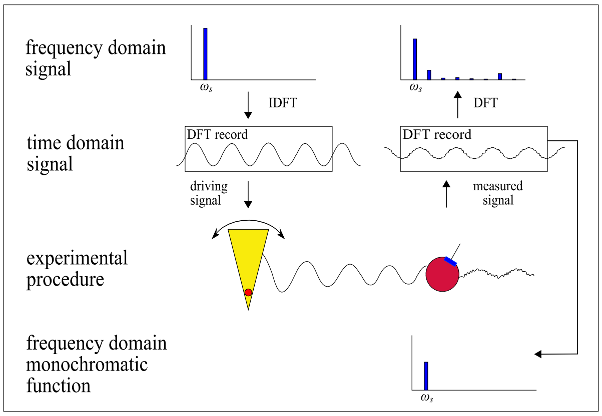

Figure 3: Measurement of monochromatic parameters: Frequency domain spectra define the waves used as excitation. A single parameter, which does not involve Fourier transformation, is plotted against the frequency of excitation.

2.4 Fourier transform considerations

There are four members of Fourier transform family [10], including the commonly used continuous Fourier transform for continuous aperiodic time signals, and the DFT for discrete periodic time signals, Additionally, all members of Fourier transform family have a real form, which uses sinusoidal basis functions, and a complex form, which uses complex exponentials as basis functions [10]. The real transform is conventionally used for describing wave elevation and radiation and diffraction hydrodynamic coefficients. When synthesising a time domain signal from a spectrum, either the real or complex forms can be used. The inverse Fast Fourier transform (FFT) uses the complex version. If the original spectrum was arrived at using a real Fourier transform, it must first be adjusted to make it suitable for use with the FFT. Conventionally the conversion between time and frequency domain systems assumes the complex version.

2.5 Restrictions on use of the frequency domain

Frequency domain representation of a system only applies to linear, time-invariant behaviour. By definition, time and frequency domain representations of a linear, time-invariant system are equivalent methods of finding post-transient response. Thus if a time domain system is a suitable model of polychromatic post-transient WEC behaviour, then so too is the frequency domain version. If a time domain system is only a suitable model of a WEC undergoing sinusoidal behaviour, then so too is the frequency domain version. Several models that are restricted to sinusoidal motion shall be considered. These restrictions are due to limitations of the models, rather than frequency domain representation.

Fig.s 3 and 4 indicate that there are similar experimental procedures for measuring frequency domain functions that apply only to sinusoidal inputs, and the frequency response (transfer function). Both use sinusoidal inputs, and take measurements at a number of frequencies. The difference is whether the Fourier transform is used to find the result.

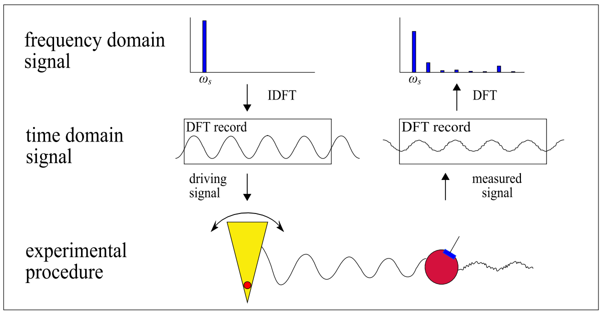

Figure 4: Measurement of frequency response: Frequency domain spectra define the waves used as excitation. Time domain response is converted to the frequency domain. For the frequency response, only the fundamental frequency (ω_s) is recorded; harmonics are ignored.

There is a further restriction when the time and frequency domain signals are discrete. The DFT is used, so the input signals, and hence responses, are periodic. A frequency domain system only gives the same post-transient response as the time domain system when the input signal is indeed periodic. Note that a common application of the DFT is to estimate the frequency content of aperiodic signals such as wave elevation. For experimental measurement of frequency response (Fig. 4), periodic excitation is required, as in Fig. 1D.

