RASP Wave-Energy Conversion Project Final Report

February 28, 2013

Abstract

Contents

List of Figures

List of Tables

Chapter 1

Introduction

In marine-related engineering problems... Google

1.1 Ocean Wave Basics

The approach for creating a wave absorbing boundary condition used here rests on a technique of domain decomposition. The entire fluid domain is split into an “inner” region encompassing the domain of the fluid near the body of interest, and an “outer” region encompassing the remainder of the fluid domain which is usually infinite in lateral extent. With this decomposition, the flow in the outer region can be considered independently of the interior flow and a matching at the interface performed. If a general solution to this outer flow is found, matching the general outer solution to a particular interior flow effectively provides an outer boundary condition for the flow in the inner region.

1.2 Thesis Outline

In summary, the aim of this thesis is to develop a general solution of the linearized free-surface external potential flow problem and apply it as an outer boundary condition to inviscid and viscous interior flows. Chapter 2 develops the general “shell-function” solution of the linear flow in the exterior region. This solution is obtained utilizing a new spectral collocation technique to solve an integral equation representing the outer flow; this is done in a general way so that the solution can be computed once and applied to a variety of interior problems. Chapter 3 applies the outer shell-function solution as an outer boundary condition for several interior region solution techniques for solving inviscid flow problems. A new matching technique is developed that allows the shell-function formulation to be applied to field discretization methods for solving the interior inviscid flow problem. Chapter 4 examines the mathematical description of a viscous-inviscid fluid interface in the presence of a free-surface with waves. The boundary conditions developed in chapter 4 form the basis of the numerical viscous-inviscid matching technique developed in chapter 5. Chapter 5 applies the shell-function outer solution as an outer boundary condition to an interior solution of a viscous-flow problem. Two matching techniques for the viscous-inviscid case are explored, the first is an extension of work appearing in the literature and the second is a newly developed technique based upon the results of chapter 4. Numerical results are presented and the promises and limitations of the technique are explored. A concluding chapter summarizes the work and outlines promising extensions of the techniques developed in this thesis. Several appendixes present details of the computational scheme and particularly the Green function computation, storage, and recall techniques which allows the computations associated with the outer solution to be re-used for any problem in the interior domain.

PowerBuoy PTO diagram - 03/15/2012

1.3 Ocean Wave Basics

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1.4 Thesis Outline

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Appendix A

Frequency Domain Analysis of Floating Bodies

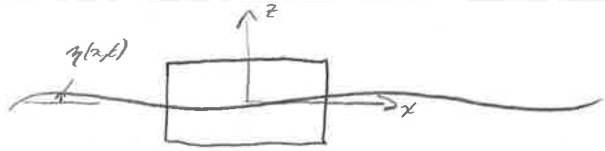
This appendix contains the frequency domain analysis utilizing linear wave theory for the heave motion of floating bodies in three configurations.

1. Single untethered floating body
2. Single floating body connected to the seafloor with a spring/damper tether.
3. Two-body system with floating body connected to a submerged body with a spring/damper tether.

   Because this analysis is performed in the frequency domain, it is neccessarily linear. Drag forces on the submerged body are therefore linearized. All motions are assumed to be sinesoidal and represented by a complex motion amplitude as follows:

$$\begin{aligned} z(t) &= Re[\xi e^{i\omega t}] \\ \dot{z}(t) &= Re[\xi i\omega e^{i\omega t}] \\ \ddot{z}(t) &= Re[-\xi\omega^2 e^{i\omega t}] \end{aligned}$$

FREQ DOMAIN HEAVE RESPONSE OF FLOATING BODY



BODY POSITION. $z(t) = x_3(t) = Re\left[\xi_3 e^{i\omega t}\right]$

VELOCITY: $\dot{z}(t) = \dot{x}_3(t) = Re\left[i\omega\,\xi_3 e^{i\omega t}\right]$

ACCEL: $\ddot{z}(t) = \ddot{x}_3(t) = Re\left[-\omega^2 \xi_3 e^{i\omega t}\right]$

ξ_3 = COMPLEX HEAVE AMPLITUDE.

FORCES ACTING ON BODY

WEIGHT $F_3^W(t) = -mg$

HYDROSTATIC $F_3^{HS}(t) = \rho\forall g - \rho g S\, Re\left[\xi_3 e^{i\omega t}\right]$

RADIATION $F_3^R(t) = Re\left[\xi_3 e^{i\omega t} f_{33}\right]$ } ASSUMES NO CROSS COUPLING BETWEEN MODES.

$$f_{33} = \omega^2 a_{33} - i\omega b_{33}$$

a_{33} = ADDED MASS

b_{33} = DAMPING

REAL

$$= Re\left[\xi_3 e^{i\omega t}\left[\omega^2 a_{33} - i\omega b_{33}\right]\right]$$

EXCITING $F_3^E(t) = Re\left[A X_3 e^{i\omega t}\right]$

X_3 = COMPLEX WAVE EXCITING FORCE.

