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OPTIMUM CONTROL OF OSCILLATION OF WAVE-ENERGY CONVERTERS

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Abstract

The power output from wave energy converters (WECs) may be increased by controlling the oscillation in order to approach an optimum interaction between the WEC and the incident wave. Optimally controlled WECs, designed to operate at full capacity a rather large fraction of their lifetime, may improve the economic prospects for wave power significantly. Most of the WECs discussed here, utilise just one mode of oscillation. An upper bound is given to the ratio between the converted power from a given wave, and the geometrical volume of the converter. One control strategy for maximising the converted power is based on measuring the incident wave, whereas another strategy utilises measurement of the WEC's own oscillation as input to the controller. In either case the measured quantity has to be predicted some seconds into the future because of non-causal control functions.

Introduction

Most of the proposed devices for conversion of wave energy are oscillating systems with a frequency-dependent response showing the phenomenon of resonance. At resonance, that is when the wave period agrees with the natural (eigen) period, the energy conversion is most ample.

With wave periods off resonance the conversion is less ample, in particular so if the resonance bandwidth is narrow. Wave energy converters of large horizontal extension, so-called "terminators" and "attenuators", have rather broad bandwidths, whereas "point absorbers", for which the extension is very small compared to the predominant wavelength, have rather narrow bandwidth. On the other hand, an advantage with point absorbers is that the smaller the structural volume of the converter is, the larger is the ratio between the potentially converted power and the mentioned volume.[1] With the reality of our wave climates a point absorber would have to operate mostly off resonance. Hence, for a point absorber it is imperative that means are provided for optimum control of the oscillatory motion in order to achieve a maximum of power conversion. For larger conversion structures, such as terminators and attenuators, with their broader bandwidths, the benefit of applying optimum control may be more marginal.[2]

In initial studies, optimum control was considered with regular (harmonical or sinusoidal) waves. The purpose of the control is then to obtain optimum phase and optimum amplitude of the oscillation in order to maximise the converted power. Subsequently, when optimum control was considered with real, irregular waves, the need for prediction became apparent.

The first problem to resolve are the conditions for optimum. Secondly, we need to discuss the general principles how to approach optimum. Thirdly, designs have to be proposed and components developed, in order to implement the optimum control in practice. The present paper will mainly address the two first-mentioned problems.

Historical review

Use of control engineering to optimise wave energy conversion was first proposed in the mid 1970s by Budal[3,4] and independently by Salter.[5,6] For the practical implementation it was proposed to use a controllable power take-off device, for instance a combined generator-and-motor. With this kind of continuous control the aim is to achieve optimum phase and optimum amplitude of the oscillation. Expressed in an other way, optimum phase control means to control the reactive power in order to maximise the active power. For this purpose it may be necessary that the instantaneous power conversion through the power take-off device is reversed during small fractions of the oscillation cycle. For this reason the term "reactive control" has been used[7] for continuous phase control.

Later Budal[8,9] proposed that approximate optimum phase control may be conveniently achieved by latching the wave absorber in a fixed position during certain intervals of the oscillation cycle. With this method, proposed independently also by Jones[10] and by French,[11] control action is made at discrete instants of the cycle. This is an alternative to the continuous phase control realised through a combined generator-and-motor or turbine-and-pump.

It was soon realised[1] that in order to apply discrete control (latching) it is necessary to predict an irregular wave some distance into the future, a time of the order of a quarter or a half of the eigen period of the wave-absorbing oscillator. Several years later a more general study was made on the relation between causality and optimum control of wave energy converters.[13] It can be shown that also with continuous reactive control, prediction is needed on the future variation of relevant physical quantities.

