DIMENSIONAL ANALYSIS OF SUBMERGED PLATE TANK TESTING

$\bar{\rho}_w\ [kg/m^3]$ $\quad \bar{F}\left[kg\frac{m}{s^2}\right]$ $\quad \bar{\theta}_L[I]$

$\bar{\rho}_p\ [kg/m^3]$ $\quad \bar{g}\ [m/s^2]$ $\quad \bar{\theta}_R[I]$

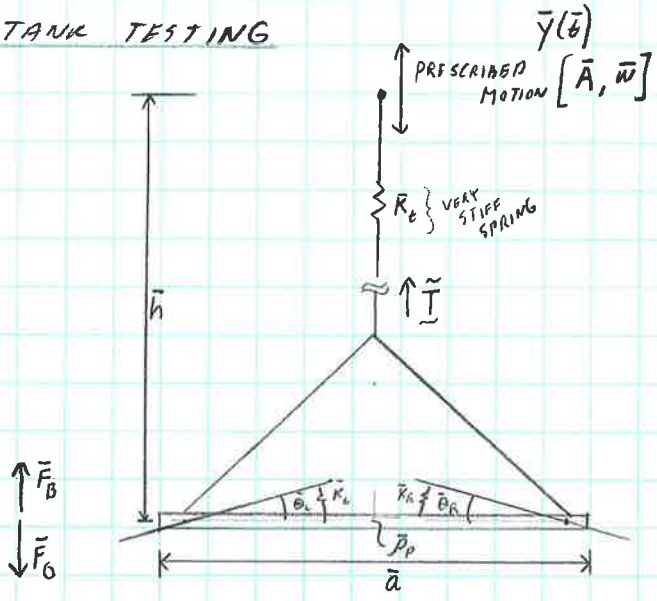
$\bar{a}[m]$ $\quad \bar{K}_t\left[\frac{kg}{s^2}\right]$ ie $[F/m]$

$\bar{t}[s]$ $\quad \bar{x}[m]$ $\quad \bar{K}_R\left[\frac{N\cdot m}{rad}\right]\left[\frac{kg\,m^2}{s^2}\right]$

$\bar{\alpha}[I]$ $\quad K_L\left[\frac{kg\,m^2}{s^2}\right]$

$\bar{u} = \frac{d\bar{x}}{d\bar{t}}\ [m/s]$ $\quad \bar{A}[m]$ $\quad \bar{\nabla}[m^3]$

$\dot{\bar{u}} = \frac{d\bar{u}}{d\bar{t}}\ [m/s^2]$ $\quad \bar{w}[1/s]$



$\bar{x}, \bar{a}$ PLATE POSITION (VECTOR) $\quad \bar{\rho}_w, \bar{\nu},$

PRINCIPAL VARIABLES $(\bar{a}, \bar{g}, \bar{\rho}_w)$

$[L] = \bar{a} \quad [T] = \sqrt{\bar{a}/\bar{g}} \quad [M] = \bar{\rho}_w \bar{a}^3$ $\qquad g = \bar{g}/\bar{a}\,\frac{\bar{a}}{\bar{g}} = 1 \qquad a = \bar{a}/\bar{a} = 1 \qquad \rho_w = \frac{\bar{\rho}_w}{\bar{\rho}_w \bar{a}^3/\bar{a}^3} = 1$

$\nu = \bar{\nu}/\bar{a}^2\sqrt{\bar{g}/\bar{a}} = \bar{\nu}/\sqrt{\bar{a}^3\bar{g}}$

$u = \bar{u}/\bar{a}\sqrt{\bar{g}/\bar{a}} = \bar{u}/\sqrt{\bar{a}\bar{g}}$ } FROUDE # $= F_R$

} $\Rightarrow Re = \frac{u}{\nu} = \frac{\bar{u}}{\sqrt{\bar{a}\bar{g}}} \cdot \frac{\sqrt{\bar{a}^3\bar{g}}}{\bar{\nu}} = \frac{\bar{u}\,\bar{a}}{\bar{\nu}}$

$t = \bar{t}/\sqrt{\bar{a}/\bar{g}}$

$F_G = \bar{F}_G/\bar{\rho}_w\bar{a}^3\bar{g} \qquad F_B = \bar{F}_B/\bar{\rho}_w\bar{a}^3\bar{g} \qquad T = \bar{T}/\bar{\rho}_w\bar{a}^3\bar{g}$

$F_D = \bar{F}_D/\bar{\rho}_w\bar{a}^3\bar{g} \Rightarrow C_D = \frac{F_D}{F_R^2} = \bar{F}_D/\bar{\rho}_w\bar{u}^2\bar{a}^2$ } DRAG COEFF IS NONDIM VISCOUS DRAG DIVIDED BY F_R^2.