$$m\ddot{x}_3 = \sum F_3$$

THESE ARE EQUAL FOR FLOATING BODY

$$Re\left[-m\omega^2 \xi_3 e^{i\omega t}\right] = Re\left[\cancel{-mg} + \cancel{\rho\forall g} - \rho g S \xi_3 e^{i\omega t} + \xi_3 e^{i\omega t} f_{33} + A X_3 e^{i\omega t}\right]$$

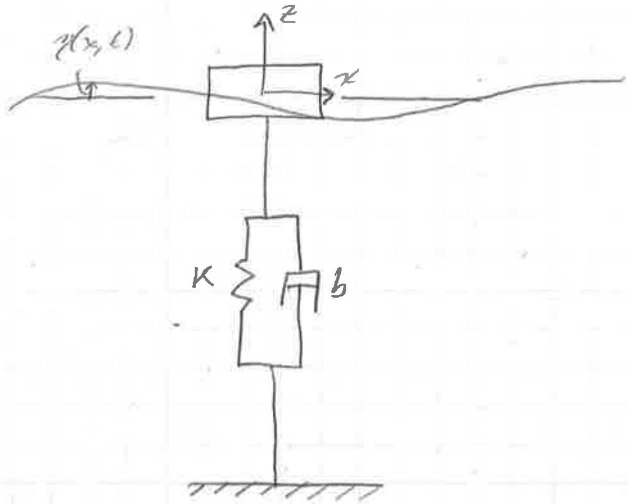
$$\xi_3\left[-m\omega^2 + \rho g S - f_{33}\right] = A X_3$$

$$\frac{\xi_3}{A} = \frac{X_3}{-m\omega^2 + \rho g S - f_{33}} = \underline{\underline{\frac{X_3}{-\omega^2(m+a_{33}) + i\omega b_{33} + \rho g S} = \xi_3}}$$

RESONANCE $\left\{ -\omega^2(m+a_{33}) + \rho g S = 0 \right.$

$$\omega = \sqrt{\frac{\rho g S}{(m+a_{33})}}$$

FREQ DOMAIN HEAVE RESPONSE OF TETHERED FLOATING BODY



ξ_3 = COMPLEX HEAVE AMPLITUDE.

BUOY POSITION: $z(t) = x_3(t) = Re\left[\xi_3 e^{i\omega t}\right]$

VELOCITY: $\dot{z}(t) = \dot{x}_3(t) = Re\left[i\omega \xi_3 e^{i\omega t}\right]$

ACCEL: $\ddot{z}(t) = \ddot{x}_3(t) = Re\left[-\omega^2 \xi_3 e^{i\omega t}\right]$

FORCES ACTING ON BODY:

WEIGHT: $F_3^W(t) = -mg$

HYDROSTATIC: $F_3^{HS}(t) = \rho \forall g - \rho g S\, Re\left[\xi_3 e^{i\omega t}\right]$

SPRING: $F_3^S(t) = -K z(t) = -K\, Re\left[\xi_3 e^{i\omega t}\right]$

DAMPER: $F_3^D(t) = -b \dot{z}(t) = -b\, Re\left[i\omega \xi_3 e^{i\omega t}\right]$

RADIATION:

$F_3^R(t) = Re\left[f_{33} \xi_3 e^{i\omega t}\right]$ $\quad f_{33} = \omega^2 a_{33} - i\omega b_{33}$ } ASSUMES NO CROSS COUPLING BETWEEN MODES

$a_{33} = a_{33}(\omega)$ = ADDED MASS
$b_{33} = b_{33}(\omega)$ = DAMPING } REAL

$$= Re\left[\xi_3\left[\omega^2 a_{33} - i\omega b_{33}\right] e^{i\omega t}\right]$$

EXCITING:

$F_3^E(t) = Re\left[A X_3 e^{i\omega t}\right]$ $\quad X_3 = X_3(\omega)$ = COMPLEX WAVE EXCITING FORCE.

$$m \ddot{x}_{33} = \sum F_3$$

$$Re\left[-m\omega^2 \zeta_3 e^{i\omega t}\right] = Re\Big[\overbrace{-mg + \rho \forall g}^{\text{CANCEL FOR FLOATING BODY}} - \rho g S \zeta_3 e^{i\omega t} - k\zeta_3 e^{i\omega t} - i\omega b \zeta_3 e^{i\omega t}$$

$$+ \zeta_3\left[\omega^2 a_{33} - i\omega b_3\right] e^{i\omega t} + A X_3 e^{i\omega t}\Big]$$

$$Re\left[A X_3 e^{i\omega t}\right] = Re\left[e^{i\omega t}\left(-m\omega^2 \zeta_3 + \rho g S \zeta_3 + k\zeta_3 + i\omega b \zeta_3 - \zeta_3\left(\omega^2 a_{33} - i\omega b_{33}\right)\right)\right]$$

$$A X_3 = \zeta_3\left[-\omega^2(m + a_{33}) + i\omega(b + b_{33}) + (\rho g S + k)\right]$$

$$\zeta_3 = \frac{A X_3}{-\omega^2(m + a_{33}) + i\omega(b + b_{33}) + (\rho g S + k)}$$