3 Wet and dry oscillator models

Two commonly used models of WECs are now described. They differ in their treatment of radiation, so the PTO is not included in the description of these models. As no energy is converted, these are oscillators rather than WECs. Nevertheless, the principles demonstrated here can be applied to models with PTO. Likewise, the principles apply to all types of WECs, not just the semi-submerged heaving sphere given as an example. Fig. 7 shows the difference between the two models. The dry oscillator is a mass-spring-damper. The wet oscillator contains similar components, but there is a mass and a damper with a shaded background. This is a new diagrammatical symbol introduced here to indicate that the mass and damper oppose present and past values of acceleration and velocity, which gives the system a memory of past motion.

3.1 Wet oscillator model

A typical frequency domain governing equation for an immersed body oscillating in one degree of freedom is of the following form:

$$F_e(\omega) = i\omega[m + M(\omega)]U(\omega) + B(\omega)U(\omega) + \frac{c}{i\omega}U(\omega) \tag{3}$$

Here $F_e(\omega)$ is the exciting force, $M(\omega)$ and $B(\omega)$ are the added mass and damping, and c is linearised buoyancy spring. Losses have not been modelled. The solution of this equation gives the Fourier transform of the post-transient velocity, $U(\omega)$. For an equation that corresponds term by term to the time domain equation, it is

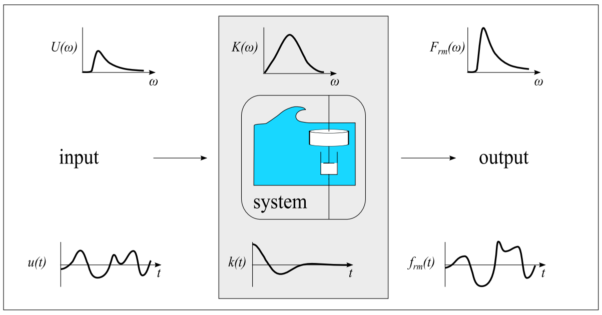

Figure 5: A wet oscillator: Middle row: the input to the causal system is velocity, and the output is radiation memory force. Top row: frequency domain. Bottom row: time domain.

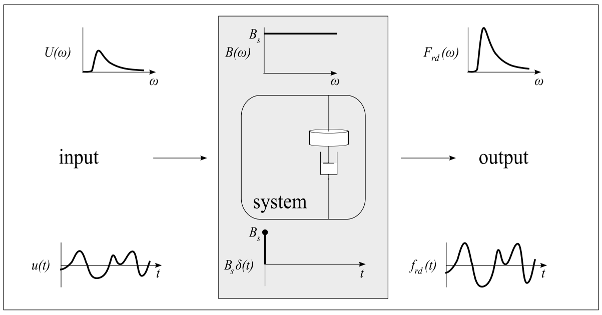

Figure 6: A dry oscillator: Middle row: the input to the memoryless system is velocity, and the output is radiation damping force (8). Top row: frequency domain. Bottom row: time domain.

necessary to rearrange (3) into the less common form:

$$F_e(\omega) = i\omega[m+m_\infty]U(\omega)+K(\omega)U(\omega)+\frac{c}{i\omega}U(\omega) \tag{4}$$

where m_∞ is the added mass at infinite frequency, and $K(\omega) = B(\omega)+i\omega[M(\omega)-m_\infty]$ the radiation impedance with the contribution from m_∞ removed. An inverse Fourier transform can then be applied to each term to give the typical time domain governing equation:

$$f_e(t) = [m+m_\infty]a(t)+\int_0^t k(\tau)u(\tau)(t-\tau)d\tau+cx(t) \tag{5}$$

The solution of this equation, $u(t)$, has transient and post-transient parts. The convolution arises because $K(\omega)U(\omega)$ is a product of functions of frequency. The integral limits arise because the excitation, and hence response, are assumed to be zero before $t=0$, and because it is known that $k(t)$ is causal (of the form shown in Fig. 2 b). In Fig 7 it can be seen that the convolution is represented by a mass and a damper with a shaded background. This is because $k(t)$ is the inverse Fourier transform of $K(\omega)$, and (4) shows this is made up of terms that resemble a mass and damper.