$\underset{\sim}{x} = \underset{\sim}{\bar{x}}/\bar{a} \qquad \alpha = \bar{\alpha} \qquad \theta_R = \bar{\theta}_R \qquad \theta_L = \bar{\theta}_L$

$A = \bar{A}/\bar{a} \qquad w = \bar{w}\sqrt{\frac{\bar{a}}{\bar{g}}}$

$K_t = \bar{K}_t/\rho_w\bar{a}^3/\bar{a}/\bar{g} = \bar{K}_t/\bar{\rho}_w\bar{g}\bar{a}^2 \qquad K_R = \bar{K}_R/\frac{\bar{\rho}_w\bar{a}^3\cdot\bar{a}^2}{\bar{a}/\bar{g}} = \bar{K}_R/\bar{\rho}_w\bar{g}\bar{a}^4$

$\dot{u} = \dot{\bar{u}}/\bar{a}/\bar{a}/\bar{g} = \dot{\bar{u}}/\bar{g}$

$\rho_p = \bar{\rho}_p/\bar{\rho}_w \qquad \nabla = \bar{\nabla}/\bar{a}^3$

NON-DIMENSIONALIZATION OF EQNS OF MOTION

SINGLE RIGID BODY (NO FLAPS)

$\bar{T}$ is a function of body position, spring constant, and PRESCRIBED POINT POSITION.

$$\sum \bar{F} = (\bar{m} + \bar{\nu}) \ddot{\bar{x}} = \bar{F}_G + \bar{F}_B + \bar{F}_D(\dot{\bar{x}}) + \bar{T}(\bar{x}, \bar{y}, \bar{K}_t)$$

($\bar{\nu} = \bar{\rho}_p \bar{\nabla}$)

$$(\rho_p \bar{\rho}_w \nabla \bar{a}^3 + \nu \bar{\rho}_w \bar{a}^3)\, \dot{x}' \bar{g} = F_G \bar{\rho}_w \bar{a}^3 \bar{g} + F_B \rho_w \bar{o}^3 \bar{g} + F_D \bar{\rho}_w \bar{o}^3 \bar{g} + \bar{\rho}_w \bar{a}^3 \bar{g}\, T(x\bar{a}, y\bar{a}, K_t \bar{\rho}_w \bar{g} \bar{a}^2)$$

$$(\rho_p \nabla + \nu) \ddot{x} = F_G + F_B + F_r^2 C_D(Re) + T(x, y, K_t)$$

EQNS OF MOTION INVARIANT IF

$F_R = 1$

$C_D(Re) = \text{CONST.}$

SELECTION OF MODEL SCALE SPRING CONSTANTS

→ FOR THE EQUATIONS OF MOTION TO BE THE SAME REGARDLESS OF SCALE

LINEAR SPRINGS:

$$\frac{\bar{F}_t}{\bar{\rho}_w \bar{g} \bar{a}^2} = \frac{k_t'}{\rho_w' g' a'^2} \Rightarrow k_t' = \bar{K}_t \left(\frac{a'}{\bar{a}}\right)^2$$

SPRING FORCE/METER SCALES AS SCALE RATIO SQUARED.

↳ LINEAR SPRING IS 25 TIMES WEAKER @ $s = \frac{1}{5}$

TORSION SPRINGS:

$$\frac{\bar{K}_R}{\bar{\rho}_w \bar{g} \bar{a}^4} = \frac{K_R'}{\rho_w' g' a'^4} \Rightarrow K_R' = \bar{K}_R \left(\frac{a'}{\bar{a}}\right)^4$$

TORSION SPRING STIFFNESS $\left[\frac{N \cdot m}{rad}\right]$ SCALES AS SCALE RATIO TO THE POWER 4!

$s = \frac{1}{5}$ → TORSION SPRING IS 625 TIMES WEAKER!