POWER DISSIPATED IN DAMPER

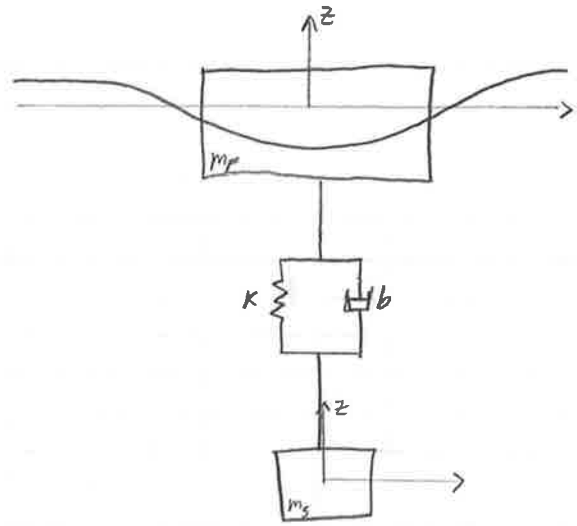
$$\bar{P} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{b}{T}\int_0^T |V(t)|^2\,dt = \frac{b}{T}\int_0^T \left\{Re\left[i\omega \zeta_3 e^{i\omega t}\right]\right\}^2 dt$$

$$= \frac{b}{T}\int_0^T \left[-\omega\left(Re(\zeta_3)\sin\omega t + Im(\zeta_3)\cos\omega t\right)\right]^2 dt =$$

$$= \frac{b\omega^2}{T}\int_0^T \left[Re(\zeta_3)^2 \sin^2\omega^2 + Im(\zeta_3)^2 \cos^2\omega t + 2Re(\zeta_3)Im(\zeta_3)\cos\omega t \sin\omega t\right] dt$$

$$= \frac{b\omega^2}{T}\left[\frac{T}{2}Re(\zeta_3)^2 + \frac{T}{2}Im(\zeta_3)^2\right] = \frac{b\omega^2}{2}\left[Re(\zeta_3)^2 + Im(\zeta_3)^2\right] = \bar{P}$$

FREQ DOMAIN HEAVE RESPONSE OF SUBMERGED BODY BELOW FLOATING BODY



FLOATING BODY MOTION (RELATIVE TO EQUILIBRIUM)

$$Z_3^F(t) = x_3^F(t) = Re[\xi_3 e^{i\omega t}]$$
$$\dot{Z}_3^F(t) = \dot{x}_3^F(t) = Re[i\omega \xi_3 e^{i\omega t}]$$
$$\ddot{Z}_3^F(t) = \ddot{x}_3^F(t) = Re[-\omega^2 \xi_3 e^{i\omega t}]$$

SUBMERGED BODY MOTION (RELATIVE TO EQUILIBRIUM)

$$Z_3^S = x_3^S(t) = Re[\varsigma_3 e^{i\omega t}]$$
$$\dot{Z}_3^S = \dot{x}_3^S(t) = Re[i\omega \varsigma_3 e^{i\omega t}]$$
$$\ddot{Z}_3^S = \ddot{x}^S(t) = Re[-\omega^2 \varsigma_3 e^{i\omega t}]$$

FORCES ON FLOATING BODY

WEIGHT: $F_3^{F:W}(t) = -m_F g$

HYDROSTATIC: $F_3^{F:HS}(t) = \rho \forall_F g - \rho g S_F Re[\xi_3 e^{i\omega t}]$

RADIATION: $F_3^{F:R}(t) = Re[\xi_3 e^{i\omega t}[\omega^2 a_{33}^F - i\omega b_{33}^F]]$

EXCITING: $F_3^{F:E}(t) = Re[A X_3^F e^{i\omega t}]$

TETHER:
$$F_3^{F:T}(t) = K(z_3^S(t) - z_3^F(t)) + b(\dot{z}_3^S(t) - \dot{z}_3^F(t)) + (\rho \forall_S g - m_S g)$$
$$= Re[\{K(\varsigma_3 - \xi_3) + b\,i\omega(\varsigma_3 - \xi_3)\}e^{i\omega t} + (\rho \forall_S g - m_S g)]$$
$$= Re[\{\varsigma_3(K + i\omega b) - \xi_3(K + i\omega b)\}e^{i\omega t} + (\rho\forall_S g - m_S g)]$$

FORCES ON SUBMERGED BODY

WEIGHT: $F_3^{S:W}(t) = -m_S g$

HYDROSTATIC: $F_3^{S:HS}(t) = \rho \forall_S g$

RADIATION: $F_3^{S:R} = Re[\varsigma_3 e^{i\omega t}[\omega^2 a_{33}^S - i\omega b_{33}^S]]$

AS SUBMERGENCE $\to \infty$
$b_{33}^S \to$ viscous drag and $a_{33}^S \to a_{33}^S(\omega \to \infty)$