If the convolution is called the radiation memory force, then this can be considered in the time domain $u(t)*k(t) = f_{rm}(t)$ (Fig. 5 bottom row) and the frequency domain $U(\omega)K(\omega) = F_{rm}(\omega)$ (Fig. 5 top row). The input (first column) is the velocity, and the output (last column) is the radiation memory force. Note that the input is polychromatic, and that a causal $k(t)$ is associated with a $K(\omega)$ that varies in value over frequency. The value of $K(\omega)$ is zero at high frequencies because $B(\omega)$ and $[M(\omega)-m_\infty]$ have high frequency asymptotes of zero.

3.2 Dry oscillator model

When using a dry oscillator to model a WEC, the values of the mass, spring and damper are those that would be found in the wet oscillator model at one given frequency, ω_s. Thus the added mass is $M_s = M(\omega_s)$, and the radiation damping is $B_s = B(\omega_s)$ (see Fig. 7 - right). The frequency domain representation of the governing equation of motion is:

$$F_e(\omega) = i\omega[m+M_s]U(\omega)+B_sU(\omega)+\frac{c}{i\omega}U(\omega) \tag{6}$$

Whereas in (3) the added mass and damping, $M(\omega)$ and $B(\omega)$, have different values at each frequency, in (6) the added mass and damping, M_s and B_s, are constants. Plotted against frequency, they would be a horizontal straight line. The inverse Fourier transform of a constant is a constant. Hence the time domain equation is:

$$f_e(t) = [m+M_s]a(t)+B_su(t)+cx(t) \tag{7}$$

Fig. 7 depicts the differences between the wet (5) and dry (7) oscillators. In the wet oscillator, part of the added mass acts like additional system mass (m_∞) and part is included in $k(t)$. In the dry oscillator both components of mass combine and act like additional system mass, to give a force directly proportional to instantaneous acceleration. In the dry oscillator, as M_s is a horizontal line when plotted against frequency, the high frequency asymptote of added mass is $m_\infty = M_s$. So (6) has B_s in the position where (3) has $K(\omega)$. Treating B_s as a function of frequency, $B(\omega) = B_s$, which is the horizontal line in Fig. 6 (top row, shaded background), ensures that the inverse Fourier transform of $B(\omega)U(\omega)$ is a convolution. Now the inverse Fourier transform of a horizontal line is a scaled delta function: $\mathscr{F}^{-1}\{B(\omega) = B_s\} = B_s\delta(t)$, and convolution with a scaled delta function is the same as multiplication:

$$\begin{aligned} U(\omega)B(\omega) &= U(\omega)B_s &= F_{rd}(\omega) \\ u(t)*[B_s\delta(t)] &= u(t)B_s &= f_{rd}(t) \end{aligned} \tag{8}$$

Fig. 6 shows (8) as a system. Writing (7) as:

$$f_e(t) = [m+M_s]a(t)+[B_s\delta(t)]*u(t)+cx(t) \tag{9}$$

makes a direct comparison to (5) easier.

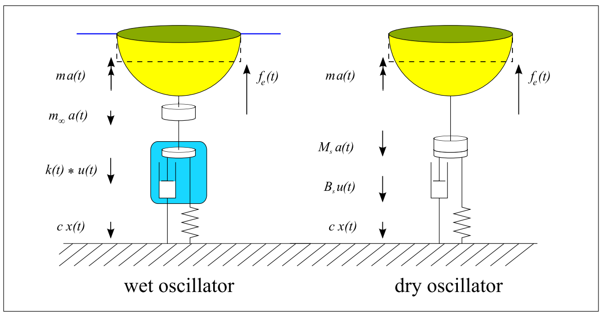

Figure 7: Wet and dry oscillators: Time domain models of a wet oscillator (left) and a dry oscillator (right).