During the first half of the 1980s several theoretical and experimental studies on optimum wave power conversion were carried out, not only for devices of the oscillating-body type,[2,14,15,16,17] but also of the oscillating-water-column (OWC) type.[17,18,19] More recent works include a study by Korde,[20] who apparently concluded, incorrectly, that optimum oscillation was possible in irregular waves without the necessity of predicting the future state of the system. Not long ago a review[21] mentioned some other recent studies of optimum control which uses the latching principle, while others implement continuous reactive control. In order to avoid prediction, a sub-optimal control strategy using a causal control function has been proposed.[22]

Optimum conditions

The conditions for optimum oscillation of wave energy converters responding to a regular wave will be discussed in this section. At first we consider a wave energy converter (WEC) utilising only one mode of oscillation. It may be an OWC in a fixed structure or a floating body oscillating in one mode, heave, say. Then one condition for maximum converted energy is the phase condition that the oscillating air pressure of the OWC, or oscillating velocity of the body, is in phase with the excitation volume flux or the excitation force of the body, respectively. (For a WEC utilising several modes of oscillation, the optimum phase condition is, in general, more complicate.) In order to obtain a maximum of converted power, also an optimum amplitude condition has to be satisfied. This condition depends on whether or not the amplitude is unconstrained. For wave amplitudes below a certain level, the amplitude condition depends on whether the absorbed wave power or the converted useful power is to be maximised.

Unconstrained amplitudes

From early theoretical studies it is well-known[23,24,25] that for a resonant point absorber or for any axisymmetrical wave-absorbing device at resonance, the maximum power that can be absorbed equals the incident wave power associated with a wave front of width one wavelength divided by 2π, when the device is placed in the open sea. For a two-dimensional case[24,25,26] a symmetrical system oscillating in one mode can absorb half of the incident wave energy. Almost all incident energy can be absorbed if the two-dimensional system is sufficiently non-symmetric, as for instance the Salter Duck.[27]

To obtain a maximum of absorbed power from a regular wave, when only one mode of oscillation is utilised, the following two conditions have to be satisfied. Firstly, the oscillating velocity must be in phase with the excitation force (the wave force acting the oscillating body when it is held fixed) a condition which is automatically satisfied if the body is in resonance with the wave. Secondly, the amplitude of the oscillation must be adjusted to the optimum value, where the absorbed power is maximum when it equals the power reradiated into the sea by the oscillating system. We may express this in the following way: Maximum power is absorbed from the wave when the destructive interference between the reradiated wave and the incoming wave is largest. We may refer to the above two conditions as (i) the condition for optimum phase and (ii) the condition for optimum amplitude.

In practice only a certain fraction of the absorbed power can be converted to useful power, while the remaining power is lost by viscous effects, friction and other dissipative effects. Assume, for simplicity, that these losses may be represented by a mechanical loss resistance. Then the conditions for maximum useful power is the same phase condition as above, while the optimum amplitude is reduced.[28] This qualitative statement is probably true even in the more general case when both the power take-off and the loss effects are rather non-linear.

The above-mentioned two optimum conditions (i and ii) may be expressed alternatively as a single optimum condition in terms of complex mechanical impedance. (Remember that a complex number is composed of two real numbers.) Let the system's internal impedance Z_i (which has dimension force divided by velocity) be the sum of the oscillating body's mechanical impedance Z_m (representing its hydrostatic buoyancy stiffness as well as its real mass), of the loss impedance Z_f (which is usually real, that is, it is a loss resistance), and of the radiation impedance Z_r (composed of a real part, the radiation resistance - also termed radiation damping coefficient -, and an imaginary part, the radiation reactance, which is the product of the angular frequency and the so-called added mass). The optimum condition for maximum converted useful power may be expressed simply as follows:

The optimum load impedance Z_u presented by the power take-off device has to equal the complex conjugate of the internal impedance. Thus

$$(Z_u)_{opt} = Z_i^* = (Z_r + Z_m + Z_f)^* \, .$$

which is similar to a well-known analogous result in elementary theory for electrical a.c. circuits.

