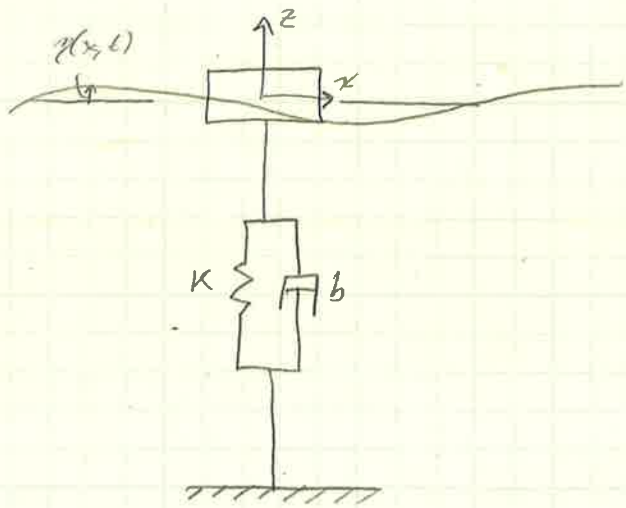
FREQ DOMAIN HEAVE RESPONSE OF TETHERED FLOATING BODY



ξ_3 = COMPLEX HEAVE AMPLITUDE

BUOY POSITION : $z(t) = x_3(t) = Re[\xi_3 e^{i\omega t}]$
VELOCITY : $\dot{z}(t) = \dot{x}_3(t) = Re[i\omega \xi_3 e^{i\omega t}]$
ACCEL : $\ddot{z}(t) = \ddot{x}_3(t) = Re[-\omega^2 \xi_3 e^{i\omega t}]$

<u>FORCES ACTING ON BODY:</u>

WEIGHT: $F_3^W(t) = -mg$

HYDROSTATIC: $F_3^{HS}(t) = \rho \forall g - \rho g S\, Re[\xi_3 e^{i\omega t}]$

SPRING: $F_3^S(t) = -K z(t) = -K\, Re[\xi_3 e^{i\omega t}]$

DAMPER : $F_3^D(t) = -b\, \dot{z}(t) = -b\, Re[i\omega \xi_3 e^{i\omega t}]$

RADIATION:

$F_3^R(t) = Re[f_{33} \xi_3 e^{i\omega t}]$ $\quad f_{33} = \omega^2 a_{33} - i\omega b_{33}$ } ASSUMES NO CROSS COUPLING BETWEEN MODES

$a_{33} = a_{33}(\omega)$ = ADDED MASS
$b_{33} = b_{33}(\omega)$ = DAMPING } REAL

$= Re\left[\xi_3 [\omega^2 a_{33} - i\omega b_{33}] e^{i\omega t}\right]$

EXCITING:

$F_3^E(t) = Re[A X_3 e^{i\omega t}]$ $\quad X_3 = X_3(\omega)$ = COMPLEX WAVE EXCITING FORCE.

$$m\ddot{x}_{33} = \sum F_3$$

$$Re\left[-m\omega^2 \xi_3 e^{i\omega t}\right] = Re\Big[\underbrace{-mg + \rho \forall g}_{\text{CANCEL FOR FLOATING BODY}} - \rho g S \xi_3 e^{i\omega t} - k\xi_3 e^{i\omega t} - i\omega b \xi_3 e^{i\omega t}$$

$$+\xi_3\left[\omega^2 a_{33} - i\omega b_3\right]e^{i\omega t} + A X_3 e^{i\omega t}\Big]$$

$$Re\left[A X_3 e^{i\omega t}\right] = Re\left[e^{i\omega t}\left(-m\omega^2\xi_3 + \rho g S\xi_3 + k\xi_3 + i\omega b\xi_3 - \xi_3(\omega^2 a_{33} - i\omega b_{33})\right)\right]$$

$$A X_3 = \xi_3\left[-\omega^2(m + a_{33}) + i\omega(b + b_{33}) + (\rho g S + k)\right]$$

$$\xi_3 = \frac{A X_3}{-\omega^2(m + a_{33}) + i\omega(b + b_{33}) + (\rho g S + k)}$$

POWER DISSIPATED IN DAMPER

$$\bar{P} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{b}{T}\int_0^T |V(t)|^2\,dt = \frac{b}{T}\int_0^T \left\{Re\left[i\omega\xi_{3,0}e^{i\omega t}\right]\right\}^2 dt$$

$$= \frac{b}{T}\int_0^T \left[-\omega\left(Re(\xi_3)\sin\omega t + Im(\xi_3)\cos\omega t\right)\right]^2 dt =$$

$$= \frac{b\omega^2}{T}\int_0^T \left[Re(\xi_3)^2\sin^2\omega^2 + Im(\xi_3)^2\cos^2\omega t + 2Re(\xi_3)Im(\xi_3)\cos\omega t\sin\omega t\right]dt$$

$$= \frac{b\omega^2}{T}\left[\frac{T}{2}Re(\xi_3)^2 + \frac{T}{2}Im(\xi_3)^2\right] = \frac{b\omega^2}{2}\left[Re(\xi_3)^2 + Im(\xi_3)^2\right] = \bar{P}$$