	Monochromatic post-transient	Monochromatic transient	Polychromatic post-transient	Polychromatic transient	Time varying
Wet oscillator - time domain	✓	✓	✓	✓	✓
- frequency domain	✓	×	✓	×	×
Dry oscillator - time domain	✓	×	×	×	×
- frequency domain	✓	×	×	×	×

Table 1: The validity of the time and frequency domain versions of wet and dry oscillator models: validity indicated with a tick.

3.3 The validity of wet and dry oscillators

Table 1 shows the validity of the time and frequency domain representations of the wet and dry oscillators.

The time and frequency domain representations of both the wet and dry oscillator models give the correct post-transient response of an immersed body, in the presence of monochromatic excitation.

A dry oscillator is not suitable for modelling an immersed body with polychromatic excitation, as it cannot model the memory inherent in the radiation of waves. Neither the frequency (6) nor time (7) domain equations are suitable. The wet oscillator model correctly represents the memory due to wave radiation. Both the time and frequency domain equations are suitable for polychromatic excitation.

Frequency domain representations of linear systems are suitable only for modelling post-transient response. The time domain wet oscillator correctly models transient response to polychromatic excitation. The time domain dry oscillator correctly models the transient response of a mass-spring-damper system. This transient response would result in a velocity that was not sinusoidal, so the time domain dry oscillator does not give the transient response of an immersed body, even for monochromatic excitation.

Any model that can suitably represent transients can be adjusted to describe time varying response. Thus only the time domain wet oscillator is capable of representing an immersed body with time varying behaviour such as latching control [2]. An example of a time varying system is a WEC controlled using latching.

The duration (length of time for which there is a non-zero value) of the radiation impulse response increases as a device is scaled up. This is clear from any normalised plot of added mass or radiation impulse response [11]: the frequency and time axes are scaled by a function of the geometry. For very small geometries, the impulse response of a wet oscillator (Fig. 5 bottom row, shaded box) has a very short duration, and is very similar to the delta function of the dry oscillator model (Fig. 6 bottom row, shaded box). As the size of the geometry approaches zero, the wet oscillator approaches the dry oscillator model. The dry oscillator can be considered a model of an infinitely small point absorber.

4 A model restricted to monochromatic motion - the dry oscillator

All of the early theory describing optimal power capture for a WEC assumed sinusoidal motion [12–14]. It is thus not surprising that first published equations of motion also assumed sinusoidal motion, and used a dry oscillator model [12, 15]. Many recent papers can be found [16–24] that model WECs as dry oscillators. Correct interpretation of the models in these papers is important. A dry oscillator only describes the dynamics for sinusoidal motion, and is thus not a general equation of motion. Use of such a model with a control technique that results in velocity that is not sinusoidal, such as in [18, 20, 23, 24], is not strictly correct, as the model no longer describes the system dynamics. Likewise, the use of such a model in polychromatic waves with values of hydrodynamic coefficients that are averaged over the frequency range of interest, as in [17], is incorrect.

Here correctness is qualitative, rather than quantitative. For WECs that are small compared to typical wavelengths (point absorbers), a dry oscillator may simulate polychromatic behaviour to an acceptable degree of accuracy. For instance [23] reported that a dry oscillator predicted results that were "*typically less than 10%*" out from experimentally measured values. The non-linear models presented in [25] are interesting: equations are given for both a wet oscillator, and a dry oscillator using the value of radiation damping at the average frequency. [25] concluded that the results from a dry oscillator fitted reasonably well to the measured data. While a dry oscillator may be used to estimate response to reasonable accuracy, it nevertheless does not adequately describe the quality of the response and the true nature of the underlying process.