In terms of the applied-pressure description[29] of an OWC, analogous optimum conditions are applicable. For instance, the condition for optimum phase is that the dynamic air pressure (above the internal water surface) is in phase with the excitation volume flow (that is, the volume flow rate by the internal water surface when the dynamic air pressure is zero).

Constrained amplitudes

It is inherent with the design of a wave energy converter that there is an upper bound on the oscillation amplitude. Moreover, the energy converting device has a limited power capacity.

The above optimum conditions apply when these upper bounds, due to the design, do not come into play, that is when the height of the incoming wave is below a certain value (which depends on the design and also on the wave period). For higher waves we have to optimise under constraints. Then the optimum oscillation amplitude will necessarily be the design amplitude, while the optimum phase condition will be as before, provided the converted power does not exceed the rated capacity of the power take-off device. To avoid surpassing the power capacity in very high waves, another (i.e.sub-optimal) phase difference between oscillation and excitation might be desirable (and achievable by a suitable controllable power take-off device).

The wave height for which the constraints come into play is smaller for a small oscillating body (or "water body" of an OWC) than for a large one. The full design amplitude will thus be utilised in a larger fraction of the converter's lifetime for the smaller oscillating body.

When the wave height is relatively large and the optimum unconstrained amplitude would be much larger than the design amplitude, the reradiated power is much less than (and not, as in the unconstrained case, equal to) the absorbed power. In this situation the incident wave energy transport is relatively large, but just a rather small fraction of it is absorbed. The absorbed power is, however, large in relation to the design amplitude.

It is easy to show that the ratio P/V between absorbed wave power P and the volume V of the oscillating body has an upper bound[1],

$$P/V < (\pi/4)\rho g H/T$$

where ρ is the mass density of sea water, g the acceleration due to gravity, T the wave period and H the wave height. Thus for a typical wave with $H = 2$ m and $T = 10$ s we have $P/V < 1.6$ kW/m^3. This upper bound is approached, provided the following five conditions are satisfied:[30]

(a) The oscillating body is relatively small ($V \to 0$).
(b) Most of the incident (free) wave power remains in the ocean.
(c) The oscillation has an optimum phase.
(d) The oscillation amplitude equals the design amplitude.
(e) The oscillation mode has a source-like wave radiation.

Only two kinds of oscillators can satisfy the last condition (e), namely an OWC and a floating body utilising the heave mode. (Other modes such as surge and pitch - and even heave for a submerged body - have dipole-like wave radiation.) The conditions (c) and (d) put certain requirements on the power take-off device.

Assume that a wave-power plant, for instance an array of point absorbers, is designed according to conditions (a) and (b). Hence, the plant can convert only a rather small fraction of the natural power transported by the incident waves, when the wave height is moderate or large. However, when the wave height is sufficiently small, that is when the unconstrained optimisation is applicable, a more substantial fraction of the incident wave energy may be captured by the wave-power plant. A result of such a design principle is that there is relatively less variation in the power output than there is in the natural wave energy transport. An advantage with this is that it reduces the required size of the short-time energy store which, due to the variation of wave groups, is needed for any WEC delivering electricity to a grid. In other words, the primary mechanical energy delivered to the energy store (e.g. rotating flywheel or gas accumulator), has a higher quality, and hence higher value, than for a similar larger WEC which operates at its full design capacity a smaller fraction of its lifetime.

A design study, including economic considerations, is required in order to determine the design dimensions (maximum amplitude, power capacity etc.) for a wave-power plant with a given location and wave climate. The result of such a study will probably be a design where the design constraints come into play a significant portion of the lifetime of the wave-power plant.

Several modes of oscillation

A floating body has, in general, six modes of oscillation; surge, sway, heave, roll, pitch and yaw. Another example with more than one mode of oscillation is a fixed structure containing two, or more, OWCs. A floating body with one OWC is a system having up to seven modes of oscillation. A number (N) of interacting bodies, each of them oscillating in just one mode, is a system with an equal number (N) of modes of oscillation.