EXCITING: $F_3^{S:E}(t) = Re[A X_3^S e^{i\omega t}]$ ($X_3 \to 0$ AS SUBMERGENCE INCREASES)

TETHER: $F_3^{S:T} = -F_3^{F:T}$

LINEARIZATION OF VISCOUS DRAG

$$b_{33} = \frac{1}{2} C_D S \rho$$

FLOATING BODY EQUATION OF MOTION

$$m_F \ddot{x}_3^F = \sum F_3^F$$

$$Re\left[-m_F w^2 \xi_3 e^{iwt}\right] = Re\left\{-m_F g + \rho g \forall - \rho g S_f \xi_3 e^{iwt} + \xi_3 (w^2 a_{33}^F - iwb_{33}^F) e^{iwt} + A X_3 e^{iwt} + (K + iwb)\varsigma_3 e^{iwt} - (K+iwb)\xi_3 e^{iwt} + (\rho \forall_S g - m_S g)\right\}$$

(1) $$Re\left\{\left[-m_F w^2 + \rho g S_f - (w^2 a_{33}^F - iwb_{33}^F) + (K+iwb)\right]\xi_3 e^{iwt} - (K+iwb)\varsigma_3 e^{iwt}\right\} = Re\left\{\underbrace{-m_F g + \rho g \forall_F - m_S g + \rho \forall_S g}_{=0} + A X_3^F e^{iwt}\right\}$$

SUBMERGED BODY EQUATIONS OF MOTION

$$m_S \ddot{x}_3^S = \sum F_3^S$$

$$Re\left[-m_S w^2 \varsigma_3 e^{iwt}\right] = Re\left\{\cancel{-m_S g} + \cancel{\rho \forall_S g} + \varsigma_3 e^{iwt}(w^2 a_{33}^S - iwb_{33}^S) + A X_3^S e^{iwt} - (K+iwb)\varsigma_3 e^{iwt} + (K+iwb)\xi_3 e^{iwt} - (\cancel{\rho \forall_S g} - \cancel{m_S g})\right\}$$

(2) $$Re\left\{-(K+iwb)\xi_3 e^{iwt} + \left[-m_S w^2 - (w^2 a_{33}^S - iwb_{33}^S) + (K+iwb)\right]\varsigma_3 e^{iwt}\right\} = Re\left\{A X_3^S e^{iwt}\right\}$$

(1) and (2) ARE TWO EQNS FOR TWO UNKNOWNS (ξ_3 and ς_3)

FROM (1)

$$\xi_3 = \frac{1}{(K-i\omega b)}\left\{\left[-m_s\omega^2 + \rho g S^s - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K-i\omega b)\right]\zeta_3 - A x_3^s\right\} \qquad (3)$$

INSERTING INTO (2)

$$-(K-i\omega b)\zeta_3 + \frac{\left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K-i\omega b)\right]\left[-m_s\omega^2 + \rho g S^s - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K-i\omega b)\right]}{(K-i\omega b)}\zeta_3$$

$$- \frac{A x_3^s}{(K-i\omega b)}\left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K-i\omega b)\right] = A x_3^s$$

$$\zeta_3 = \frac{A x_3^s + \frac{A x_3^s}{(K-i\omega b)}\left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K-i\omega b)\right]}{-(K-i\omega b) + \frac{\left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K-i\omega b)\right]\left[-m_s\omega^2 + \rho g S^s - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K-i\omega b)\right]}{(K-i\omega b)}}$$

$$\zeta_3 = \frac{A x_3^s (K-i\omega b) + A x_3^s\left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K-i\omega b)\right]}{-(K-i\omega b)^2 + \left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K-i\omega b)\right]\left[-m_s\omega^2 + \rho g S^s - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K-i\omega b)\right]} \qquad (4)$$

INSERT (4) INTO (3) TO FIND ξ_3

<u>LOW FREQUENCY LIMIT</u>

$\omega \to 0$

$$\zeta_3 = \frac{A x_3^s K + A x_3^s K}{-K^2 + [K][\rho g S^s] + K} = \frac{AK[x_3^s + x_3^F]}{K \rho g S^F} = \frac{A[x_3^s + x_3^F]}{\rho g S^F} \Rightarrow \frac{[m]\left[\frac{K_g m}{m_s^2}\right]}{\left[\frac{K_g}{m^3}\cdot\frac{m}{s^2}\cdot m^2\right]}$$