5 A model restricted to monochromatic motion - a WEC with optimal control

It is well known [12–14] that the optimal PTO impedance is the complex conjugate of the intrinsic impedance (also known as impedance matching):

$$\tilde{Z}_{pto}(\omega) = Z(\omega)^* \tag{10}$$

The *tilde* indicates imedance matching at all frequencies. Using the notation of (4), the intrinsic impedance is $i\omega[m+m_\infty]+K(\omega)+\frac{c}{i\omega}$. Thus the optimal PTO force of the form $F_{pto}(\omega) = Z_{pto}(\omega)U(\omega)$ contains $K(\omega)^*$, which can be written $K(-\omega)$:

$$\begin{aligned}\tilde{F}_{pto}(\omega) &= \left(-i\omega[m+m_\infty]+K(-\omega)-\tfrac{c}{i\omega}\right)U(\omega) \\ &= \left(-i\omega[m+M(\omega)]+B(\omega)-\tfrac{c}{i\omega}\right)U(\omega)\end{aligned} \tag{11}$$

Inverse Fourier transformation gives the PTO force described implicitly in [3]:

$$\tilde{f}_{pto}(t) = -[m+m_\infty]a(t)+k(-t)*u(t)-cx(t) \tag{12}$$

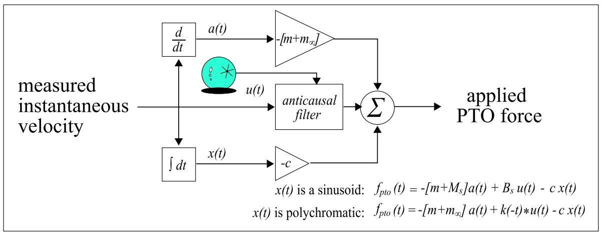

Figure 8: Impedance matching at all frequencies: This circuit gives optimal control for polychromatic excitation. Future velocity, indicated by the crystal ball, as well as instantaneous velocity, are fed into a linear anticausal filter. Summation of the filtered velocity, and the scaled displacement and acceleration, gives a signal proportional to the required PTO force.

As $k(t)$ is causal, $k(-t)$ is anticausal (Fig. 2 c). This means the optimal control force requires future knowledge. Fig. 8 shows (12) displayed as a system, with $u(t)$ as the input and $\tilde{f}_{pto}(t)$ as the output. The future values of $u(t)$ originate from the icon of a crystal ball in Fig. 8. This indicates that they cannot be derived from real time system information.

Adding the optimal control force (12) to a wet oscillator (3) gives an acausal system (Fig. 10 - left). To avoid acausality, which requires future knowledge, a model of a WEC with optimal control is valid only when behaviour is sinusoidal. There is one situation where the optimal PTO force is no longer anticausal: when the velocity is sinusoidal. In this case $U(\omega) = U_s\delta(\omega)$, and consequently $K(\omega)U(\omega) = K_sU_s\delta(\omega)$. This is a sinusoid with amplitude and phase determined by the values of $K(\omega)$ and $U(\omega)$ at ω_s. This term is memoryless and does not require future knowledge.

6 Optimal control restricted to sinusoidal motion - a mass-spring-damper PTO

Fig. 10 (left) shows that the optimal control force described by (12) mirrors the wet oscillator model (3). It is also interesting to consider a control force that mirrors the dry oscillator (6): this is the mass-spring-damper-system in Fig. 10 (right). With a mass-spring-damper system, it is only possible to meet the ideal control condition (10) at one frequency, ω_p.

For a PTO impedance of the form $Z_{pto}(\omega) = i\omega m_{pto} + b_{pto} + \frac{c_{pto}}{i\omega}$, the best implementation of PTO force [26] requires $\breve{m}_{pto} = -[m+M_p]$, $\breve{b}_{pto} = B_p$ and $\breve{c}_{pto} = -c$, which gives:

$$\breve{f}_{pto}(t) = -[m+M_p]\,a(t) + B_p u(t) - cx(t) \tag{13}$$

The *breve* indicates that the impedance is matched (10) at one frequency only. Note the similarity of (11) to the frequency domain version of (13):

$$\breve{F}_{pto}(\omega) = \left(-i\omega\,[m+M_p] + B_p - \frac{c}{i\omega}\right)U(\omega) \tag{14}$$