The optimisation problem of maximising the absorbed wave power is more complicate for a system of several modes of oscillation than for the previously considered system with a single degree of freedom. For instance, one cannot in general say (as in the case of one mode) that resonance provides optimum phase condition. Theories have been developed for the case of unconstrained amplitudes both for the case of oscillating bodies[31,32] and for the case of OWCs[29] as well as for the case of interacting OWCs and bodies[33].

Optimisation with constraints is even more complicated. Some preliminary studies were made in the early 1980s. A rather simple optimisation problem under one global constraint (upper bound on the sum of all amplitudes squared) was solved mathematically.[34] If the waves are sufficiently high, one may assume that all amplitudes have reached their design limits, and it remains to optimise the phases.[35] At more moderate wave heights, when some of the amplitudes are still unconstrained, while other of the amplitudes have reached their design limits, numerical optimisation procedures may be adopted.[36]

There may be additional types of constraints with a system of several modes of oscillation when the power take-off device does not provide a load damping for every mode of oscillation. For instance, two oscillating bodies may have only one common power take-off device utilising the relative motion between the bodies. Or a system of two interacting OWCs may have just one air turbine run by the pressure difference between the two air chambers above the OWCs.

The case of more than one mode of oscillation will not be discussed in great detail in the present paper.

Control methods and control strategies

Unconstrained continuous control

In formulating the optimum conditions of the preceding section it was assumed that the incident wave is regular ("monochromatic", "harmonical" or "sinusoidal"), which is an idealised situation. For real sea waves, which are irregular, a modified formulation, of the optimum conditions for maximum power conversion, is required.

Applying Fourier transform on the functions of time representing physical quantities of the wave, and also of the oscillation, those quantities are decomposed into harmonical components. In the situation of unconstrained amplitudes, linearity may be assumed. In this case the preceding optimum conditions are applicable to every harmonical component.[3] Then, applying the inverse Fourier transform, the optimum conditions are expressed in terms of convolutions in the time domain. Such a convolution is a linear transformation between two physical quantities. Transformations representing this optimum are non-causal, which means that some future information is required on at least one of the physical quantities. The better this quantity can be predicted, the closer the converted power may approach the theoretical maximum. What is required is a prediction so far into the future as the oscillator's impulse response "remembers" into the past. This time span, corresponding to the duration of the system's transient response, usually does not exceed many seconds. Hence, since this is shorter than the coherence time of sea waves, a reasonably good prediction ought to be possible.

If the wave spectrum is relatively narrow, the previously mentioned phase condition and amplitude condition are still approximately valid. However, the optimum condition is more precisely represented by a convolution. To apply these conditions information is needed on the excitation force, during a time interval of length extending a few seconds into the past as well as into the future.

Instead of measuring the excitation force directly, one may measure the incident wave and apply a known (not necessarily causal)[37] transfer function. When measuring the wave, it may be necessary to correct for the reradiated wave, which may be significant in cases where a substantial fraction of the incident wave power is to be absorbed by the converter. With the known (and predicted) excitation force as input, the controller has to provide, as output, the optimum oscillating velocity.

An alternative control procedure, to be described in more detail below, uses as input signals measurement of the WEC's oscillation instead of measurement of the waves.

One way to express the optimum condition is in terms of an optimum load impedance $(Z_u)_{opt}$, as explained previously. In the frequency domain this means that the optimum load force provided by the power take-off is the product of this optimum load impedance and the velocity. Noting that the internal mechanical impedance Z_i is determined by stiffness, inertia and (radiation and loss) damping, the optimum load force is composed, correspondingly, of three components, two of which, the stiffness and inertia components, are reactive. Providing these two terms of the optimum load force means "reactive control" or "phase control". For example, reactive control applied to the Salter Duck means that[6] "a force proportional to angular displacement (θ) is required" and "a force proportional to angular acceleration $(\ddot{\theta})$ is required". In addition the optimum active component of the load force, proportional to the angular velocity is required. For regular waves, of off-resonant periods, it is that simple. For irregular waves both prediction of measured signals into the future and convolution are implied as part of the optimum operation.