$\frac{F}{m}$

$$= [m] \checkmark$$

POWER DISSIPATED IN DAMPER

$$P(t) = F(t)\cdot V(t) = b\left(\dot{z}_3^{\,\zeta}(t) - \dot{z}_3^{\,\xi}(t)\right)^2$$

$$= b\left(Re\left\{ i w \zeta_3 e^{iwt} - i w \xi_3 e^{iwt} \right\}\right)^2$$

$$= b\left(Re\left\{ i w e^{iwt}(\zeta_3 - \xi_3)\right\}\right)^2$$

$\Delta v = \zeta_3 - \xi_3$

$\hookrightarrow$ COMPLEX

$$= b\,Re\left\{\left[-w\sin wt + iw\cos wt\right]\left[Re(\Delta v) + i\,Im(\Delta v)\right]\right\}^2$$

$$= b\,Re\left\{\left(-Re[\Delta v]\,w\sin wt - Im[\Delta v]\,w\cos wt\right) + i\left(Re(\Delta v)\,w\cos wt - Im(\Delta v)\,w\sin wt\right)\right\}^2$$

$$= b\left(-Re[\Delta v]\,w\sin wt - Im[\Delta v]\,w\cos wt\right)^2$$

$$= b\left[\left(Re(\Delta v)\,w\sin wt\right)^2 + \left(Im[\Delta v]\,w\cos wt\right)^2 + 2Re[\Delta v]Im(\Delta v)\,w^2\cos wt\sin wt\right]$$

$$P(t) = b\left[Re[\Delta v]^2 w^2\sin^2 wt + Im[\Delta v]^2 w^2\cos^2 wt + 2Re[\Delta v]Im[\Delta v]\,w^2\cos wt\sin wt\right]$$

AVERAGE POWER OVER ONE CYCLE

$$\bar{P} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{bw^2}{T}\int_0^T\left(Re[\Delta v]^2\sin^2 wt + Im[\Delta v]^2\cos^2 wt + 2Re[\Delta v]Im[\Delta v]\cos wt\sin wt\right)dt$$

$$= \frac{bw^2}{T}\left[\frac{T}{2}Re[\Delta v]^2 + \frac{T}{2}Im[\Delta v]^2\right] = \underline{\frac{bw^2}{2}\left[Re[\Delta v]^2 + Im[\Delta v]^2\right] = \bar{P}}$$

ALTERNATIVELY, FORMULATE RESULT OF PAGE 3 AS 2 EQNS FOR 2 UNKNOWNS.

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \zeta_3 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$A \quad x = B \quad \Rightarrow \quad x = A^{-1} B$$

$$A_{11} = -m_f w^2 + \rho g S_F - (w^2 a_{33}^f - i w b_{33}^f) + (k + i w b)$$

$$A_{12} = -(k + i w b)$$

$$A_{21} = -(k + i w b)$$

$$A_{22} = -m_s w^2 - (w^2 a_{33}^s - i w b_{33}^s) + (k + i w b)$$

$$B_1 = A X_3^f$$

$$B_2 = A X_3^s$$

RELATION OF WAMIT REPORTED QUANTITIES TO THESE DOCUMENTS

$a_{33} = \overline{A}_{33}\rho$

$b_{33} = \overline{B}_{33}\rho w$

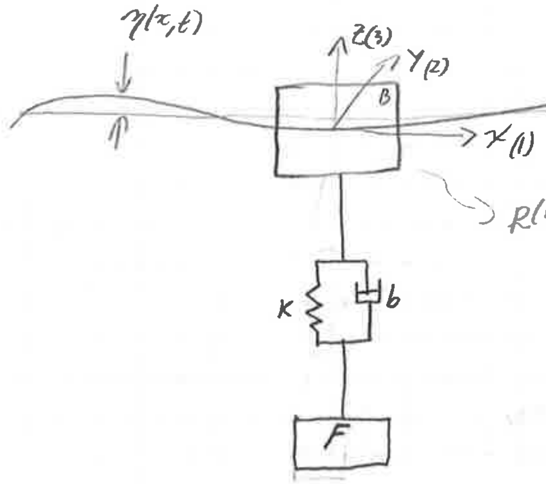
$X_3 = \rho g \overline{X}_3$

OVERBAR QUANTITIES ARE REPORTED FROM WAMIT IN .1 AND .3 FILES

Appendix B

Time Domain Analysis of Floating Bodies

TIME DOMAIN ANALYSIS OF FLOATING BODY MOTIONS



$$R(t) = \begin{bmatrix} p_x \\ p_y \\ p_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} \qquad \underset{\sim}{V}(t) = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \dot{v}_{\alpha_x} \\ \dot{v}_{\alpha_y} \\ v_{\alpha_z} \end{bmatrix}$$