Fig. 9 shows (13) as a system. This represents an electrical circuit that is analogous to a mass-spring-damper system. When the input $u(t)$ is polychromatic, the control force is sub-optimal. However when $u(t)$ is sinusoidal, the control force equals the optimal control force

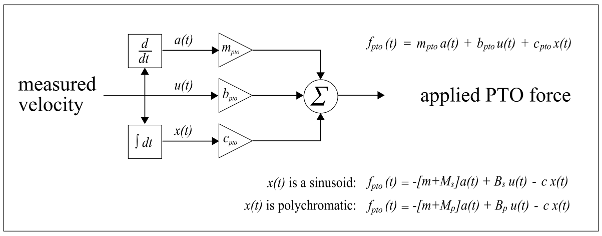

Figure 9: Impedance matching at one frequency: This circuit gives optimal control for monochromatic excitation and sub-optimal control for polychromatic excitation. Summation of scaled acceleration, velocity and displacement signals gives a signal proportional to the required PTO force.

(12). Fig. 8 and Fig. 9 are equivalent when the input is sinusoidal. The relationship between optimal control and mass-spring-damper control is similar to that of wet and dry oscillators, which are also equivalent for sinusoidal motion.

7 The distinction between monochromatic and polychromatic experiments

The issue of acausality is the reason why it is important to clearly distinguish between monochromatic and polychromatic experimental conditions. Response due to optimal causal control of sinusoidal motion and optimal acausal control (11) of polychromatic motion look identical when plotted in the frequency domain.

7.1 Estimations of power production should not use performance optimised at each frequency

Calculating absorbed power using PTO impedance that has been optimised at each frequency will give an over-estimation [26]. The power calculated in this manner is that which would have been absorbed if acausal optimal control were possible.

It is important to remember that the potential ambiguities described here have not always been widely known. Sometimes some detective work is required to determine whether experiments were conducted in monochromatic or polychromatic conditions. In Skyner's influential internal report [27], it is not stated whether values of efficiency calculated for monochromatic waves were used to estimate absorbed power in known (South Uist [5]) wave spectra. This is indeed the case, as can be seen from careful examination of Fig. 7.4, and from the statement on page 86 that the resulting control would be acausal. However, the section on the estimates of absorbed power from the South Uist spectra gives no indication that these levels of power capture rely on future knowledge.

7.2 Optimal control of monochromatic behaviour does not require future knowledge

The lack of a clear distinction between acausal and causal control can lead to misinterpretation, as exemplified in Nebel's thesis [28]. The introduction of *Part I* describes how complex conjugate control is optimal control for polychromatic seas and requires future knowledge (pages 7 – 8). The experimental method is then

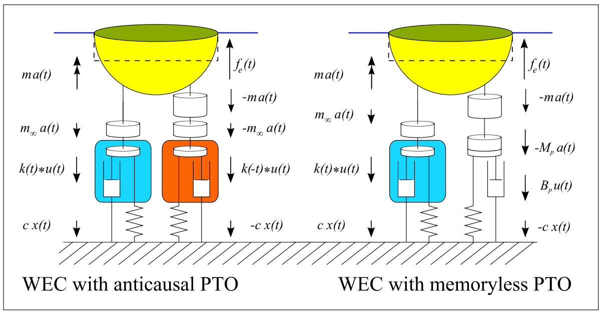

Figure 10: Optimal and sub-optimal control : Left - A dry oscillator with an anticausal PTO force that matches impedance at all frequencies. The blue background indicates a causal force and the orange background indicates an anticausal force. Right - A dry oscillator with a memoryless PTO force that matches impedance at one frequency.

described (page 9) in a way that gives the impression that acausal optimal control is being simulated by using knowledge of future waves. The remainder of *Part I* gives experimental results for complex conjugate control. There is no indication as to whether monochromatic or polychromatic waves were used. If *Part I* is extracted from the thesis and read in entirety, the reader is left with a very strong impression that the experiments were carried out for polychromatic waves. However, the first sentence of *Part II* leaves no doubt that monochromatic waves had been used.