In general, for irregular waves the optimum load force, given by a product in the frequency domain, is represented by a convolution in the time domain. A corresponding optimum operation is based on the following principle. The input signal to the controller is in this case (not the excitation force due to the incident wave, but) the measured and predicted velocity (and/or acceleration and/or displacement) of the oscillating body. Based on an on-line computation of convolution integral(s), the controller then provides the optimum load force as output.

In formulating the above optimum control strategies a WEC utilising a body oscillating in one mode without amplitude constraints was assumed. For a WEC utilising one OWC without constraints the two different control strategies are as follows. For a strategy based on measuring the wave, correcting for the reradiated wave, and then computing and predicting (on-line) the excitation volume flux, the controller has to compute (by convolution) the optimum air pressure, as output. For the alternative control strategy it is required to measure and predict the air chamber pressure as input to the controller. Based on convolution computation the optimum air volume flux to the power take-off (for instance a blade-pitch-controlled air turbine)[38] is provided as output from the controller.

"Reactive control" or "phase control" is one of the necessary conditions in order to obtain optimum. It is then required that the total reactance vanishes. This is accomplished by the controllable power take-off system if it provides for a reactance contribution which cancels the internal reactance $\mathrm{Im}(Z_r + Z_m + Z_f)$. Off resonance this means that during certain parts of the cycle energy has to be delivered from the power take-off system to the oscillating system including the waves. In other words, the instantaneous power delivered from the waves to the power take-off is sometimes negative. Relative to the produced (average) useful power, the reactive power may be quite large, in particular for point absorbers due to their rather narrow resonance bandwidth. For this reason a power take-off with high conversion efficiency is needed.

Since high-pressure hydraulic energy-converting machineries may be developed to have efficiencies in the region 90 to 100 percent, this kind of mechanical technology is well suited for power take-off with optimum control. Such a power take-off should comprise a combined motor-and-pump and an energy storage. The energy flow into the storage is made up of the difference between energy delivered from the waves and energy returned to the waves. Using further machinery, such as an electrical generator a relatively even flow of energy from the store is converted into useful form.

Causal control function

Above we have described optimum control in order to obtain the theoretically maximum converted power. In practice this maximum can not be achieved exactly, but only approached, because prediction of physical variables, some seconds into the future, is imperfect.

Some alternative control strategies, somewhat sub-optimal, have been proposed, for instance a strategy with a causal control function, which do not need prediction of physical quantities.[22] With this strategy, the input to the controller is the excitation parameter, that is, the excitation volume flux for an OWC or excitation force in the case of an oscillating body. The output from the controller is the desired air pressure or body velocity, respectively. In the optimum control, aimed at maximising the absorbed wave power as described earlier, when wave prediction is involved, the (non-causal) transfer function of the controller is the inverse of twice the OWC's radiation conductance or the body's radiation resistance, respectively. Contrary, in the present case, where only the instantaneous and past values from the measurement are utilised, a transfer function of the controller is chosen as to maximise the absorbed power under the constraint of the transfer function being causal. Other constraints, for instance requiring the transfer function to be a rational function,[22] may be included for mathematical convenience. Because of these constraints the absorbed power will necessarily be somewhat lower than the theoretically maximum absorbed power. There may, however, still be a considerable gain in the performance, as compared to a converter without any form of optimum control.

Discrete control

The strategies described above may be classified as continuous control, which means that the controller can act at any instant of time. An alternative strategy is discrete control, where the controller can act on the system only a finite number of instants during each wave cycle. In practice this number is small. The author is not aware of any proposal where the number exceeds four. (In the present classification, "continuous" control includes the application of digital computation and measurements data-logged with a reasonably high sampling rate, typically 5 to 50 Hz.)