EQNS OF MOTION

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{R}}(t) = \sum \underset{\sim}{F}$$

$$\begin{bmatrix} m & 0 & 0 & & & \\ 0 & m & 0 & & 0 & \\ 0 & 0 & m & & & \\ & & & I_{11} & I_{12} & I_{13} \\ & 0 & & & I & I_{22} \\ & & & & & I_{33} \end{bmatrix} \begin{bmatrix} \ddot{p}_x \\ \ddot{p}_y \\ \ddot{p}_z \\ \ddot{\alpha}_x \\ \ddot{\alpha}_y \\ \ddot{\alpha}_z \end{bmatrix} = \begin{bmatrix} \sum F_x \\ \sum F_y \\ \sum F_z \\ \sum M_x \\ \sum M_y \\ \sum M_z \end{bmatrix} \Rightarrow \dot{\underset{\sim}{x}} = \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \\ \dot{\alpha}_x \\ \dot{\alpha}_y \\ \dot{\alpha}_z \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \\ \dot{v}_{\alpha x} \\ \dot{v}_{\alpha y} \\ \dot{v}_{\alpha z} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} I \end{bmatrix} & \\ & \begin{bmatrix} M^{-1} \end{bmatrix} \end{bmatrix} \begin{bmatrix} x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ \sum F_x \\ \sum F_y \\ \sum F_z \\ \sum M_x \\ \sum M_y \\ \sum M_z \end{bmatrix}$$

$$(1) \qquad \dot{x} = f(\underset{\sim}{x}, t)$$

SYSTEM OF 1st OR ODE'S TO BE SOLVED NUMERICALLY.
[EULER METHOD
R-K METHOD,
ETC]

REQUIRES INITIAL CONDITION.

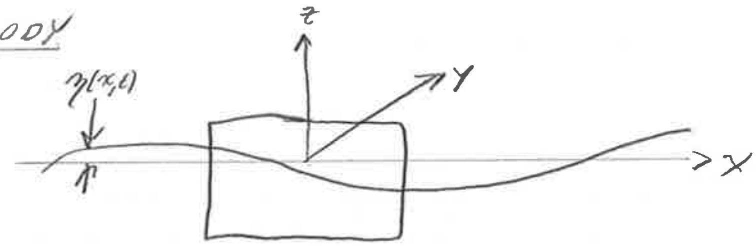
MULTIPLE BODIES (i.e. BUOY + FOIL)

$$\begin{bmatrix} \begin{bmatrix} \dot{p}_x^B \\ \dot{p}_y^B \\ \vdots \\ \dot{v}_x^B \\ \vdots \end{bmatrix} \\ \begin{bmatrix} \dot{p}_x^F \\ \vdots \\ \dot{v}_y^F \\ \vdots \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} I \end{bmatrix} \\ \begin{bmatrix} M_B^{-1} \end{bmatrix} \end{bmatrix} & \\ & \begin{bmatrix} \begin{bmatrix} I \end{bmatrix} \\ \begin{bmatrix} M_F^{-1} \end{bmatrix} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} x_6^B \\ \vdots \\ \sum F_y^B \\ \vdots \end{bmatrix} \\ \begin{bmatrix} x_6^F \\ \vdots \\ \sum F_y^F \end{bmatrix} \end{bmatrix}$$

WITH MORE THAN ONE BODY, THESE EXPRESSIONS MAY DEPEND ON THE POS, VEL, ACCEL OF MORE THAN ONE OF THE BODIES. OTHERWISE,

$$\dot{\underset{\sim}{x}} = f(\underset{\sim}{x}, t) \quad \text{STILL.}$$

ΣF FOR SINGLE FLOATING BODY



FULLY LINEARIZED THEORY

- → WAVE AMPLITUDE SMALL RELATIVE TO WAVE-LENGTH
- → BODY MOTIONS SMALL RELATIVE TO WAVE-LENGTH
- → SUPER POSITION OF SOLUTIONS
  - → EACH OF THE 6 MODES OF MOTION CAN BE CONSIDERED INDEPENDENTLY
  - → WAVE FORCES CAN BE DECOMPOSED AS WAVE EXCITING FORCES AND RADIATION FORCES.

CONSIDER FORCES IN Z DIRECTION

$$\sum F_z = F_z^W + F_z^{HS} + F_z^E + F_z^R$$

→ $F_z^W = -mg$ } CONST

→ $F_z^{HS} = \rho g \nabla$ ← DISPLACEMENT = FCN (p_z)

$$= \rho g \forall + \rho g S p_z \quad \left[p_z = \zeta(x,t) \right]$$

$\forall$ ← EQUILIBRIUM DISPLACEMENT; S ← WATERPLANE ARE

→ F_z^E = WAVE EXCITING FORCE IN Z-DIRECTION.

↳ THIS IS THE FORCE ON THE BODY IN EQUILIBRIUM POSITION DUE TO INCOMING AND DIFFRACTED WAVE FIELD. DOES NOT DEPEND ON BODY POSITION OR VELOCITY

$$= \int_{-\infty}^{\infty} K_z(\tau)\, \eta_0(t-\tau)\, d\tau$$

η_0 ← INCIDENT WAVE HEIGHT AT $x=0$

K_z ← EXCITING IMPULSE RESPONSE FUNCTION, DEPENDS ONLY ON BODY GEOMETRY. [CHARACTERIZES BODY]

F_z^R = WAVE RADIATION FORCE IN Z-DIRECTION

↳ THIS IS THE FORCE DUE TO WAVES CREATED BY THE BODIES MOTION

$$= \underbrace{-\mu_{a_{33}}}_{\text{ADDED TO M MATRIX}} \ddot{x}_3(t) - \int_0^t L_{33}(t)\, \ddot{x}_3(t-\tau)\, d\tau \qquad = f(\underset{\sim}{x}, t)$$