When using monochromatic waves, optimal control is no longer acausal: advance knowledge of future wave elevation is not required. Information about the frequency of the monochromatic wave is required, but this is a fundamentally less difficult problem. Only careful reading of Nebel's thesis in entirety shows that he did *not* use future knowledge to perform complex conjugate control in polychromatic experiments. This is not well known in the wave energy community [29, 30].

8 The suitability of frequency domain models to polychromatic behaviour

If a time domain model is suitable for polychromatic behaviour, then so is the corresponding frequency domain model. However, when a model is restricted to sinusoidal behaviour, this restriction applies to both the time and frequency domain representations.

8.1 The wet and dry oscillator models appear similar in the frequency domain

One reason why critics of the dry oscillator have had difficulty in explaining the limitations of this model, the same mathematical notation and associated terminology are conventionally used for both the wet and dry oscillator models [31]. For instance, in the dry oscillator model, $B(\omega)$ is commonly used to indicate a constant, denoted B_s in the present paper, which equals the radiation damping at the frequency of sinusoidal oscillation, ω_s. As B_s depends on the frequency of oscillation, the terminology "*a function of frequency*" is used to describe it. For the wet oscillator model, the notation $B(\omega)$ is used to indicate that the radiation damping has different values at each frequency, and as such could be described as *a function of frequency*.

Thus conventional notation blurs the distinction between the frequency domain descriptions of the wet and dry oscillators. A reader's interpretation of *a function of frequency* and the notation $B(\omega)$ inevitably depends on whether they interpret a frequency domain model of a WEC as a wet or dry oscillator.

8.2 The dry oscillator model is not the frequency domain model of a WEC

The common notation and terminology describing the frequency domain versions of the wet and dry oscillators are responsible for another inconsistency in the literature. Occasionally the dry oscillator is associated with the frequency domain model of a WEC, while the wet oscillator model is associated with the time domain model of a WEC, as shall now be shown.

The association of the dry oscillator model with the frequency domain may have had its origins in the similar form of the frequency domain descriptions of the wet oscillator (3), and the time (7) and frequency (6) domain descriptions of the dry oscillator. However, at present this misconception continues because there are some well-known papers that may be interpreted as advocating this point of view [1–3]. To dispel this misconception, it is not sufficient to state that the wet and dry oscillators are distinct models and that each can be represented in the time or frequency domains; it is also necessary to identify these texts, and to demonstrate that they are describing a dry oscillator, *rather than* the frequency domain model.

The paper that most clearly shows the association of the dry oscillator model with the frequency domain is [2] by Greenhow and White. It aims to show that the dry oscillator is not appropriate for modelling the non-sinusoidal motion resulting from the non-linear control technique used: latching. Perhaps to make the paper more accessible to those using the dry oscillator model, the authors wait until the end of the paper to state that they consider this to be "*an incorrect hydrodynamic model*". However, when the model is introduced, the time domain equation of the dry oscillator is given, and it is referred to as "*the frequency domain equation*". It is written in such a way that it could be mistaken for a frequency domain equation, namely the constants M_s and B_s are written in a form that was conventional at the time, $M_a(\omega)$ and $b(\omega)$, and none of the variables are explicitly written as functions of time. The only indications of time domain terms is the overdot notation, which is used to denote time domain functions that are the first and second time derivatives of the displacement, $\dot{x}$ and $\ddot{x}$. However, this convention is not followed universally by wave energy engineers, some of whom use the overdot notation for frequency domain variables.

Despite being called a frequency domain equation, the *incorrect hydrodynamic model* presented in [2] is a time domain equation. This is clear from the development and substitution of the equations that follow, and from that fact that the equation is used as a time domain equation to solve for displacement at each time step in the simulations presented later in the paper. Comparison to (7) shows this is a dry oscillator model.