An example where this strategy is applied is a heaving buoy where approximate optimum phase is obtained by means of latching, as proposed by Budal[9] and, independently, by Jones[10] and by French.[11] The natural period of the buoy is shorter than the predominate wave periods. Thus, even outside resonance, approximate optimum phase is obtained by latching the buoy at an instant when its velocity is zero, and then releasing it at an instant such that the phase of its velocity will have a best possible match with the phase of the predicted excitation force.

The damping of the oscillation must be properly adjusted in order to obtain approximate optimum amplitude. This is accomplished if, for each cycle, an appropriate amount of energy is taken up by the power take-off system. A very flexible power take-off is required, due to the large variety of real ocean waves. If the desired flexibility is limited, this may represent an extra constraint on the optimum control.

One advantage with latching is that any negative energy flow to the power take-off is eliminated. The requirement on high conversion efficiency of the power take-off is correspondingly eased, as compared to the case of continuous optimum control. However, with latching the phase condition is fulfilled only to a certain approximation.

Latching may be achieved by a mechanism, such as a friction coupling or a clutch.[15] If the power take-off includes a hydraulic cylinder-and-piston directly connected with an oscillating body's motion, latching may be achieved by closing a valve.[1] It is assumed here that the working fluid is incompressible.

With a pneumatic power take-off, which is typical with wave energy converters of the OWC type, discrete control may be achieved by opening or closing air valves. Then the motion is not completely latched. However, operation of the valves has an influence on the oscillatory motion. Optimum oscillation for maximum absorbed power may, to some extent be approached in this way.

If an air valve in series with an air turbine is used for this discrete control, there is a drawback that the air will flow in relatively large peaks between time intervals of zero flow. A consequence is increased variation of air flow through the turbine, resulting in a reduced efficiency of the conversion from pneumatic energy to mechanical energy in the rotating turbine.

In order to avoid this drawback, Budal[39] proposed to operate two air valves for discrete control of a twin-OWC device, instead of a single-OWC device. The air turbine, placed between the air chambers above two adjacent OWCs, is supplied with a more even flow of air. The optimum control function is somewhat difficult to determine, both because it is a two-mode oscillating system (two OWCs) and because the control is discrete. Experimental or numerical studies are necessary to determine the best operation of the air valves.

Conclusion

We have here presented a review of methods to increase the power output from wave energy converters (WECs) by controlling the oscillation in order to approach an optimum interaction between the WEC and the incident wave. Such methods represent an essential step in furthering a practical use of ocean wave energy in a large scale. Applying optimum control on WECs designed to operate at full capacity a rather large fraction of their lifetime, may improve the economic prospects for wave power significantly.

We may classify optimum control methods into continuous or discrete methods. In the former case the control can operate at any instant, while in the latter case control action can be made just one or a few times during every oscillation cycle. With discrete control, for instance by latching the oscillation during two time intervals of each cycle, the optimum oscillation can be achieved only to a certain approximation.

In another way we may distinguish between two different control strategies, where one is based on measurement of the wave, while the other is based on measurement of the WEC's oscillation. There is a need for further research to compare the two strategies, in order to determine which is the best one to implement in practice, or whether both strategies should be used in combination. Both strategies require prediction, some seconds into the future, of the wave or oscillation, respectively. The coherence time of real ocean waves is probably sufficiently long to make such a prediction feasible in practice. A somewhat sub-optimal strategy based on measurement (but without need for prediction) of the wave has been proposed.[22]

Several subjects related to the optimum control need more research consideration. Methods for prediction of the wave and of the oscillation need to be improved. Computer software for the control has to be developed, as well as mechanical components for realising the desired oscillation, requiring certain velocities, accelerations or forces.

Most of the discussion in the present review is related to the simple case with a WEC oscillating in one mode only. It is necessary to make further studies including more modes of oscillation and also constraints due to bounds on excursions, accelerations, forces, pressures and power handling capacities. Means to protect the WEC in extreme wave situations have also to be included in the overall control strategy.

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