ADDED MASS ↑ DEPENDS ON BODY GEOMETRY

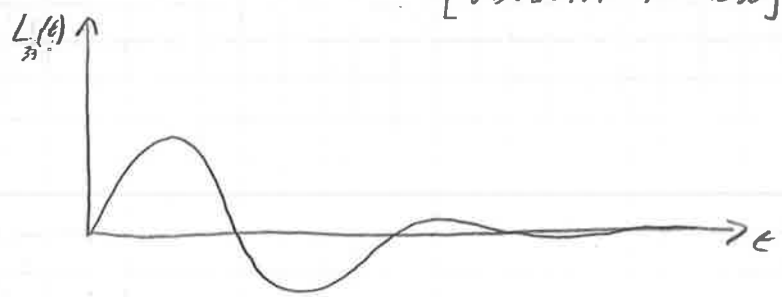
$\ddot{x}_3(t-\tau)$: PAST BODY MOTION → MEMORY EFFECT

$L_{33}(t)$: IMPULSE RESPONSE FUNCTION, DESCRIBES RADIATION FORCE AND IS A FCN OF BODY GEOMETRY.

Actually:

$$F_z = -\mu_{a_{3j}} \ddot{x}_j(t) - \int_0^t L_{3j}\, \ddot{x}_j(t-\tau)\, d\tau$$

RADIATION IMPULSE RESPONSE FUNCTION = FORCE ON BODY DUE TO A STEP CHANGE IN BODY POSITION [VELOCITY IMPULSE]

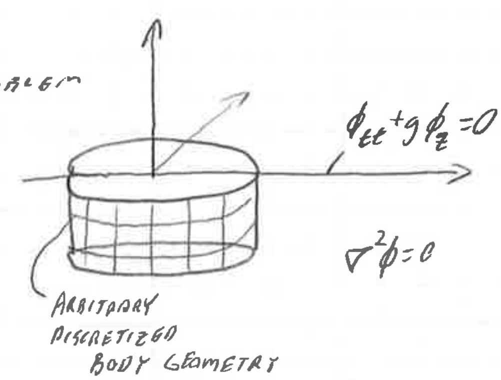


CAN BE COMPUTED FROM FREQUENCY DOMAIN HYDRODYNAMIC COEFFICIENTS [ADDED MASS / DAMPING]

PROGRAMS TO SOLVE FREQUENCY DOMAIN PROBLEM

→ WDRA3D

→ WAMIT



ANALYSIS APPROACH FOR A GIVEN BODY GEOMETRY

→ USE WDRA3D/WAMIT TO COMPUTE FREQ DOMAIN COEFFICIENTS.

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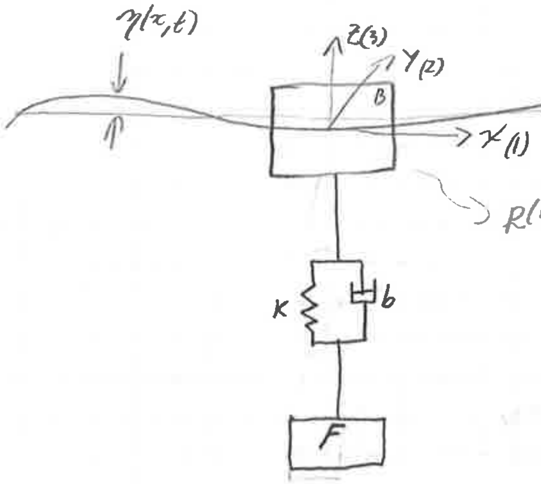
ADDING ADDITIONAL FORCES

i.e. FOR CASE OF SUBMERGED FOIL, THIS BODY WILL CONTRIBUTE FORCES THROUGH TETHER TO THE FLOATING BODIES ΣF.

FOR SUBMERGED FOILS, DIFFERENT APPROACH IS NEEDED TO COMPUTE ΣF, – NO WAVE FORCES – LIFTING FLOW } AFS CFD CONTRIBUTION.

TIME DOMAIN ANALYSIS OF FLOATING BODY MOTIONS



$$R(t) = \begin{bmatrix} p_x \\ p_y \\ p_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} \qquad \underset{\sim}{V}(t) = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \dot{v}_{\alpha_x} \\ \dot{v}_{\alpha_y} \\ \dot{v}_{\alpha_z} \end{bmatrix}$$