This paper [2] presents various arguments as to why the time domain equation is suitable for polychromatic modelling, while the "*the frequency domain equation*" is not. Some of these arguments are true for frequency domain modelling in general: it is not suitable for modelling non-linear behaviour, time varying control, or transient responses. As shown in Table 1, the time domain dry oscillator is likewise not suitable for modelling these behaviours, but for different reasons: nonlinearities, time variance and transients result in motion that is not sinusoidal. Latching control results in a time varying damping coefficient, and transients every time the damping is varied. Thus neither the frequency domain wet oscillator nor the time domain dry oscillator are suitable for polychromatic behaviour, while the time domain wet oscillator is. The reason why the contentious equation in [2] is an incorrect hydrodynamic model is not because it is a frequency domain model, but because it is a time domain representation of a dry oscillator valid only for sinusoidal motion.

It is important to recognise that the main message of [2] is to promote the correct wet oscillator model and to show that the dry oscillator is not a valid hydrodynamic model. The principal message is correct and important, but naming the dry oscillator model the frequency domain model gives the strong impression that frequency domain modelling is not suitable for representing the behaviour of a WEC in polychromatic conditions.

Another paper that appears to question the validity of frequency domain modelling for polychromatic conditions is [1]. This paper holds enormous authority in the wave energy community because it formulated the time domain equation of the wet oscillator still widely used. Before Cummins' paper, the hydrodynamic responses of ships were routinely expressed in terms of the added mass and damping at one particular frequency. This approach dates back to Stokes' [32] measurement of added mass using a pendulum, which of course oscillated at a single frequency. When Cummins discussed the advantages of his new formulation over the existing model [1], he used terminology that suggests that the model being replaced was the frequency domain version of the time domain model being presented. If it is not recognised that he is in fact describing the inadequacies of the dry oscillator model, this paper could be interpreted as a statement of the inadequacies of the frequency domain wet oscillator model.

In Fig. 9 it can be seen that a mass-spring-damper PTO force is only capable of optimal control when the velocity is sinusoidal. This is useful for interpreting another key paper in wave energy literature: Niato and Nakamura's derivation of the optimal control force [3]. There is a statement on the first page which could be interpreted as claiming that frequency domain modelling is suitable only for sinusoidal motion. However, another possible interpretation is that when using the mass-spring-damping PTO shown in Fig. 9, optimal control is only possible when motion is sinusoidal.

9 Conclusions

Various uses and applications of the frequency domain in wave energy engineering were discussed. There is a distinction between frequency domain functions, signals and systems. For signals and systems, the frequency domain is synonomous with the Fourier transform. There are four members of the Fourier transform family, and each has real and complex versions. Experimental procedure and representation of results is very similar for functions that do and do not involve the Fourier transform.

The concepts of wet and dry oscillators were introduced. This shows clearly the distinction between frequency domain representation and dry oscillators, and shows why the concepts may be easily confused using current mathematical notation. When WEC dynamics are represented by a mass-spring-damper (a dry oscillator), the model is correct only for sinusoidal behaviour in a WEC. When the PTO signal is represented by a mass-spring-damper (instantaneous), optimal power absorption is restricted to sinusoidal behaviour. However, the WEC model remains causal for polychromatic behaviour. When the control signal gives optimal power absorption for polychromatic behaviour, it is anticausal. When causal behaviour is required, this model should be restricted to sinusoidal behaviour. The use of experimental results optimised at one frequency to calculate power absorbed for a polychromatic input will result in an overestimation: this is the power that could be absorbed if future knowledge were available. Future knowledge is not required for optimal control of a WEC undergoing sinusoidal motion.

When a time domain model is valid for polychromatic behaviour, the corresponding frequency domain model is also valid for polychromatic behaviour. Papers that appeared to limit the use of frequency domain models to sinusoidal behaviour were in fact describing the limitations of mass-spring-damper models.

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