EQNS OF MOTION

$$\underset{\sim}{M}\, \ddot{\underset{\sim}{R}}(t) = \sum \underset{\sim}{F}$$

$$\begin{bmatrix} m & 0 & 0 & & & \\ 0 & m & 0 & & 0 & \\ 0 & 0 & m & & & \\ & & & I_{11} & I_{12} & I_{13} \\ & 0 & & & I & I_{22} \\ & & & & & I_{33} \end{bmatrix} \begin{bmatrix} \ddot{p}_x \\ \ddot{p}_y \\ \ddot{p}_z \\ \ddot{\alpha}_x \\ \ddot{\alpha}_y \\ \ddot{\alpha}_z \end{bmatrix} = \begin{bmatrix} \sum F_x \\ \sum F_y \\ \sum F_z \\ \sum M_x \\ \sum M_y \\ \sum M_z \end{bmatrix} \Rightarrow \dot{\underset{\sim}{x}} = \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \\ \dot{\alpha}_x \\ \dot{\alpha}_y \\ \dot{\alpha}_z \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \\ \dot{v}_{\alpha x} \\ \dot{v}_{\alpha y} \\ \dot{v}_{\alpha z} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} I \end{bmatrix} \\ \begin{bmatrix} M^{-1} \end{bmatrix} \end{bmatrix} \begin{bmatrix} x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ \sum F_x \\ \sum F_y \\ \sum F_z \\ \sum M_x \\ \sum M_y \\ \sum M_z \end{bmatrix}$$

$$(1) \qquad \dot{x} = f(\underset{\sim}{x}, t)$$

} SYSTEM OF 1^{st} OR ODE'S TO BE SOLVED NUMERICALLY.
[EULER METHOD
R-K METHOD,
ETC]
REQUIRES INITIAL CONDITION.

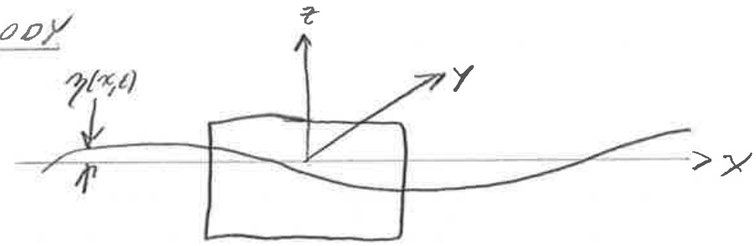
MULTIPLE BODIES (i.e. BUOY + FOIL)

$$\begin{bmatrix} \begin{bmatrix} \dot{p}_x^B \\ \dot{p}_y^B \\ \vdots \\ \dot{v}_x^B \\ \vdots \end{bmatrix} \\ \begin{bmatrix} \dot{p}_x^F \\ \vdots \\ \dot{v}_y^F \\ \vdots \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} I \end{bmatrix} \\ \begin{bmatrix} M_B^{-1} \end{bmatrix} \end{bmatrix} \begin{bmatrix} x_6^B \\ \vdots \\ \sum F_y^B \\ \vdots \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} I \end{bmatrix} \\ \begin{bmatrix} M_F^{-1} \end{bmatrix} \end{bmatrix} \begin{bmatrix} x_6^F \\ \vdots \\ \sum F_y^F \\ \vdots \end{bmatrix} \end{bmatrix}$$

WITH MORE THAN ONE BODY, THESE EXPRESSIONS MAY DEPEND ON THE POS, VEL, ACCEL OF MORE THAN ONE OF THE BODIES. OTHERWISE,

$$\dot{\underset{\sim}{x}} = f(\underset{\sim}{x}, t) \quad \text{STILL.}$$

$\sum F$ FOR SINGLE FLOATING BODY



FULLY LINEARIZED THEORY

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$$= \rho g \underbrace{\forall}_{\text{EQUILIBRIUM DISPLACEMENT}} + \rho g \underbrace{S}_{\text{WATERPLANE ARE}} p_z \qquad \left[\underbrace{p_z}_{} = f(\underset{\sim}{x}, t) \right]$$

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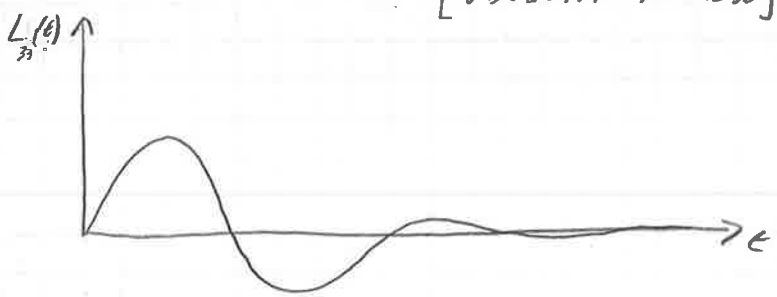

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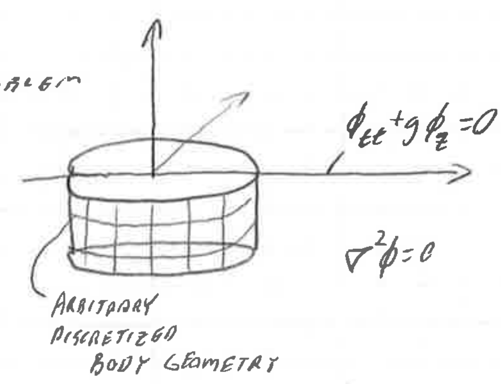

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